

Universal Gravity and Harmonic Oscillators.

1) Whither the Moon?

a) TIDES: The water (and to some extent the liquid core) of the earth bulge out toward and away from the moon to the fact that its gravitational attraction is stronger on the side of the earth toward the moon than on the center of the mass of the earth itself, so the water is pulled up relative the earth. On the far side of the earth (away from the moon) the moon's gravitational force is weaker and this water is left behind this accelerating earth, producing another tidal bulge. Therefore there are two high tides per day (in most places on the earth).

The height of the tide is scaled by $\sim \frac{a_{tidal}}{g}$.

$$a_{tidal} = \Delta g_{moon} = GM_m \left(\frac{1}{(D_{em} + R_e)^2} - \frac{1}{D_{em}^2} \right) \approx -\frac{2GM_m R_e}{D_{em}^3} \approx 10^{-7} g$$

Where $D_{em} = 3.84 \times 10^8$ m.

This produces tides about 1 m high (Since $R_e = 6.378 \times 10^6$ m) in mid-ocean but it can be magnified by over an order of magnitude by resonances with bays like the Irish Sea and the Bay of Fundy.

There are no torques on the earth moon system, so its total angular momentum is conserved:

As the earth rotates underneath the tidal bulges, energy is lost. The angular momentum of the earth decreases as well. Moreover, the tides are carried along by the earth rotation. This means that the bulge closer to the moon pulls the moon ahead in its orbit, increasing the angular momentum of the moon.

The initial angular momentum is:

$$L_i = I_e \omega_e + M_m D_{em}^2 \omega_m = M_m D_{em}^2 \omega_m \left(1 + \frac{I_e \omega_e}{M_m D_{em}^2 \omega_m}\right) = 1.203 M_m D_{em}^2 \omega_m$$

Note that the spin angular momentum of the earth is much smaller than that of the moon in its orbit.

b) FINAL CONFIGURATION:

The tidal dissipation will slow the earth until one of two things happens:

- 1) The moon is driven away from the still spinning earth.
- 2) The earth slows until one side faces the moon all the time. At this point the length of the day and the length of the lunar month will be the same.

The actual outcome is # 2 due to the much smaller angular momentum of the earth's spin relative to the moon's orbital angular momentum. The final angular momentum is easy to find as the earth-moon system all rotates at ω_f and so one simply needs the combined moment of inertia:

$$L_f = (I_e + M_m D_f^2) \omega_f$$

In fact one can approximate the final angular momentum by neglecting the final angular momentum of the earth to first approximation:

$$L_f \approx M_m D_f^2 \omega_f$$

From:

$$L_f = L_i \quad (\text{no torque on earth-moon system})$$

We get:

$$M_m D_f^2 \omega_f = 1.203 M_m D_{em}^2 \omega_m$$

From *Kepler's* law ♣ we know that:

$$\omega = \text{const. } D^{-\frac{3}{2}}$$

$$\therefore \text{const. } D_f^2 D_f^{-\frac{3}{2}} = \text{const. } D_{em}^2 D_{em}^{-\frac{3}{2}} \times 1.203 \Rightarrow \boxed{D_f = (1.203)^2 D_{em} = 1.45 D_{em}}$$

$$T_{final} = \left(\frac{D_f}{D_{em}}\right)^{\frac{3}{2}} T_{orbit} = 1.74 \times 27.3 \text{ days} = 47.5 \text{ days}$$

♣:

Kepler: $T \sim a^{\frac{3}{2}}$ ($a \equiv$ semi major axis $\sim D$)

$$\omega \sim \frac{1}{T} \sim D^{-\frac{3}{2}} \quad D \equiv (\text{distance to moon})$$

$$\text{Scaling via Kepler's law} \Rightarrow T_f = \left(\frac{D_f}{d}\right)^{\frac{3}{2}} T_{orbit}$$

2) **13.58** Oscillating Car and Passengers.

Denote the mass of the passengers by m and the (unknown) mass of the car by M . The spring constant from the Newton's second law for the passengers at *equilibrium* ($\sum F_y = k\Delta l - mg = 0$) is:

$$k = \frac{mg}{\Delta l} \quad (1)$$

The period of the empty car (Equation 13.12 for simple harmonic oscillator) is:

$$T_E = 2\pi\sqrt{\frac{M}{k}} \quad (2)$$

The loaded car has the mass $M + m$. So:

$$T_L = 2\pi\sqrt{\frac{M + m}{k}} \quad (3)$$

Write (3) in terms of T_E and Δl by (1) and (2):

$$T_L = \sqrt{T_E^2 + (2\pi)^2 \frac{\Delta l}{g}} \quad (4)$$

Solving (4) for T_E :

$$T_E = \sqrt{T_L^2 - (2\pi)^2 \frac{\Delta l}{g}} \quad (5)$$

Plugging in the numbers given in the problem ($T_L=1.08$ s; $\Delta l=4.00$ cm) you'll get:

$$\boxed{T_E = 1.003 \text{ s}}$$

3) **13.60** Spring vs. Pendulum Period.

The general relation for the period of spring oscillations is:

$$T_S = 2\pi\sqrt{\frac{m}{k}} \quad (6)$$

With the information given in this problem you can find the period of the vertical mass-spring system in terms of L and g. The mass has been moved down slowly so the final *stretched* length is also the *equilibrium* length which is L:

$$0 = ma_y = \sum F_y = kL - mg \Rightarrow \frac{m}{k} = \frac{L}{g}$$

Substitute this result into equation (6) you'll get:

$$T_S = 2\pi\sqrt{\frac{L}{g}}$$

which is the same as

$$T_P = 2\pi\sqrt{\frac{L}{g}}$$

the period of a pendulum with length L (from Eq. 13.34).