

Harmonic Oscillators and Relative Motion.

1) Vibration Isolation System

a) Let's denote the table displacement from equilibrium with \mathcal{Y} . The Newton's second law $\sum f_y = ma_y$ for the box will read:

$$M \frac{d^2 \mathcal{Y}}{dt^2} = k(y - \mathcal{Y}) \quad (1)$$

Where y denotes the displacement of ceiling from the situation that table is in equilibrium. The form of y is *given*:

$$y = A \cos \omega t$$

Replace this in (1) you'll get:

$$M \frac{d^2 \mathcal{Y}}{dt^2} = -k \mathcal{Y} + kA \cos \omega t$$

Which is equivalent to an additional force - other than the spring $-kA \cos \omega t$ acting on the table.

b) Substitute $\mathcal{Y} = C(\omega) \cos \omega t$ in (1) and divide two sides by M:

$$-\omega^2 C(\omega) \cos \omega t = \omega_0^2 (A - C(\omega)) \cos \omega t$$

Where I used the fact that $\frac{k}{M} = \omega_0^2$.

This gives:

$$C(\omega) = \frac{A}{1 - \frac{\omega^2}{\omega_0^2}}$$

c) Set $\omega_0 \sim 2\pi$ Hz and $\omega = 15 \times 2\pi$ Hz

$$\frac{C(\omega)}{A} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2}} \sim \frac{1}{1 - 15^2} \sim -0.005$$

So the amplitude is *reduced* by a factor ~ 200 .

2) **13.88** A Rod's Oscillation via Spring

Please refer to the figure 13.88. There is a horizontal force which acts at the end of the metal rod which points to the *left* and it comes from the stretched spring. The magnitude of this force for small θ is $k(l/2)\theta$.

The second step is to set up the torque equation $\sum \tau_z = I \frac{d^2\theta}{dt^2}$. The I around the center of mass is $I_{cm} = \frac{1}{12}Ml^2$

$$\sum \tau_z = -\left(\frac{l}{2}\theta k\right)\frac{l}{2}$$
$$\therefore \frac{1}{12}Ml^2 \frac{d^2\theta}{dt^2} = -k\frac{l^2}{4}\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{3k}{M}\theta}$$

So the rod will oscillate with the $\omega^2 = \frac{3k}{M}$:

$$\boxed{T = 2\pi\sqrt{\frac{M}{3k}}}$$

3) **37.1** Simultaneity

Please refer to the figure (37.4).

The person is in the middle of sparks, but moves to the right. If the sparks were simultaneous to the ground observer she would encounter the light from the right hand end sooner than the light from the left. She sees them simultaneously - therefore the left side sparked first if viewed from the ground's perspective.

You can do it *alternatively* using **Lorentz transformation** :

Use the Lorentz coordinate transformation derived from relativity principles:

$$t' = \gamma(t - ux/c^2)$$

Where $\gamma = \frac{1}{1-u^2/c^2}$.

This transformation is *linear* so you can easily write:

$$t'_B - t'_A = \gamma[(t_B - t_A) - u(x_B - x_A)/c^2] \quad (2)$$

The *condition* given is that events A and B is simultaneous in the " ' " frame so:

$$\text{given condition : } t'_B - t'_A = 0$$

Combine this condition with (2) you'll get:

$$t_B - t_A = u(x_B - x_A)/c^2$$

Here the $x_B - x_A > 0$ (not \geq !) so $t_B - t_A > 0$ which means that first the events in the rest frame is NOT simultaneous and second events A (which is *at* the left end of the train) will occur *sooner*.

NOTE: If you want to be more rigorous you can use the Lorentz transformation in

$$x = \gamma(x' + ut')$$

and consequently

$$x_B - x_A = \gamma[(x'_B - x'_A) + u(t'_B - t'_A)]$$

Use $t'_B - t'_A = 0$ you'll get:

$$x_B - x_A = \gamma(x'_B - x'_A) = \gamma L$$

Where L is the *length* of the train as seen by a person in the train.
So the rigorous result is:

$$\boxed{t_B - t_A = \frac{u\gamma L}{c^2}}$$

Note as u increases γ increases and so does $t_B - t_A$ so the events become more and more non-simultaneous.

4) Michelson-Morley Experiment with ether.

Please refer to the figure (2) attached to this file.

Denote " ' " for the lab variables when it moves with velocity v .

From the figure (2) and using hypotenuse theorem we have:

$$t'_y = \frac{2\sqrt{L^2 + (L/c)^2 v^2}}{c} = \frac{2L}{c} \sqrt{1 + v^2/c^2} \simeq \frac{2L}{c} \left(1 + \frac{v^2}{2c^2}\right)$$

Subtract this from $t_y = \frac{2L}{c}$ you'll get:

$$\Delta t_y = \frac{2L}{c} \left(1 + \frac{v^2}{2c^2} - \frac{2L}{c}\right)$$

$$\boxed{\Delta t_y = \frac{Lv^2}{c^3}}$$

The situation in x axis is simpler (use the relative velocity of light with respect to apparatus which is $c \pm v$):

$$t'_x = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2Lc}{c^2 - v^2} = \frac{2Lc/c^2}{1 - v^2/c^2} \simeq 2L/c(1 + v^2/c^2)$$

Subtract this from $t_x = \frac{2L}{c}$ you'll get:

$$\boxed{\Delta t_x = \frac{2Lv^2}{c^3}}$$

So we have:

$$\delta t = \Delta t_x - \Delta t_y = \frac{Lv^2}{c^3}$$

$$\delta L = \delta tc = Lv^2/c^2$$

From optics we know that the fringe shift $\Phi = \delta L/\lambda$ where λ is the spacing of fringes in the experiment, $\lambda = 0.25\mu$

The earth's velocity v in it's orbit is about 30 km/s:

$$\Phi = \frac{11}{0.25 \times 10^{-6}} \left(\frac{30 \times 10^3}{3 \times 10^8} \right)^2 = 0.44$$

Observed fringe shift is $2 \times$ this., since $x \leftrightarrow y$ applies the shift one way and then the other.

$$\boxed{\Phi_{observed} = 0.88}$$

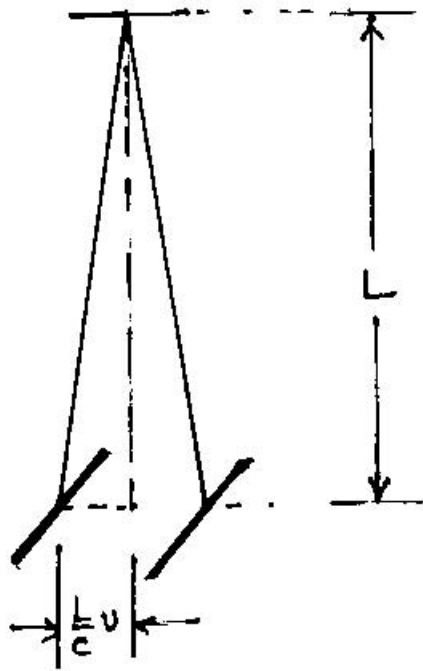


Figure 1: Michelson-Morley