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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 29

You see here the topics that will be on your plate on Monday.

It's clearly not possible for me in one exam to cover all these topics, so I will have to make a choice Monday.

Today I also have to make a choice.

I cannot do justice to all these topics in any depth, so I will butterfly over them and some of them I won't even touch at all.

However, what is not covered today may well be on the exam.

You will get loads of equations.

Almost every equation that I could think of will be on your exam.

There's also special tutoring this weekend you can check that on the Web--

Saturday and Sunday.

Let's start with a completely inelastic collision.

Completely inelastic collision.

We have mass m1, we have mass m2.

It's a one-dimensional problem.

This one has velocity v1, and this has velocity v2.

They collide, and after the collision they are together--

because that's what it means when the collision is completely inelastic--

and they have a velocity, v prime.

If there is no net external force on the system, momentum must be conserved.

So I now have that m1 v1 plus m2 v2 must be m1 plus m2 times v prime.

One equation with one unknown--

v prime follows immediately.

You may say, "Gee, you should really have put arrows over here." Well, in the case that it is a one-dimensional collision, you can leave the arrows off because the signs automatically take care of that.

Kinetic energy is not conserved.

Before the collision, your kinetic energy equals one-half m1 v1 squared plus one-half m2 v2 squared.

After the collision, kinetic energy equals one-half m1 plus m2 times v prime squared.

And you can easily prove that this is always less than that in case of a completely inelastic collision.

There's always kinetic energy destroyed, which then comes out in the form of heat.

Let's do now an elastic collision.

And I add the word "completely" elastic, but "elastic" itself is enough because that means that kinetic energy is conserved.

I start with the same initial condition: m1 v1, m2 v2, but now, after the collision, m1 could either go this way or this way, I don't know.

So this could be v1 prime, this could be v1 prime.

m2, however, will always go into this direction.

That's clear, because if you get hit from behind by object one, after the collision, you obviously go in this direction.

Again, we don't have to put the arrows over it, because the signs take care of it in a one-dimensional case.

This will be plus, then.

That could be... you could adopt that as your convention, and if it goes in this direction, then it will be negative.

So if you find, for v1 prime, minus something, it means it's bounced back.

So now we can apply the conservation of momentum if there is no external force on the system.

Internal force is fine.

All right, so now we have m1 v1 plus m2 v2 equals m1 v1 prime plus m2 v2 prime--

conservation of momentum.

Now we get the conservation of kinetic energy, because we know it's an elastic collision.

One-half m1 v1 squared, one-half m2 v2 squared--

that's before the collision.

After the collision, one-half m1 v1 prime squared plus one-half m2 v2 prime squared.

Two equations with two unknowns and in principle, you can solve for v1 prime and for v2 prime, except that it could be time-consuming.

And so on exams, what is normally done when you get a problem like that...

Normally you get a problem whereby either m1 is made m2 or m1 is much, much larger than m2 or m1 is much, much smaller than m2, like banging a basketball onto a Ping-Pong ball or a Ping-Pong ball onto a basketball.

I will do a very simple example whereby I will take now for you m1 equals m2, and I will call that m and I will even simplify the problem by making v2 zero, so the second object is standing still.

One-dimensional collision, one hits that object.

Very special case.

So the conservation of momentum now becomes much simpler.

m times v1--

there is no v2--

equals m v1 prime plus m v2 prime.

Conservation of kinetic energy is one-half m v1 squared equals one-half m v1 prime squared plus one-half m v2 prime squared.

Notice that now I lose my "m"s, which is very convenient.

Here I lose my one-half "m"s, even and this is very easy to solve.

If you square this equation you get something that will look very similar to this.

If you square it, you get v1 squared equals v1 prime squared plus v2 prime squared plus two v1 prime v2 prime.

And compare this equation with this equation--

they're almost identical, except for this term, so this term must be zero.

But we know that v2 prime is not zero.

It got kicked and so it'll go forward.

So what that means, then, is that v1 prime equals zero and if v1 prime equals zero, you see that v2 prime equals v1.

And this is that classic case whereby a ball hits another ball.

This one has no speed.

It hits it with a certain velocity, v1.

They have the same mass.

After the collision, this one stands still and this one takes over the speed.

Famous Newton's Cradle.

You see it often with pendulums.

It is the logo on the 8.01 home page, and I showed you a demonstration here when we discussed that in lectures.

All right, let's move on, and let's do something now on torques, angular momentum, rotation and let's discuss the Atwood machine.

The Atwood machine is a clever device that allows you to measure, to a reasonable degree of accuracy, the gravitational acceleration.

Here's a pulley, and the pulley has mass m, has radius R.

It's solid, so it's a solid disk--

rotates, frictionless, about point P, radius R.

And there is a rope here, near massless--

we ignore the mass.

Mass m2 is here and mass m1 is here, and let's assume that m2 is larger than m1.

So this will be accelerated in this direction, this will be accelerated in this direction, and this will start to rotate with angular velocity omega, which will be a function of time.

And now the first thing we want to do
is to make up "free-body" diagrams.

"Free-body" diagrams, for this one, is easy.

m1 g down... and T1 up.

For this one, we have m2 g down and we have T2 up.

For the pulley, it's a little bit more complicated.

This is that point P.

If here is a tension T1 that's pulling down on the pulley--

so this is T1--

and this T2 is pulling down on the pulley--

so there's T2--

it has a mass, so it has weight mg.

The sum of all forces on the pulley must be zero; otherwise it would accelerate itself down, which it doesn't.

And so there has to be a force up--

I'll call it n.

And that force n has to cancel out all these three forces.

So that must be T1 plus T2 plus mg.

We will not need it any further in our calculations, but there has to be a force to hold that in place, so to speak.

So now we're going to calculate the acceleration under the condition that the rope does not slip.

That means there is friction with the pulley--

not friction here, but here.

Otherwise, the rope would slip, the rope would slip.

What it means, if there is no slip, that if the rope moves one centimeter that the wheel also turns at the circumference one centimeter.

That's what it means when there is no slip.

That means the velocity of the rope--

v of r, v of the rope--

must be omega times R of the pulley.

That's what "no slip" means.

So the acceleration--

which is the derivative of that velocity of the rope--

A, is omega dot times R, which is alpha times R.

Omega is the angular velocity and alpha is the angular acceleration.

This is a condition... it is an important condition for no slip.

So let's now start at object number one and write down Newton's second law.

I call this the positive direction for object one and I call this the positive direction for object two--

just easier for me.

So we get T1 minus m1 g must be m1 a--

one equation.

I don't know what T1 is, I don't know what a is.

Second equation, for this one.

I call this the positive direction.

m2 g minus T2 must be m2 a--

second equation.

One unknown has been added, so I need more.

Of course I need more.

I also have to think about the pulley.

The pulley... the net force on the pulley is zero.

That's why this stays in place, but it is going to rotate because this force T2 is larger than this T1.

There is a torque relative to that point P, and torque is defined as r cross F.

The torque relative to point P, the magnitude is this position vector times this force.

That is a torque in the blackboard.

What is in the blackboard, I will call positive.

The torque, due to this force, is out of the blackboard, and I will call that negative.

Since this angle is 90 degrees, I simply get that the torque relative to point P equals the radius of the cylinder--

the radius of that pulley--

times T2.

That's the positive part, and the negative part is the radius times T1.

Notice that this force and this force go through P, do not contribute to the torque.

And that now equals the moment of inertia about that point P, times alpha.

But since we have no slip, alpha is a divided by R.

So it's the moment of inertia about point P times a divided by R.

But since it is a rotating disk which is rotating about its center of mass...

I know the moment of inertia, I looked that up--

if you need it during your exam, you will find that in the exam--

that is one-half MR squared...

one-half MR squared times a divided by R and I lose one R, and so I find, then, that T2 minus T1 equals one-half Ma.

Notice I also lose my second R.

And so now I have a third equation and I can solve for T2, I can solve for T1 and I can solve for a.

And you can do that as well as I can.

If you find the result, you should always do a little bit of testing to make sure that your result makes sense.

And what you should do is you should say...

You should make sure that m2 g is larger than T2.

That's a must--

otherwise, it's not being accelerated down.

You should also check that T1 comes out larger than m1 g.

That's also a must.

And you should also check that T2 be larger than T1.

Otherwise, the pulley wouldn't rotate in clockwise direction.

It would also be useful, which is a trivial check, to stick in your results m1 equals m2.

That should give you that the acceleration should be zero and it should give you that T1 equals T2.

Those are obvious things and that can be done very simply.

It takes you no more than ten seconds.

And if it's... any one of these is not met, then somewhere you slipped up and it will give you an opportunity to go over the problem again.

All right, let's now do another problem--

simple harmonic oscillation of a physical pendulum.

I have here a rod--

mass M, length l--

rotating here about an axis perpendicular to the blackboard without friction.

Here is the center of the rod, and let this angle be theta.

There is a torque relative to point P.

There is also a force that goes through point P.

I'm not even interested in that force.

I know that it's here, mg.

Since I'm going to take the torque relative to point P, I don't worry about the force, but there has to be a force through point P.

Otherwise this ruler would be accelerated down with acceleration g, if this is the only force there were.

But we know that's not the case, it's going to swing.

So there has to be a force to P.

I don't want to know what it is, but there has to be one.

So the torque relative to point P--

the magnitude--

is the position vector, r of P, from here to here crossed with this force.

And so that makes it one-half l times mg times the sine of the angle, and that's... the angle is theta so that is times the sine of theta.

The cross product has the sine of the angle in it.

So this is the torque relative to point P which also must be the moment of inertia about point P times alpha.

No different from what we just had with the pulley.

So alpha equals omega dot.

It's also theta double dot.

This omega is the angular velocity--

it's d theta/dt and the derivative gives me the angular acceleration.

There has to be a minus sign here, that's important because the torque is restoring--

the same situation we had when we had a spring.

We were oscillating a spring, the spring force is minus kx.

The minus sign here plays exactly the same role.

So I can write down here minus I of p theta double dot.

Now, if we take a small angle approximation--

"small angle approximation"--

then the sine of theta is approximately theta, if theta is in radians.

And so I can replace now the sine of theta here by theta and so now I find, if I bring this to the other side, I get theta double dot plus one-half lMg divided by the moment of inertia about point P times theta equals zero.

Needless to say that I am happy like a clam at high tide because I see here an equation which clearly tells me that we have a simple harmonic oscillation--

theta double dot plus a constant times theta is zero.

And so we must get as a solution that theta must be some maximum angle times the cosine of omega t plus or minus phi.

This omega has nothing to do with that omega.

This is the angular frequency.

This is related to the period T of the oscillation, which is two pi divided by omega.

This is a constant.

This omega is not a constant.

It is unfortunate, in physics, that we use the same symbol.

The angular velocity is zero when the object stands still; is a maximum when the object is here.

That omega is always the same.

It's related to how long it takes to make one oscillation.

Both are called omega, both are radians per second.

Couldn't be more confusing.

All right, if we find I of P then we can solve for the frequency, angular frequency and we can solve for the period.

So let's do the... calculate the moment of inertia about point P.

In order to do that, we have to apply the Parallel-Axis Theorem because you will probably be given the moment of inertia for rotation about the axis through the center of mass parallel to this axis.

And then you will have to add the mass times the distance between these two axes squared to apply the Parallel Axis Theorem.

So this is the moment of inertia about the center of mass plus m times the distance squared.

And the distance between these two axes is one-half l, so this is one-quarter l squared.

I look up in a table what the moment of inertia is for rotation of a rod about its center--

a perpendicular rod is perpendicular to the axis--

and that is 1/12 Ml squared plus one-quarter Ml squared give me one-third Ml squared.

So I know now what I of P is, and so now I can solve for the value for omega--

the angular frequency.

I'll do that here so that we keep everything on one blackboard.

So this term here, which is omega squared...

Omega squared equals one-half lMg divided by one-third Ml squared.

I lose one l, I lose one M--

very common, always lose your "M"s in gravity--

and so this is three-halves g divided by l.

And so the period of one oscillation is two pi divided by omega--

omega is the square root of this--

so that becomes the square root of two-thirds l over g.

And that is the period of the oscillation of this ruler.

We worked on something similar in lectures and we even measured the period and we found very, very good agreement with the theoretical prediction.

You can ask, now, what is the kinetic energy of rotation of this rod, which changes with time? Kinetic energy of rotation is one-half I omega squared.

Remember, the linear kinetic energy is one-half mv squared.

The m becomes I when you go to rotation, and v becomes omega.

So the kinetic energy of rotation equals one-half I omega squared--

you'll find this equation on your exam.

This is I about that point P.

You know what theta is, as a function of time.

So omega equals d theta/dt, so you can find what omega is.

You know what I of P is, we just calculated it.

And so you know what the kinetic energy of rotation is at any moment in time.

It will change.

It will be zero when this comes to a halt, and it will be a maximum when it is here.

It's a continuous conversion from gravitational potential energy, which is a maximum here, to kinetic energy, which is a maximum here.

And so this will obviously change with time--

the kinetic energy of rotation.

I thought problem 8-1 was a nice example of how to apply Kepler's laws, Newton's laws of gravity in the case that we have an elliptical orbit.

Here we have a planet, mass M, radius R.

It's not rotating and it has no atmosphere.

Grant you, a little bit artificial.

We launch a satellite and the satellite gets a velocity here which was called v zero as it is right here at the surface and this angle, 20 degrees.

And we were told that the point farthest away was five planets' radii away.

So if I try to sketch the elliptical orbit--

assuming that the velocity is instantaneously reached; that means, this point must also be a point of the orbit.

If I try to make a sketch, then that elliptical orbit would look something like this.

Something like this.

And then, this distance is five R.

I call this point A.

It's the point that is farthest away from the center.

It's clear that the satellite will crash back onto the planet, but that's no concern to us.

When I draw this ellipse, I made the assumption that all the mass of the planet was inside the ellipse--

for instance, somewhere here.

Now, that is not the case, so only this part of the ellipse is realistic and this part, of course, is not.

What you're being asked is to calculate what v zero was only based on the information of the 5R and the 20 degrees and on the mass.

And I can add to it what is the velocity here at A, when it is farthest away, and I can even add to it what is the semimajor axis where this is 2a? All of that comes for free with that initial condition and the knowledge that it goes out five planets' radii.

The angular momentum, l, equals R cross P.

Angular momentum is conserved for this object in orbit--

mass m--

but only if you take the angular momentum relative to this point.

I don't know what I called this point.

It looks like I called it P.

I can hardly read it, I'll call it C for now.

So angular momentum of this object in this orbit is not conserved relative to any point, but it's only conserved relative to this point.

So the angular momentum relative to point C at this moment in time

is the position vector--

which is R--

times the velocity, but I have to multiply it by the sine of 20 degrees.

So it is m... it is mv, right? so we get a v zero, here.

We get a capital R here, and we get the sine of 20 degrees.

R times... indeed.

That must also be the angular momentum right here.

The fact that the angular momentum is in the blackboard is no concern to me; I just want to know the magnitude.

When the object is here, the angular momentum is the position vector which has length 5R.

The velocity is v of A, but the angle is now 90 degrees, so the sine of the angle is one.

So I get m times vA times five R.

I lose my little m, and I have one equation with two unknowns.

I don't know what v zero is and I don't know what v of A is.

I need another equation.

Well, these are conservative forces--

we're dealing with gravity--

so mechanical energy must be conserved or, what we used to say--

the total energy of the system is conserved.

The total energy of the system is kinetic energy plus potential energy...

must be conserved, must be the same here as it is there.

What is the total energy here? The kinetic energy is one-half m v zero squared.

What is the potential energy here? It is minus mMG divided by its distance to the center, which is capital R.

That is the total energy right here.

The total energy here could not have changed.

It must therefore be one-half m vA squared minus mMG, but now the distance equals 5R.

Apart from the fact that I lose my little m, notice I have a second equation.

I didn't add any unknowns, so I have two equations with two unknowns, v zero and v of A.

And you can solve for that, you can find both.

Interestingly enough, you can also find the semimajor axis because the total energy of the system is also minus mMG divided by 2A.

This equation is also on your exam.

And so if you know v zero, or you know v A, then you can...

The m cancels and you can also calculate the semimajor axis.

Okay, Doppler shift.

Let's do something about Doppler shift.

First, Doppler shift of electromagnetic radiation: light, radio waves, X rays.

There is a star moving relative to us with a velocity v.

Since we're dealing with electromagnetic radiation, we don't have to ask whether the star is moving relative to us or we are moving relative to the star.

That question is a meaningless question in special relativity.

You are here.

You are receiving the frequency F prime, and here is that star and that star is moving with a certain velocity--

let's say this velocity, v, and this angle is theta.

So this component of the velocity in our direction is v cosine theta.

We call that the radial component.

And whether the star is moving to us or we to the star makes no difference.

If the velocity of the star is much, much smaller than C, then F prime, the one that you will receive, equals F times one plus v over C times the cosine of theta.

This equation is on your exam.

C is three times ten to the fifth kilometers per second.

If cosine theta is positive, the object is approaching and you observe a higher frequency.

If cosine theta is negative, the object is going away.

The radial component is away from us and the frequency is lower.

In optical astronomy, we cannot measure frequencies.

We can only measure wavelength.

And the connection between wavelength and frequency...

Lambda equals the speed of light divided by the frequency.

So we can substitute in that equation F equals C divided by lambda and F prime equals C divided by lambda prime.

When we do that and we also make use of this approximation--

that one divided by one plus x is approximately one minus x as long as x is much, much smaller than one--

this is the first order term in the Taylor Expansion.

Then you can find that now lambda prime becomes lambda times one minus--

it becomes a minus sign; the plus changes to a minus sign, for that reason--

times v divided by C times cosine theta.

Notice that here, you have that radial velocity.

And you have that here.

If lambda prime is larger than lambda, the object is going away from us.

We call that "red shift."

If the object is receding--

if lambda prime is less than lambda--

we call that "blue shift," because the wavelengths become shorter in this time--

move towards the blue end of the spectrum.

Here it becomes larger, shifts to the red part of the spectrum.

The blue shift means the object is approaching you.

During the lecture that we discussed this, we took an example where lambda prime over lambda was 1.00333.

If you substitute that in this equation, you'll find that v cosine theta equals minus 100 kilometers per second.

So what it means--

all you know is the radial velocity; you never get any information about the angle--

that the radial velocity is 100 kilometers per second away from us, or we are moving 100 kilometers per second away from the star.

That's the same thing in special relativity.

If we apply this to sound, then we get a very similar equation.

Suppose you sit still in 26.100 and I move a sound source to you, but you don't move.

Then you get the similar equation that f prime--

that's the frequency that you will hear--

equals f times one plus v over v sound times the cosine of theta, and the sound's speed is about 340 meters per second at room temperature.

In other words, if I twirl something around in a circle and you are in the plane of that circle--

you are here--

then when the object comes to you, you hear a maximum frequency--

f prime is larger than f.

When it goes away from you, you hear a decrease in frequency--

smaller than f.

And when the object is here, and when it is here--

when the angle is 90 degrees and the cosine theta is zero--

you'll find f prime equals f, f prime equals f.

So when I twirl something around...

let's suppose I twirl it around with an orbital velocity.

If I call that orbital velocity of 3.4 meters per second--

just to get some nice numbers, then this is .01...

And so if it comes to you, you hear an increase of one percent in the frequency.

When it goes away from you, you hear a decrease of one percent.

I have here a sound source which produces roughly 1,500 hertz.

[machine emits high-pitched tone]

I could twirl it around.

I can do it once per second around, which gives you even twice this speed.

And so you would hear a two percent increase when it comes to you; two percent decrease when it goes away from you.

[tone wavers between higher and lower pitch]

Can you hear the Doppler shift? It's always difficult in a lecture hall because you get reflections from the wall.

Can you hear it, when it comes to you, that the pitch is higher than when it goes away from you?

Okay, rolling objects.

Let's roll something down the hill.

Classic problem.

Very often you see that on exams.

We take a ball and we roll it down an incline.

This is the incline and the angle is beta.

Here is the ball, it's a sphere.

It's a solid sphere, it has mass m with radius R and it is solid.

This is the center of that ball.

I call this point P.

I put all the forces on that I can think of.

There is mg here...

at this point P, the normal force from the surface...

and there is friction.

This is the frictional force.

The sum of Ff and the normal force is what we call the contact force.

That is the force with which the incline pushes back onto the ball, but we normally decompose that in a direction along the slope and perpendicular to the slope.

So these are the only three forces that this object is experiencing and I can ask you now what is "a"--

the acceleration at which it goes down the slope--

and I can ask you what is the frictional force.

I will ask you that under the conditions of "pure roll." What does pure roll mean? It means that if this--

the circumference here--

moves one centimeter that the center of mass has also moved one centimeter.

So it means that the velocity of the center--

so this velocity, v of c--

at all moments in time will be omega R.

And omega is the angular velocity which is changing with time.

That is the condition of pure roll.

At all moments in time if I have a rotating sphere or cylinder always will the circumferential speed be omega R.

It could be standing still and just rotating like this, slipping like hell and not moving forward--

then the circumferential speed is always omega R.

But if the center also moves with that same speed, only then do we call it pure roll.

If you take the derivative of this, you get a equals omega dot times R.

That equals alpha R, alpha being the angular acceleration.

All right, we are going to write down now the torque relative to that point C.

If I take point C, then I lose the force mg and I lose the force N.

There is only one force that I have to deal with.

For the torque, relative to point C...

The fact that the torque is in the blackboard is of no concern to me.

I just want to know the magnitude--

that is R times Ff, R times the frictional force.

That must be the moment of inertia about that point C times alpha.

We've seen this now twice already.

And so, since alpha is a divided by R, this is I of c times a divided by R.

This is one equation and I have a as an unknown and I have Ff as an unknown.

There is not enough--

I need more.

Newton's second law always holds--

the center of mass...

For the center of mass, it must always hold that F equals Ma.

So all I do now is I go to the center of mass and I say, What are the forces which are acting down the slope? I am not interested in the ones perpendicular to the slope--

only the ones down the slope.

That must be Ma--

it's nonnegotiable, because that is the quality, characteristic of the center of mass.

Well, this one is uphill and the only one that is downhill is the component of mg along the slope, which is Mg sine beta.

I call that the plus direction, so I get Mg sine beta minus that frictional force equals M times a.

It's my second equation.

I have two equations with two unknowns, and I can solve.

a is an unknown, and the frictional force is an unknown.

You can do that as well as I can--

I will just give you the results.

I don't want to waste your time on the algebra.

a, the acceleration, under the conditions of pure roll--

but only under the conditions of pure roll--

equals MR squared times g times the sine of beta divided by MR squared plus the moment of inertia about that point C.

And the frictional force--

you can see that from this equation--

is the moment of inertia times a divided by R squared.

And I will leave "a" as it is because it becomes too complicated otherwise.

And so the frictional force becomes the moment of inertia about C times a divided by R squared.

So this a has to be substituted in here.

The acceleration is independent of the mass and independent of the radius--

very nonintuitive.

It doesn't matter whether you take a ball this big or this big.

It doesn't matter what the mass is.

The reason for that is that, as you will see shortly, that the moment of inertia also contains the term MR squared, and so all the MR squares will disappear.

Let's first take a look at beta related to a.

Notice when beta equals zero, that a equals zero.

It better be.

If there is no slope, there cannot be any acceleration.

Notice if a equals zero that the frictional force is also zero.

It better be.

So that is an easy check that you can do--

internal consistency check.

So now let's look at I of C.

What is I of C? I look it up in the table.

A ball, sphere rotating about an axis through the center of mass.

It's one of the few that I happen to remember--

is two-fifths MR squared.

If I substitute that in here, I'll find that the acceleration under the conditions of pure roll is five-sevenths g times the sine of beta.

And I find that the frictional force, F of f, equals two-sevenths, I believe, times Mg sine beta.

Indeed.

These are the results by substituting the moment of inertia in there.

Let's look at these results a little closer.

If there were no friction, then we know that the acceleration would have been g sine beta.

We remember that--

a sliding object with no friction would be g sine beta.

Why is it now lower? Five-sevenths is lower than one.

It's obvious.

Because when this object comes rolling down, the kinetic energy, the total kinetic energy is kinetic energy of rotation plus the linear portion of the kinetic energy--

the forward motion due to this one-half Mv squared.

When you have a sliding object, there is only the linear component.

There is no rotational kinetic energy.

So now the linear kinetic energy must be lower because some gravitational potential energy goes into rotational kinetic energy.

And so it must go slower and so the acceleration must be less.

It's completely intuitive that it is less than g sine beta.

Let's look at the frictional force.

This is a situation of pure roll.

We know that the frictional force can never exceed the maximum value possible.

That is a no-no.

If the frictional force becomes the maximum one, it would start to slip.

It would no longer be pure roll.

In other words, there is a necessary condition that the frictional force, which is two-sevenths Mg sine beta, must be less than the maximum frictional force.

But the maximum frictional force is mu s--

static friction coefficient times this value N--

and this value N is mg cosine theta... beta.

And so this is mu s times Mg cosine beta.

I lose my Mg--

no surprise--

and so a necessary condition is that the static friction coefficient must be larger than two-sevenths times the tangent of alpha in order to have pure roll.

Not alpha--

I don't know what is wrong with me--

beta.

And this, of course, is pleasing in a way, because what this is telling you--

and we all know that by our instinct, you feel it in your stomach--

that if you make the angle too large, if you tilt that, there's no way that you're going to get pure roll.

It will start rolling, of course, but it also starts slipping.

For instance, if you freeze the friction coefficient and you say the friction coefficient is 0.2, then it means that beta--

the angle beta--

must be less than 35 degrees.

That's just an example.

Well, that's intuitively pleasing.

Remember that during one of the lectures we calculated the period of oscillation of an object that was sliding on an air track without friction.

And it was a curve of a circle with a huge radius.

It was hundreds of meters.

And we had a sliding object here and it was sliding without friction.

And we derived that the period was two pi times the square root of R divided by g, as if this were a pendulum with length R, and that's in fact what it is.

We did that with the air track and the results were fantastically in agreement with our prediction, unbelievable accuracy.

And then we did it with a smaller radius--

with a ball, a rolling ball, this one.

The radius of that curvature was 85 centimeters, and we predicted a period of 1.85 seconds--

this equation.

And we measured it, and what did we find? Way higher.

We observed something like, I think, 2.3 seconds.

And I asked you at the end of the lecture, why? And some of you said, "Well, the radius is smaller." That was no reason, because the angle--

the displacement angle--

was still very small.

This maximum angle, theta maximum, was so small that that could never explain why the period was so much larger.

But now you know, because when this object rolls down, the kinetic energy is partially in the linear term and partially in rotation--

partially in the linear term and partially in rotation.

If it is sliding, it is all in the linear term, so it clearly goes faster.

Now it has to share with the rotation.

And so that's the reason why we found the higher period.

And that's precisely the situation that you have here.

I want to repeat that this is what you have on your plate on Monday.

There is no way that I can cover all of that during one exam.

Please hold it--

we have one minute left.

There's no way I can cover all of that during one exam.

I'll have to make a choice.

I will pick only a few.

Neither could I cover today, at any depth, all these topics.

The ones that I did not cover doesn't mean at all that you will not see them on the exam.

There is a special tutoring schedule this weekend that you can make use of and I believe that this lecture will be on PIVoT tomorrow morning.

I wish you luck and I'll see you Monday.