HW Solutions 4 - 8.01 MIT - Prof. Kowalski

Newton's third law, forces and motion with pulleys.

`Note: All the figures are at the END!`

1) **5.13**

The natural choice of axis is x and y which is along the incline and perpendicular to the incline respectively because there is no acceleration and two forces , $\overrightarrow{\mathbf{T}}$ and $\overrightarrow{\mathbf{N}}$, out of 3 forces present are along these axes. Writing $\sum \overrightarrow{\mathbf{F}} = 0$ for both masses:
a) $\sum F_x$ acting on B is 0 $\Rightarrow$ $T_{AB} - w \sin\alpha = 0 \Rightarrow T_{AB} = w \sin\alpha$.

b) $\sum F_x$ acting on A is 0 $\Rightarrow$ $T_A - T_{AB} - w\sin\alpha = 0 \Rightarrow T_A = w\sin\alpha + w\sin\alpha = 2w\sin\alpha$.

c) $\sum F_y = 0$ for both A and B. $\Rightarrow$ $N_A - w\cos\alpha = 0$ and $N_B - w\cos\alpha = 0$ $\Rightarrow N_A = N_B = w\cos\alpha$.

2) **5.14**

a) In level flight, the thrust and drag are horizontal and the lift and weight are vertical. At constant speed, the net force is zero:

$$\sum F_x = F - f = 0 \tag{1}$$

$$\sum F_y = L - w = 0 \tag{2}$$

so F=f and w=L.

b) When the plane attains the new speed, it is again in equilibrium and so the new values of the thrust and drag, F' and f' are related by $F' = f'$; if $F' = 2F$, $f' = 2f$.

c) The drag force f $\propto v^2$ $\Rightarrow \frac{f'}{f} = \frac{v'^2}{v^2} \Rightarrow v' = v\sqrt{\frac{f'}{f}}$ So in order to increase the the magnitude of the drag force by a factor of 2, the speed must increase by a factor of $\sqrt{2}$.

3) **5.15**

a)
The tension is related to the masses and acceleration(it's best to take y up for both blocks) by

$$T - m_1 g = m_1 a_1 \tag{3}$$

$$T - m_2 g = m_2 a_2 \tag{4}$$

b)For the bricks, mass m_1, accelerating upward, let $a_1 = -a_2 = a$(the counterweight will accelerate down at the same rate as m_1 goes up since the string does not stretch). Subtracting the two equations to eliminate T gives

$$(m_2 - m_1)g = (m_1 + m_2)a \tag{5}$$

$$a = g\frac{m_2 - m_1}{m_1 + m_2} = 9.80(\frac{28.0 - 15.0}{15.0 + 28.0}) = 2.96 \text{ m/s}^2. \tag{6}$$

c) The result of part **b** may be substituted into either of the above expressions to find the tension T=191 N. As an alternative the expressions may be manipulated to eliminate a algebraically by multiplying the equation (1) by m_2 and the equation (2) by m_1 and adding (with $a_2 = -a_1$) to give

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 191 \text{ N} \tag{7}$$

In terms of the weights, the tension is

$$T = w_1\frac{2m_2}{m_1 + m2} = w_2\frac{2m_1}{m_1 + m2} \tag{8}$$

If, as in this case, $m_2 > m_1$, $2m_2 > m_1 + m_2$ and $2m_1 < m_1 + m_2$. So the tension is greater than w_1 and less than w_2; this must be the case, since the load of bricks rises and the counter weight drops.

4) **5.64**

$210\mu g = 210 \times 10^{-6} \times 10^{-3}\ kg$
mg=(210 $\times 10^{-6} \times 10^{-3}) \times 10 = 2.1\ \mu$N.

a)

$$a_{initial} \approx 62g. \tag{9}$$

$$(\sum F_y)_{initial} = ma_{initial} \approx 62\ mg \approx 130\mu N. \tag{10}$$

The net force in y direction, $\sum F_y$, is just ma or mg$\times(\frac{a}{g})$. So

$$\frac{\sum F_y}{mg} = \frac{a}{g} \tag{11}$$

$$\frac{(\sum F_y)_{initial}}{mg} = (\frac{a}{g})_{initial} \approx 62 \tag{12}$$

So the net force is 62 times its weight.

b)We should find the maximum $\frac{a}{g}$ from the graph which is around 140. Similar to above calculations we get

$$(\sum F_y)_{max} \approx 294\mu N. \tag{13}$$

You can read the time that the function takes to reach its maximum - which is around 140 - from the graph: $T_{max} \approx 1.2$ ms

c)The acceleration is always positive so the flea adds to its speed and the maximum speed is just the total area under the a(t) curve or the total area A under the a/g v.s t curve times g. $v_{max} = A \times g$. The area of each square is 0.25×25 ms which times g (10 m/s^2) gives approximately 60 mm/s. The next step is to count the number of these squares which is around 21. So the maximum speed would be 21×60 which gives 1260 mm/s, so $v_{max} \approx 1$ m/s.

d) The flea will accelerate at $a_y = -g$ for a time $t_H = \frac{v_{max}}{g}$ which is $\approx$ 120 ms. To accuracy 3 percent we can neglect the displacement during the 1ms acceleration period. Using

$$v_f^2 - v_i^2 = 2(-g)H \tag{14}$$

$$v_f = 0 \tag{15}$$

$$v_i = v_{max} \tag{16}$$

we get:

$$H = \frac{v_{max}^2}{2g} \approx \frac{1\ m^2/s^2}{2 \times 10\ m/s^2} = 1/20\ m. \tag{17}$$

$$H \approx 5\ cm. \tag{18}$$

5) **5.119**

Referring to the figure of free body diagram:

$$\sum F_y = 0 \Rightarrow f_s \cos\beta + N \sin\beta - mg = 0 \tag{19}$$

$$\sum F_x = \frac{mv^2}{R} \Rightarrow N\cos\beta - f_s \sin\beta = \frac{mv^2}{R} \tag{20}$$

where

$$R = h \tan\beta. \tag{21}$$

the cases of **extreme** f_s gives rise to the v_{max} and v_{min}:

$\mathbf{v_{min}}(\mathbf{T_{max}})$:

The friction will oppose the motion of the block inward and down relative to the cone so (from the free body diagram attached):

$$\mu_s N \cos\beta + N \sin\beta - mg = 0 \tag{22}$$

$$N\cos\beta - N\mu_s \sin\beta = \frac{mv_{min}^2}{R} \tag{23}$$

By eliminating N we get

$$v_{min} = \sqrt{gh \tan\beta \frac{\cos\beta - \mu_s \sin\beta}{\sin\beta + \mu_s \cos\beta}} \tag{24}$$

$\mathbf{v_{max}}(\mathbf{T_{min}})$:

The friction will oppose the motion of the block outward and up relative to the cone. The free body diagram in this case gives us:

$$-mg - N\mu_s \cos\beta + N \sin\beta = 0 \tag{25}$$

$$N\cos\beta + N\mu_s \sin\beta = \frac{mv_{max}^2}{R} \tag{26}$$

Eliminating N gives

$$v_{max} = \sqrt{gh \tan\beta \frac{\cos\beta + \mu_s \sin\beta}{\sin\beta - \mu_s \cos\beta}} \tag{27}$$

$$v = \frac{2\pi R}{T} \quad or \quad T = \frac{2\pi R}{v} \tag{28}$$

so

$$T_{max} = 2\pi \sqrt{\frac{h\tan\beta}{g} \frac{\cos\beta + \mu_s \sin\beta}{\sin\beta - \mu_s \cos\beta}} \tag{29}$$

$$T_{min} = 2\pi \sqrt{\frac{h\tan\beta}{g} \frac{\cos\beta - \mu_s \sin\beta}{\sin\beta + \mu_s \cos\beta}} \tag{30}$$

6) **5.126**

The forces acting on the pulley is F upward and 2T downward and because pulley is massless the $\sum \overrightarrow{\mathbf{F}}$ acting on it should be zero so

$$T = \frac{F}{2}. \tag{31}$$

It's crucial to understand that once you know F, T will be determined from the above equation. This simplifies the problem into two independent F=ma problem for each block i.e. the motion of each block is unaffected by the motion of the other block.

$w_A = 200$ N $\quad w_B = 100$ N.

a) $T = \frac{F}{2} = 62$ N.
T is less than both w_A and w_B. A and B will stand on the floor and the Normal forces N_A and N_B will compensate the difference between w's and the T.

b) $T = \frac{F}{2} = 147$N which is greater than w_B and smaller than w_A. We know that the normal force can't be negative so B will accelerate upward (the normal force becomes Zero) with acceleration $a_B = \frac{T-w_B}{m_B} = 4.7m/s^2$ and for the same reason as part **a**; a_A=0.

c) $T = \frac{F}{2} = 212$N which is greater than both w_B and w_A. So both will accelerate upward with accelerations $a_B = \frac{T-w_B}{m_B} = 11.2m/s^2$ and $a_A = \frac{T-w_A}{m_A} = 0.6m/s^2$.

7) **Three Masses and Two Pulleys**

In scientific research it happens a lot that you will be given an answer ,say, from a computer simulation and you need to know if that answer makes sense or not. One typical way of doing it is to check the answer in special cases which we know the answer.

$\mathbf{M_1 = 0}$:
The right pulley will have a free fall. So the $a_1 = +g$ so **c**, **e** and **f** are **out**.

$\mathbf{M_2 = 0}$:
M_3 will have a free fall but $T_{LL} = T_{LR} = 0 \Rightarrow a_1 = -g$. There is an equivalent situation for $\mathbf{M_3 = 0}$. So **g** and **b** are **out**.

The **only option** is **d**.

For another check; it should balance (with a=0) if

$\mathbf{M_2 = M_3 = \frac{M_1}{2} = M}$:
$T_{LL} = T_{LR} = \frac{T_L}{2} = Mg \Rightarrow a_1 = 0$ which is **consistent** with **d** $\surd$.

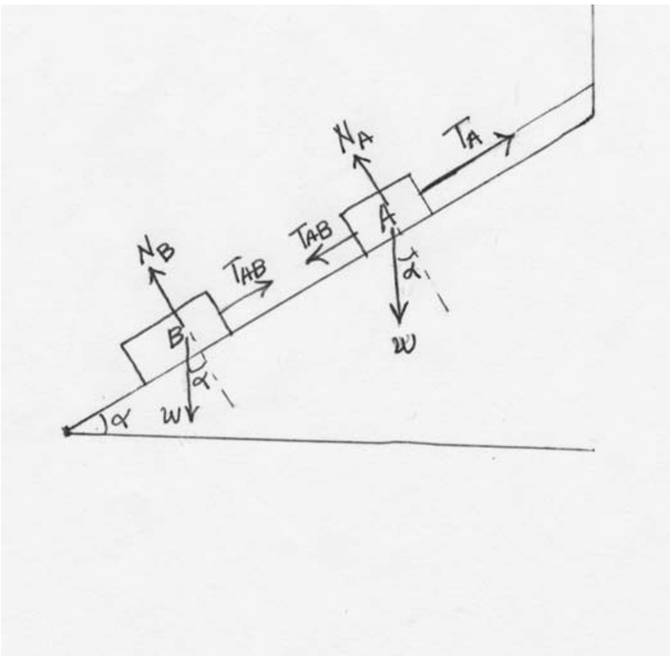

Figure 1: **5.13**

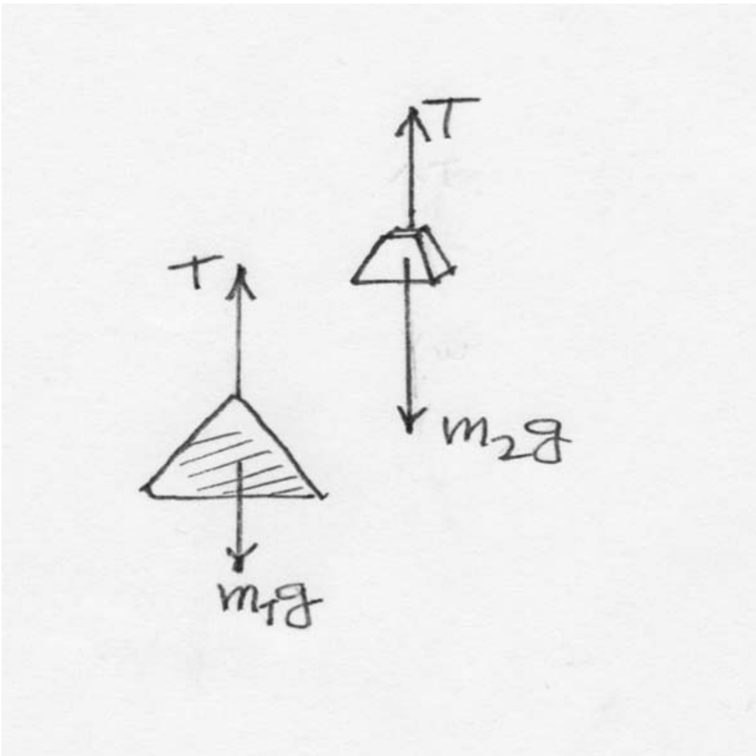

Figure 2: **5.15**

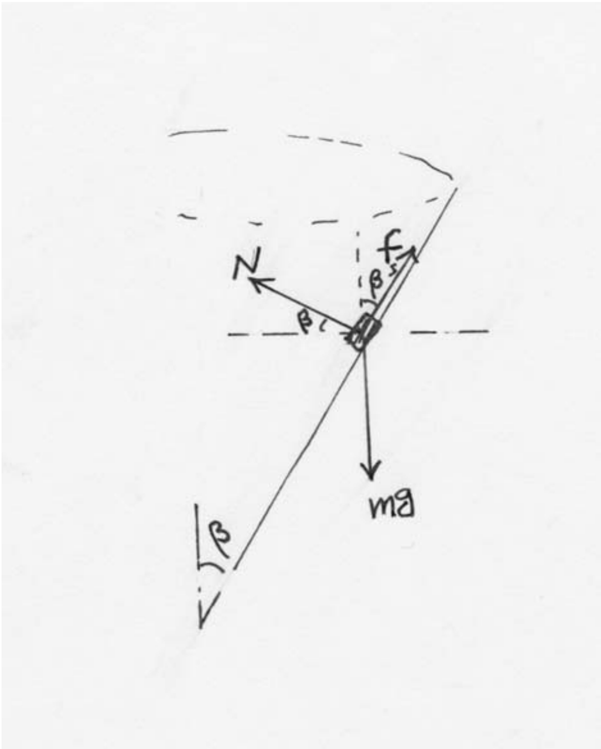

Figure 3: **5.119**