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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 21

Last lecture, I introduced the concept of angular momentum and torque.

They're the most difficult concepts in all of 8.01.

And only when you get a lot of practice will you really get the hang of it.

You shouldn't feel bad if it takes a while.

This is very difficult.

I will spend the next five lectures exclusively on dealing with these concepts, and you will see many examples--

some intuitive, some nonintuitive and some even quite bizarre.

I'd like to briefly review the key things we discussed last time, and you see the related equations there on the blackboard.

We have a mass m, and let that mass in your frame of reference have a velocity v.

So then it clearly has a momentum p.

There may be a force acting upon that mass--

F.

And now I choose a point Q at random.

This is the position vector relative to Q.

Never forget to indicate what the origin is that you have chosen.

Then the definition of angular momentum relative to that point Q equals the cross product of the position vector with p, and that is my equation number one there.

The direction is perpendicular to the blackboard, and it will be in this case into the blackboard.

And the magnitude can be calculated provided that you take the angle into account.

You get the sign of theta because of the cross product.

The torque relative to point Q is defined as the position vector, cross F.

In this case, that would be out of the blackboard.

And, again, the magnitude can be found, but you have to take into account the angle between the position vector and the force.

The torque leads to a change in angular momentum.

You see that in equation three.

If there is no torque, then angular momentum won't be changing.

And if you have a system of objects--

not just one like we have here, but many interacting particles--

then as long as there is no external--

net external--

torque on that system as a whole, then angular momentum of the system as a whole will be conserved.

Today you will see various applications of equation four and of equation five.

Last time, we already discussed the idea of spin angular momentum, which is an intrinsic property of a rotating object.

We did an experiment with the ice-skater's delight when I was sacrificing there on this rotating turntable.

And we'll see some more examples of that during the next few lectures.

In case that you do have a rotation about an axis through the center of mass--

a stationary axis through the center of mass--

then the angular momentum is intrinsic in the sense that you don't have to specify the point about which you take the angular momentum.

You can take any point, and you always find the same, which is not true in this situation.

So that makes the intrinsic angular momentum quite unique.

The reason why it's so nonintuitive--

angular momentum--

is that the angular momentum depends on the point you choose.

And in one problem, you can sometimes pick a point about which the angular momentum changes, but in the very same problem, you can pick a point about which angular momentum doesn't change, and both solutions would be perfectly valid.

So you often have a choice, and that doesn't make it very intuitive; that doesn't make it very easy.

So let's start with an example, which I also had last time, whereby we have an object going around the Earth or around the Sun.

Let's take the Earth going around the Sun, and this is the location of the Sun, and here is the Earth.

It has a mass m, and it's going around with a velocity.

The magnitude doesn't change, but the direction does change.

The position vector relative to point C is r of C, and then we have a gravitational force F, which is pointed towards the center, and the angular velocity omega is in this direction.

Well, go to equation number one, and the angular momentum relative to point C--

L relative to point C--

the magnitude, because the direction is clear.

If it's going, seen from where you are, counterclockwise, then the direction of the angular momentum will be pointing out of the blackboard.

So I'm only interested in the magnitude.

That will be r of C times the mass times v.

And the reason why I don't worry about the cross now is because this angle is 90 degrees, so the sign of theta equals one.

Now, we may not like to leave v in there.

It's up to you.

You can always write v equals omega R, so you can also write, then, m r C squared, times omega.

Either one is fine.

And this r, if this is the radius of the circle, then this obviously becomes m R squared omega.

You could have chosen equation five, and you would immediately have said, "Aha! I have a rotation about point Q," which in this case is C, and so L... the magnitude of L of C equals the moment of inertia about point C times omega.

The moment of inertia about point C for this object is clearly mR squared, and I multiply that by omega, and you see you get exactly the same answer.

So this is equation one, but this will be equation five.

If I go to equation number two, then equation number two is telling me that if I choose point C, but only if I choose point C, that the torque relative to point C is zero--

equation number two--

because the force and the position vector make an angle of 180 degrees with each other.

Whether the object is here or here or here, that makes no difference, and so the torque relative to point C is zero, so I also know that the angular momentum is not changing relative to point C, but only relative to point C, because any other point that you would have chosen here or here or here, there would have been a torque, and the angular momentum would be changing, so there's something very special about this point C.

Angular momentum is only conserved, in this case, about point C.

Now I take another example whereby angular momentum is only conserved relative to one point but not to any other point.

I take a ruler or a rod... and the rod has mass M and length l, and C is the center of mass of that rod, but I force it to spin about point P...

and this distance is d.

Think of this as a horizontal frictionless plane, and I'm rotating it with an angular velocity.

Let's say we rotate it in this direction, force it about that point P.

I put a pin in there perpendicular to the blackboard and I rotate it.

I'd like to know what the magnitude is of the angular momentum relative to point P, and for that, I go immediately to equation five, so that tells me that it is the moment of inertia about that axis through point P times omega.

I remember the parallel axis theorem.

I know that the moment of inertia for rotation about a center of mass through this axis perpendicular to the blackboard--

I know that that one equals 1/12 Ml squared.

But I just looked it up in a table, because I don't remember that.

So that would be the moment of inertia about this axis, and then the parallel axis theorem tells me I have to add plus Md squared...

times omega.

I'm not interested in the direction of l, because that's immediately obvious.

If it's rotating clockwise, then the direction of the angular momentum would be perpendicular to the blackboard and into the blackboard.

I claim that at this point P, there must be a force acting on this ruler, and the force is in this direction.

And I can make you see that the best way by first showing you the case of a massless rod with two equal masses at both ends.

And I rotate about this axis perpendicular to the blackboard.

There is going to be a centripetal force here and a centripetal force here, and the two are equal, they cancel each other out, so there will be no force on that pivot point about which the two rotate.

However, if I had the situation such that this is my massless rod and here are the two equal masses, but now I rotate them about this point, then this centripetal force is larger than this one, so now I have asymmetry, and so now there will be a force on this pin.

The ruler will push on the pin, and action equals minus reaction--

the pin will push on the ruler.

And it is because of this same asymmetry that you have here that there will be a force from the pin on point P.

However, I don't care about that force because I'm going to take the torque relative to point P.

And when I take a torque relative to point P, any force through point P has no effect, because the position vector is zero.

But I want you to appreciate that there is a force, so if I take the torque relative to point P, I do not worry about this force.

Well, the torque relative to that point P is zero, and so it's clear that angular momentum relative to point P must be conserved.

Angular momentum relative to point P is conserved.

Take any other point--

it doesn't matter which one you take; take this point Q, take this point here, take this point here--

and angular momentum is not conserved.

You immediately see that if I take this point Q here, that this is the position vector, and you see that r cross F is not zero, so there is a torque relative to point Q.

Angular momentum is not conserved.

Only this point--

that point is very special.

I now take the same ruler, but I'm going to make it rotate about the center of mass.

So here is now the same ruler, and now I'm going to rotate about the center of mass.

Rotation--

same direction.

I have a stationary axis in 26.100, and it's rotating about there.

Center of mass--

C is my center of mass.

Now there is no force on this pin because of the symmetry, which I just explained, so the pin is not pushing on the rod either, so there is no force at all if this is a frictionless object.

So if there is no force at all, then the torque relative to any point must be zero--

not only relative to this point, but also relative to this point and relative to this point.

Because if the force is zero, then any cross product of r and F is zero.

So now you see the special case that I alluded to in equation number six.

Now you have the case that the angular momentum of this rotating object is the same, no matter which point you choose.

We call that the spin angular momentum.

It is an intrinsic property of a spinning object, and it is the same relative to any point that you choose.

And if you want to know how large it is, well, the magnitude of the angular momentum--

I calculated about C, which is the center of mass--

is simply equation number six, is the moment of inertia about that point C, which is the center of mass, times omega about the center of mass, so CM--

center of mass--

and C are the same point.

What is the moment of inertia of rotation about the center of mass? That is my 1/12 Ml squared.

So now I have 1/12 Ml squared times omega.

That is now the intrinsic spin angular momentum, and it is the same for any point that you choose.

Even if I choose a point here in space, you can prove that it is still the same angular momentum.

Now we're going to look at other applications.

There are a huge number of applications, and in the next four or five lectures, we will go through many of them.

In some cases, you will say, "Yeah, yeah, that's intuitive; that's obvious." In some cases, you will say, "Hmm, not so intuitive." And in some cases, you will fall off your chair--

it is completely bizarre.

It is so nonintuitive that you don't even believe it.

You won't believe it until I actually show you with a demonstration that that's the way nature works.

But that comes later, not today.

On assignment number seven, I give you a problem--

problem 7-9--

in which I have a ruler or a rod on a frictionless horizontal table, and the ruler has a mass l...

a mass M and it has a length l, and here is the center of mass at C.

And I hit that ruler--

I give it an impulse--

perpendicular to the direction of the ruler.

Here, I give it an impulse.

A force acts upon it for a certain amount of time.

And let this distance be...

I think I call it "d" in your problem.

And it's on a frictionless table.

I do this--

bang--

very short hit.

And now the question is, what will this object do? What does your instinct tell you? You will say, "Well, for sure, it will move in this direction." Remember it's frictionless, so whatever happens after the hit happens forever and ever and ever.

It will never stop.

So you all say, "Well, it's going to move in this direction." That's very vague, but that's true.

But how about rotation? Will it rotate? And if it rotates, about which point? Well, you may say, "Well, maybe where... the point of rotation may depend on where I hit it." That would be a reasonable intuition, but it's not true.

It must rotate about the center of mass.

It cannot rotate about any other point.

Suppose it rotated about this point, just for sake of argument.

So after the hit, it's rotated about this point.

That would mean that the center of mass would then do this.

And that's not allowed, because the center of mass behaves like a point in which all the mass is concentrated.

; F equals Ma holds for the center of mass.

And so after the hit, a single point--

because the center of mass can always be considered to be a single point with all the mass in it.

That point can never do this.

That would be absurd.

So from that reasoning alone, you must conclude that the only way that nature can digest that impulse is by giving the center of mass a certain velocity in this direction, which will never change.

In addition, it will give it an angular velocity about the center of mass.

And if there is no friction, that will never change.

So a little later in time, the center of mass will still be on this line, and it may have rotated one or two or three rotations.

That depends... that depends on how large I is and that depends on this distance d, but let's assume that it is now like so, and then it is happily rotating with that angular velocity, omega C, and continues all the time with that velocity v center of mass.

And in that problem, I'm asking you to calculate the velocity of the center of mass and to calculate the angular velocity about the center of mass.

I'll help you a little bit, and I hope that PIVoT will help you a little more.

For the center of mass, what must always hold...

The center of mass acts like a point source, so it must always hold that F equals Ma of the center of mass.

That is really one of the major characteristics of the center of mass.

So if we look at magnitude--

F delta t equals M times a center of mass delta T--

the force acts for a certain amount of time, and that's the impulse.

But if--

before I hit--

if this velocity is zero, then a delta t is obviously the velocity of the center of mass afterwards.

v equals at, right, and v was zero to start with.

So you see now immediately that I--

that is the impulse--

equals M times the velocity of the center of mass, and so the velocity of the center of mass equals I divided by M.

And so I have solved for you the first part of that problem.

And what is remarkable--

that it's independent of b...

or d, sorry.

We called that d.

It's completely independent of d.

Whether you hit here with the very same impulse or hit there makes no difference.

The center of mass behaves like a point source--

F equals Ma--

and the velocity of the center of mass only depends on I and on the mass of the object.

The larger I, the larger the velocity.

That's, of course, quite intuitive.

And the larger the mass, the lower the velocity.

But now comes the hardest part, and I will leave you largely with that hardest part but give you a few clues, because now I want you to calculate omega about the center of mass.

And now you have to choose an origin, because you're dealing with torques, you're dealing with changing angular momentum, and so you would probably choose C as your origin.

And you will say, "Okay, let's think about what the angular momentum is "relative to point C before the collision, after the collision--

"if you call it a collision.

What is the torque relative to point C?" Well, the torque relative to point C, there is here the position vector r of C, and the torque is r cross F, and this F acts for a certain amount of time, so this certainly is not zero.

You can clearly see that there is a torque relative to point C, and if there is a torque relative to point C, the angular momentum relative to point C must be changing.

Yeah, you bet your life it's changing, because the angular momentum before the hit is zero, but after the hit, the angular momentum is I about the center of mass times omega, center of mass.

You can see that if it rotates about the center of mass, it has an angular momentum of rotation about the center of mass which is I omega.

Before, it was zero; afterwards, it's this.

So clearly there must be a torque about the center of mass.

And so now, all you would have to do is to somehow relate that change in angular momentum with this distance d and with I.

And you will see that if d becomes larger, that indeed the omega of center of mass--

the angular velocity of our center of mass--

will increase.

It's very sensitive.

That's also intuitive because if you hit right in the middle, you didn't expect it to rotate at all.

You would just expect this object to translate.

And so clearly, the velocity of the center of mass is insensitive of where you hit, but the angular velocity around the center of mass is very sensitive, as I will show you shortly.

If you hit here, you don't expect any rotation.

Some of you may say, "I don't like to take C as my origin.

"I'm going to pick another point as my origin.

"I'm going to take this point P as my origin or any other point on this line." Angular momentum must be conserved about that point, because there is no torque relative to that point.

Because any position vector from this point to here is in the same direction as the force, so r cross F is zero.

So now we have a point--

an infinite number of points--

all these points here on this line, the torque is zero.

So the torque relative to point P is zero, so there is angular momentum L relative to point P is conserved.

It is zero before, and so it's zero after the hit.

Isn't that interesting? And you can solve that problem and find exactly the same answer for the angular velocity about the center of mass by choosing point P.

It's just a matter of taste.

Most of you would probably pick C, but if you pick point P, the problem may even be a little easier.

Both should work, and I will make sure that the solutions to both will appear on the final solutions.

So you will see that it can be done in two different ways.

I have here a ruler with a...

which I can hit.

Here's this ruler.

This is the center of mass, and this is just a reference line.

What I will try is to hit it in a direction parallel to the reference line.

If I succeed, then it may start to rotate, but the center of mass should stay on that line, depending upon where I hit.

However, if I don't hit it exactly parallel to this line, it will not stay on this line--

the center of mass--

so don't start laughing then.

It simply means that my angle is not perfectly in this direction.

If I hit like this, then obviously it will go in this direction.

So I will try to hit it first along the direction of this line.

I will hit it right in the middle, so I expect no rotation.

I only expect that the center of mass will move.

Now, there is lots of friction here, so the thing will come to a grinding halt.

It will not go on forever and ever.

So let's try this.

And you see that's fantastic.

It moved, it translates, stays on the line, but it hardly rotates at all because I hit it very close to the center of mass.

Now I'm going to hit here, far below the center of mass, so now I expect this point to stay on the line if I'm lucky enough that I hit it in the right direction, and then, of course, it will start to rotate like this.

And it will come to a halt because of the friction.

So let's try that.

Not bad.

You see it stays very close to the center, to that line.

That means my hit was almost perfectly in this direction, and you saw it rotate.

I'll do it once more.

Oh, look at that.

It goes much farther.

It rotated over a much larger angle.

The center of mass indeed is very close to this line, so I was lucky, and my direction in which I hit it was quite close to being parallel to that black line.

Now comes another application of the... use of torques.

And that has to do with simple harmonic oscillations.

We were able to calculate the period of oscillation of a pendulum.

That was relatively easy.

Most of you have even learned how to do that in high school--

a string, no mass, an object at the end, and you give it a kick, and it goes simple harmonic if the angle is not too large.

But now suppose you have a ruler like this, and you have here a little pinhole, and you're going to oscillate it like this.

What now is the period of oscillation? That looks like a mathematical headache.

And yet, with the knowledge that we have now, this can be solved quite easily, and we can make a quite accurate prediction about the period of oscillation of this rather complex system.

Even a system as bizarre as this hula hoop, which oscillates about that pin, can now be calculated very easily with the knowledge that we have, or at least that you will have very shortly if you spend the time and you study this.

So let's first do the...

We first take the ruler.

This is a ruler.

The ruler has mass M and length l.

This is the center of mass of the ruler, and I make...

I force it to rotate about point P, put a pin through there, nearly frictionless, and the separation between the center of mass and P, I call that now "b", as in "bridge." This angle is theta.

It rotates about the pin perpendicular to the blackboard.

For sure, there will be forces through that point P.

I don't care about that because now I'm not going to negotiate with you about which point I'm going to take the torque.

I will insist on taking torques exclusively through that point P, so that I get rid of any forces that go through that pin--

any forces that the pin exerts on the object.

Gone--

don't care about it.

All I worry about now is this Mg and this position vector rP.

That determines the torque.

And the magnitude of that torque--

and I'm sure that torque is going to change in time--

equals Mg times this distance b, but I have to take the sine of this angle theta because remember, it is a cross product between r and F.

And the cross product has the sine of the angle in it.

So the magnitude of that is going to be Mg times b times the sine of theta.

This is r cross F--

the magnitude of r cross F--

this being F, this being r.

I go to equation number four, and I say, aha, that must be equal to the moment of inertia about point P--

about the axis through the perpendicular to the blackboard through point P--

times alpha, which is the angular acceleration, and, of course, this may change with time and theta will change with time.

However, there is one important thing.

This torque is a restoring torque.

It wants to drive it back to equilibrium.

And therefore, we need a minus sign here for the very same reason that when we did the spring that the force was minus kX and not plus kX.

Now I say, aha, small angle approximation.

By the way, alpha equals omega dot--

first derivative of the angular velocity.

Therefore, it also equals theta double dot.

I also wrote that there.

And theta is this angle.

This is the change of this angle that you see there--

the second derivative.

I say, aha, small angle approximation, then the sine of theta is theta--

we've done that before, provided the angle is in radians.

And so I'm going to get that Mg b theta plus I relative to P times theta double dot equals zero.

And now I already begin to feel it in my bloodstream.

I already begin to smell the simple harmonic oscillation, and you, too.

Let's bring the theta double dot naked out here plus Mg b divided by I of P times theta equals zero, and this, for sure, is a simple harmonic oscillation in theta.

And the solution must be that theta in time must be some maximum angle times the cosine of omega t plus phi.

This omega has nothing to do with that omega.

This is angular velocity, and this is angular frequency.

This will never change.

That's related to the period of oscillation.

This is changing all the time.

d theta/dt is changing all the time.

When the object is here, d theta/dt has the largest value.

When the object stands still, d theta/dt is zero.

So the angular velocity is changing with time, but this omega is a constant, and it is very awkward in physics that we use the same symbol in one problem and have totally different meaning.

In any case, we know that this omega, the angular frequency equals the square root of Mg b...

divided by I of P, and so the period of an oscillation, which is 2pi divided by omega has to be this.

This, by the way, is independent of the mass of the object.

You may say, "Yeah, but there is a mass downstairs." Yeah, yeah, but there is also a mass upstairs, because the moment of inertia about point P has a mass in it.

The moment of inertia about point P we find immediately by using the parallel axis theorem.

If it were rotating about the center of mass, about this axis, then it would be 1/12 Ml squared, and so we have to add Mb squared.

We did something similar there.

So this is 1/12 Ml squared plus M times b squared--

parallel axis theorem--

and so now we find that the period is 2pi times the square root of...

I lose my M, so I get 1/12 l squared plus b squared, and I get here gb.

And that is a very simple result when you come to think of it.

I mean, this would have been a complete headache to do it in any other way.

And this always baffles me so much.

Some people... some of you think that physics is difficult, but I always think of it the other way around.

I always think of that...

physics is there to make very difficult things easy.

Look at it.

This is an incredibly complex system that is going to rotate about a pin, which is offset from the center, and here is a very straightforward prediction about the period.

In our case, a b equals 40 centimeters, and it is an exact meter stick.

So l equals 1.00, and b equals, oh, it's about 40 centimeters, but we could be off by something like maybe two millimeters.

It's not so easy to get that whole in precisely.

So it's 40 centimeters plus or minus 0.2 centimeters.

So that is an uncertainty of about half a percent.

I have upstairs l square and b square and downstairs b, and that makes the error analysis a bit complicated.

If I stick in the numbers, then I find a period which will be very close to 1.565 seconds.

That's taking the exact numbers that I gave you for l, and I take for g 9.8, and I take the values for b.

Since I prefer to be on the conservative side, I will allow for an uncertainty of about 0.01 seconds, and I would like to make a measurement and show you...

yeah, and show you what we get.

So we have here the ruler.

We're going to oscillate it...

small angle.

Well, we can be a little bit rougher, make it a little larger.

Let's stop it first, and I will start it when it stops at my side.

That is always the easiest to do.

Now--

one, two, three, four, five, six, seven, eight, nine, ten.

Look--

on the button, way within the error.

Amazing, isn't it, that such complex systems can be so easily dealt with in physics? It is 15... what is it--

.71, and my reaction time is always .1 second, so the agreement is absolutely fantastic.

Now I will show you, once you have done this with your ruler, you can now apply this knowledge on a way more interesting system, and that is the, um... the hoop.

We'll take the hula hoop, and we're going to do the same thing for the hula hoop.

And the system, the approach, will be exactly identical.

We have a pin through the hula hoop.

This is the hula hoop, this is point P, this is the center of mass--

I call it "C".

It's now like this.

A little later, the center of mass will be here, and this angle will be theta.

The mass of the hula hoop is M, and it has a radius R.

I know there will be forces through that point P, but I couldn't care less about them.

All I worry about is this Mg and this position vector rP.

And the whole thing goes parallel.

I'm going to get that the torque about that point P equals Mg...

and I'll fill in for this immediately the radius R...

times the sine of theta.

And that must be equal minus the moment of inertia about point P times theta double dot.

What is the moment of inertia about point P? Well, it is the moment of inertia of rotation about the center of mass plus MR squared--

that's the parallel axis theorem--

and the moment of inertia for rotation about this axis is clearly MR squared, because all the mass is distributed at the circumference at distance R, so it is the summation of all the masses times the radius squared.

So this is going to be MR squared plus MR squared, so that is 2MR squared.

So we bring this to one side, so we're going to get theta double dot plus MgR.

We make this theta--

small angle approximation--

divided by IP equals zero.

Simple harmonic oscillation in theta with exactly that same solution, but now we get as a period, we get 2pi times the square root of IP divided by MgR.

But IP is 2MR squared.

You see IP divided by Mg, not b, but now we have MgR...

divided by MgR, and that is 2pi times the square root of 2R/g.

And that is a cute result, because if I had a pendulum and the pendulum had a length l which is 2R and I put all the mass at the end of the pendulum and the string has no mass, I would have found exactly the same result.

Remember, the period of a pendulum is 2pi times the square root of l/g.

Notice again there is no mass in there.

It's only a matter of geometry.

Geometry is the only thing that determines the period of oscillation.

That's always the case.

With a pendulum--

a simple pendulum--

with a mass at the end is also the period is independent of the mass, not with springs, but always with gravity.

So the first thing I want to show you is that if I have here an apple on a string, and to the center of the apple is about the same to here, to this pin, is about the same distance as the diameter of the...

of the hula hoop, that they probably have very closely the same period.

You see, they track each other quite well; not precisely, because I was not able to get the length exactly right, but very closely.

So already you see that indeed that result makes sense, but now I want to go one step further.

I want to do a quantitative measurement because I know the radius of this hula hoop.

The radius, I think, is 40 centimeters, and we know that to an accuracy of about half a centimeter.

So R is 40.0 plus or minus 0.5 centimeters.

You will say, "Boy, can you not measure that any better? "I mean, half a centimeter-
-

it's almost yea big, half a centimeter." Well, the reason is, it's not a perfect circle.

So if you measure the diameter at various places, you don't always get the same, and so that's why I allow for half a centimeter uncertainty.

So that's a one percent uncertainty in R, one percent.

Under the square root, that becomes half a percent.

And so the prediction then would be, if I put in the numbers here that we have...

is 1.795, and that would become half a percent uncertainty, half of 1.79 is about one, so let's make it easy--

.01 seconds.

That is my prediction, and I'm going to make ten oscillations, which gives me an uncertainty of 0.1 seconds in ten oscillations, so that gives me an uncertainty of .01 in one oscillation.

Imagine that you come home at Thanksgiving, and you show this to your parents, and you say, "Oh, by the way, Dad and Mom, would you be able to calculate the period of this oscillation?" I mean, they would turn pale, green, purple, and you take out your pocket, and you just do it like that--

simple.

And not only that, but you can do a demonstration.

You can show it to them.

They'll be proud of you.

[students laugh]

Okay, let's try it.

There we go.

Small angle--

this is about five degrees, seven degrees.

It doesn't make that much difference as long as you don't get much above ten.

I don't like the wiggle.

I'll start it when it stops here.

Now! One, two, three, four, five, six, seven, eight, nine, ten.

Holy smoke! Look at that.

18.07... 18.07.

If I divide that by ten, then I get 1.807.

Let's call it one plus or minus 0.01.

And if you take this error into account and this...

well, you can make that a 07...

pretty impressive.

We have five minutes left, and I want to use these five minutes...

First, I'm going to ask your forgiveness for what I'm going to do the next five minutes because by now, you may be lost already and I'm going to make it a little worse by challenging you a little.

And this challenge makes life really interesting.

If I spin an object--

a top--

then clearly, there will always be external friction on that top, and the top will come to a halt.

It's obvious.

We've all seen it.

You turn a top, you wait a while, and it stops.

And if it doesn't stop like it didn't when I gave the exam review, then something stinks.

Then you're being cheated, and you were cheated.

There was something going on in that little black box which kept it going, of course.

It can't keep going forever.

So whenever you spin something in the laboratory here, different from outer space, there are always external frictional torques on it and it will come to a halt.

It is inconceivable that all by itself it would come to a halt and then turn the other way around.

That would be absurd.

It couldn't possibly happen because the moment it comes to a stop, all the external torques go away, so it will just sit there.

Imagine that you slide an object on the table--

whoosh--

a plate, and it comes to a stop because of friction and then it comes back at you--

it would be equally absurd, right? Unless you attached a spring to it, of course.

There are no other external forces on it.

You shove it on the table, friction takes out the kinetic energy, it comes to a halt, and it sits there.

It couldn't possibly come back at you--

would be a violation of basic laws of physics.

So if you rotate...

if you spin a top, then it has to come to a halt, and it couldn't possibly rotate, come to a halt and reverse its direction of rotation.

And as long as we agree to that, then you will pass this course.

Now, you already smell a rat, don't you? You already smell a rat.

Okay, I have here a piece of plastic, a nice piece of plastic.

You can come and examine it.

And this plastic behaves quite nicely.

I will give it a spin.

Friction, external torque, friction will bring it to a halt, and it stops.

No problem, right? Inconceivable that after the stop, it would reverse the direction of motion, which it did just now.

You may not believe your eyes.

In fact, when I saw this first, I did not believe my eyes.

When I saw it again, I felt sick in the stomach and couldn't sleep all night.

I mean, look at this.

I'm rotating it, it comes to a halt.

Friction brings it to a halt, and it reverses direction.

Don't have sleepless nights about it, but give it some thought.

Is this a complete violation of the conservation of angular momentum of some kind? Think about it, and see you on Wednesday.