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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 19

We have here, going back to rotating objects...

I have an object here that has a certain velocity v, and it's going around with angular velocity omega, and a little later the angle has increased by an amount theta and then the velocity is here.

We may now do something we haven't done before.

We could give this object in this circle an acceleration.

So we don't have to keep the speed constant.

Now, v equals omega R, so that equals theta dot times R.

And I can take now the first derivative of this.

Then I get a tangential acceleration, which would be omega dot times R, which is theta double dot times R, and we call theta double dot...

we call this alpha, and alpha is the angular acceleration which is in radians per second squared.

Do not confuse ever the tangential acceleration, which is along the circumference, with a centripetal acceleration.

The two are both there, of course.

This is the one that makes the speed change along the circumference.

If we compare our knowledge of the past of linear motion and we want to transfer it now to circular motion, then you can use all your equations from the past if you convert x to theta, v to omega and a to alpha.

And the well-known equations that I'm sure you remember can then all be used.

For instance, the equation x equals x zero plus v zero t plus one-half at squared simply becomes for circular motion theta equals theta zero plus omega zero t plus one-half alpha t squared--

it's that simple.

Omega zero is then the angular velocity at time t equals zero, and theta zero is the angle at time t equals zero relative to some reference point.

And the velocity was v zero plus at.

That now becomes that the velocity goes to angular velocity omega equals omega zero plus alpha t.

So there's really not much added in terms of remembering equations.

If I have a rotating disk, I can ask myself the question now which we have never done before, what kind of kinetic energy, how much kinetic energy is there in a rotating disk? We only dealt with linear motions, with one-half mv squared, but we never considered rotating objects and the energy that they contain.

So let's work on that a little.

I have here a disk, and the center of the disk is C, and this disk is rotating with angular velocity omega that could change in time, and the disk has a mass m, and the disk has a radius R.

And I want to know at this moment how much kinetic energy of rotation is stored in that disk.

I take a little mass element here, m of i, and this radius equals r of i and the kinetic energy of that element i alone equals one-half m of i times v of i squared, and v of i is this velocity--

this angle is 90 degrees.

This is v of i.

Now, v equals omega R.

That always holds for these rotating objects.

And so I prefer to write this as one-half m of i omega squared r of i squared.

The nice thing about writing it this way is that omega, the angular velocity, is the same for all points of the disk, whereas the velocity is not because the velocity of a point very close to the center is very low.

The velocity here is very high, and so by going to omega, we don't have that problem anymore.

So, what is now the kinetic energy of rotation of the disk, the entire disk? So we have to make a summation, and so that is omega squared over two times the sum of m of i r i squared over all these elements mi which each have their individual radii, r of i.

And this, now, is what we call the moment of inertia, I.

Don't confuse that with impulse; it has nothing to do with impulse.

And this is moment of inertia...

So the moment of inertia is the sum of mi ri squared.

In...

So this can also be written as one-half I, I put a C there--

you will see shortly why, because the moment of inertia depends upon which axis of rotation I choose--

times omega squared.

And when you see that equation you say, "Hey, that looks quite similar to one-half mv squared." And so I add to this list now.

If you go from linear motions to rotational motions, you should change the mass in your linear motion to the moment of inertia in your rotational motion, and then you get back to your one-half mv squared.

You can see that.

So we now have a way of calculating the kinetic energy of rotation provided that we know how to calculate the moment of inertia.

Well, the moment of inertia is a boring job.

It's no physics, it's pure math, and I'm not going to do that for you.

It's some integral, and if the object is nicely symmetric, in general you can do that.

In this case, for the disk which is rotating about an axis through the center and the axis--

that's important--

is perpendicular to the disk--

that's essential--

in that case the moment of inertia equals one-half m times R squared.

And I don't even want you to remember this.

There are tables in books, and you look these things up.

I don't remember that.

I may remember it for one day, but then, obviously, you forget that very quickly again.

Needless to say, that the moment of inertia depends on what kind of object you have.

Whether you have a disk or whether you have a sphere or whether you have a rod makes all the difference.

And what also makes the difference--

about which axis you rotate the object.

If we had a sphere, a solid sphere, then...

So here you have a solid sphere, and I rotate it about an axis through its center.

Then the moment of inertia, I happen to remember, equals two-fifths mR squared if R is the radius and m is the mass of the sphere.

My research is in astrophysics.

I deal with stars, and stars have rotational kinetic energy.

We'll get back to that in a minute--

not in a minute but today--

and this is the one moment of inertia that I do remember.

If you have a rod, and you let this rod rotate about an axis through the center, and this axis is perpendicular to the rod--

the latter is important, perpendicular to the rod--

and it is length l and it has mass m, then the moment of inertia--

which I looked up this morning; I would never remember that--

equals 1/12 ml squared.

And all these moments of inertia you can find in tables in your book on page 309.

So the moment of inertia for rotation about this axis of a solid disk is one-half mR squared.

But it's completely different, the moment of inertia, if you rotated it about this axis.

So you take the plane of the disk.

Instead of rotating it this way, you rotate it now this way.

You get a totally different moment of inertia.

And most of those you can find in tables, but not all of them.

Tables only go so far, and that is why I want to discuss with you two theorems which will help you to find moments of inertia in most cases.

Suppose we have a rotating disk, and I will make you see the disk now with depth.

So this is a disk, and we just discussed the rotation about the center of mass.

And I call this axis I.

And so it was rotating like this and was perpendicular to the disk.

This is the moment of inertia.

But now I'm going to drill a hole here, and I have here an axis I prime which is parallel to that one.

And I'm going to force this object to rotate about that axis.

I can always do that--

I can drill a hole have an axle, nicely frictionless bearing and I can force it to rotate about that.

What now is the moment of inertia? If I know the moment of inertia, then I know how much rotational kinetic energy there is.

That's one-half I omega squared.

And now there is a theorem which I will not prove, but it's very easy to prove, and that is called the parallel axis theorem.

And that says that the moment of inertia of rotation about l prime--

provided that l prime is parallel to l--

is the moment of inertia when the object rotates about an axis l through the center of mass plus the mass of the disk times the distance d squared.

So this is the mass.

And that's a very easy thing to apply, and that allows you now in many cases, to find the moment of inertia in situations which are not very symmetric.

Imagine that you had to do this mathematically, that you actually had to do an integration of all these elements mi from this point on.

That would be a complete headache.

In fact, I wouldn't even know how to do that.

So it's great.

Once you have demonstrated, once you have proven that this parallel axis theorem works, then, of course, you can always use it to your advantage.

Notice that the moment of inertia for rotation about this axis--

which is not through a center of mass--

is always larger than the one through the center.

You see, you have this md squared; it's always larger.

There is a second theorem which sometimes comes in handy, and that only works when you deal with very thin objects, and that is called the perpendicular axis theorem.

If you have some kind of a crazy object--

which of course we will never give you; we'll always give you a square or we'll give you a disk...

But it has to be a thin plate.

Otherwise the perpendicular axis theorem doesn't work.

And suppose I'm rotating it about an axis perpendicular to the blackboard through that point.

I call that the z axis.

It's sticking out to you.

That's the positive z axis.

I can draw now any xy axis where I please, at 90-degree angles, anywhere in the plane of the blackboard.

So I pick one here, I call this x, and I pick one here and I call that y.

So z is pointing towards you.

Remember, I always choose a positive right-handed coordinate system.

My x cross y is always in the direction of z.

I always do that.

And so you see that here, x cross y equals z.

Now, you can rotate this thin plate about this axis.

You can also rotate it about that axis.

And you can also rotate it about the z axis.

And then the perpendicular axis theorem, which your book proves in just a few lines, tells you that the moment of inertia for rotation about this axis here is the same as the moment of inertia for rotation about x plus the moment of inertia for rotation about the axis y.

And this allows you to sometimes...

in combination with the parallel axis theorem to find moments of inertia in case that you have thin plates which rotate about axes perpendicular to the plate or sometimes not even perpendicular.

Sometimes you can use... if you know this and you know this, then you can find that.

So both are useful, and in assignment 7 I'll give you a simple problem so that you can apply the perpendicular axis theorem.

There are applications where energy is temporarily stored in a rotating disk, and we call those disks flywheels.

And the rotational kinetic energy can be consumed, then, at a later time, so it's very economical.

And this rotational kinetic energy can then be, perhaps, converted into electricity or in other forms of energy.

And there are really remarkably inventive and intriguing ideas on how this can be done.

Of course, whether it is practical depends always on dollars and cents and to what extent it is economically feasible.

But I have always, even when I was a small boy...

I remember when I was seven years, it already occurred to me that all this heat that is produced when cars slam their brakes--

all you're doing is you produce heat; you lose all that kinetic energy of your linear motion--

whether somehow that couldn't be used in a more effective way.

And this is what I want to discuss with you now and see where we stand.

This is actually being taken seriously by the Department of Energy.

So I want to work out with you an example of a car which is in the mountains and which is going to go downhill.

And the mountains are very dangerous--

zigzag roads--

and so he or she can only go very slowly.

And the maximum speed that the person could use is at most ten miles per hour--

without killing him or herself--

which is about four meters per second.

And so here is your car, and let's assume you start out with zero speed.

And let's assume that the mass of the car--

we'll give it nice numbers--

is just 1,000 kilograms.

And so you zigzag down this road.

Let us assume that the height difference h--

let's give it a number, 500 meters...

And you arrive here at point p.

And you later have to go back up again.

What is your kinetic energy when you reach point p? Well, you have a speed of four meters per second, and as you went down, you've been braking all the time.

One way or another, you got rid of your speed and that's all burned up--

heat, you heat up the universe.

So when you reach point p, your kinetic energy at that point p is simply one-half mv squared.

m is the mass of the car, so that is 500 times 16--

v squared--

so that is 8,000 joules.

Now compare this with the work that gravity did in bringing this car down.

That work is mgh, and mgh is a staggering number.

1,000 times ten times 500--

that is five million joules! And all of that was converted to heat using the brakes.

It actually even gives you also wear and tear on the brakes.

So who needs it? Is there perhaps a way that you can salvage it or maybe not all of it, maybe part of it? And the answer is yes, there are ways.

At least in principle there are ways.

You can install a disk in your car, which I would call, then, a flywheel, And you can convert the gravitational potential energy.

You can convert that to kinetic energy of rotation in your flywheel.

And to show you that it is not completely absurd, I will put, actually, in some numbers.

Suppose you had a disk in your car which had a radius of half a meter.

That's not completely absurd.

That's not beyond my imagination.

That's a sizable disk.

And I give it a modest mass--

so that the mass of the car is not going to be too high--

200 kilograms.

That's reasonable.

That would be a steel plate only five centimeters thick, so that's quite reasonable.

And the moment of inertia of this disk if I rotate it about an axis through the center perpendicular to the disk--

that moment of inertia, we know now, is one-half m...

oh, we have a capital M--

R squared, and that equals 25.

The units are kilograms, if you're interested, kilograms/meter squared.

So we know the moment of inertia.

Now, what we would like to do is we would like to convert all this gravitational potential energy into kinetic energy of that disk.

If you think of a clever way that you can couple that--

people have succeeded in that--

then you really would like one-half I omega squared...

You would really like that to be five times ten to the six joules.

And so that immediately tells you what omega should be for that disk, and you find, then, if you put in your numbers, which is trivial...

you find that omega is about 632 radians per second, so the frequency of the disk is 100 hertz, 100 revolutions per second.

I don't think that that is particularly extravagant.

So as you would come down the hill, you would not be braking by pushing on your brake, you would not be heating up your brakes, but you would somehow convert this energy into the rotating disk and that would slow you down.

So the slowdown, the "braking" is now done because of a conversion from your linear speed--

which comes from gravitational potential energy--

to the rotation of the disk.

And when you need that energy, you tap it.

So you should also be able to get the rotational kinetic energy out and convert that again into forward motion.

And if you could really do this, then you could go back uphill and you wouldn't have to use any fuel.

All your five million joules can be consumed, then, in an ideal case, and you would not have to use any fuel.

Now, you can ask yourself the question, is this system only useful in the mountains or could you also use this in a city? Well, of course you can use it in a city.

You wouldn't be braking like this, then, but again, you would slow down by taking out kinetic energy of linear forward motion, dump that into kinetic energy of rotation of your flywheel and that would slow you down.

And when the traffic light turns green, you convert it back--

rotational kinetic energy into linear kinetic energy--

and you keep going again.

Now, of course, this is all easier said than done, but it is not complete fantasy.

People have actually made some interesting studies, and I would like to show you at least one case that I am aware of, that I found on the Web, that shows you that United States Energy Department is taking this quite seriously.

This view graph is also on the 801 home page.

And so you see here the idea of mounting such a flywheel under the car here.

And it has the location of the "flywheel energy management power plant." Wonderful word, isn't it? And here you see a close-up of this flywheel.

I didn't get any numbers on it.

I don't know which fraction of the energy can be stored in your flywheel, but it's an attempt.

People are seriously thinking about it.

And it may happen in the next decade that cars may come on the market whereby some of your energy, at least, can be salvaged.

Instead of heating up the universe, use it yourself, which could be very economical.

I have here a toy car--

I'll show it on TV first.

And this toy car has a flywheel.

Do you see it? That the flywheel itself is the wheel of the car, but the idea is there.

In this case, I cannot convert linear motion into the flywheel.

I could do that, but I'm going to do it in a reverse way.

I'm going to give this flywheel a lot of kinetic energy of rotation, and you will see shortly how I do that.

And then I will show you that that can be converted back into forward motion--

in this case, it's very easy because the flywheel itself is the wheel.

So let me try to... power this car.

I do that with this plastic... okay.

So I'm going to put some energy into this wheel, into this flywheel, and then we'll see whether the car can use that to start moving.

Great that my lecture notes were there.

So, you see, it works.

And, of course, if you could reverse that idea, that when the car...

before it stops, get it back into the flywheel, then you have the idea that I was trying to get across.

Very economical, and definitely that will happen sometime in the future.

Flywheels are used more often than you may think.

MIT, at the Magnet Lab, has two flywheels which are amazing.

They have a radius, I think, of 2.4 meters--

that is correct--

and each one of those flywheels has a stunning mass of 85 tons, 85,000 kilograms...

and they rotate at about six hertz.

You can calculate the moment of inertia.

They rotate about their center axis perpendicular to the plane.

You know now what one-half I omega square is, and so you can calculate the kinetic energy of rotation.

And that kinetic energy of rotation is, then, a whopping 200 million joules in each of those rotating flywheels.

Now, they use this rotational kinetic energy to create very strong magnetic fields on a time scale as short as five seconds.

So they convert mechanical energy of rotation to magnetic energy, which is not part of 801 so I will not go into how they do that.

This is part of 802, and I'm sure all of you are looking forward to 802, and that's when you will see how you can convert mechanical energy into magnetic energy.

We have already seen a demonstration in class whereby we converted mechanical energy when someone was rotating, into electric energy.

I think that was you, wasn't it? And we got these light bulbs on.

Well, you can also convert it into magnetic energy.

And then when they have created these strong magnetic fields that they do their research with and when they want to get rid of them, they go the other way around and they dump that energy, that magnetic energy, back into the flywheels, who then start spinning again at six hertz.

Needless to say that huge amount of rotational kinetic energy must be stored in planets and in stars, and I would like to spend quite some time on that.

It's a very interesting subject.

I will first discuss with you the sun and the earth and see how much rotational kinetic energy is stored in the earth and in the sun.

This is also on the 801 home page, so don't copy this.

Let's first look at the sun.

We have the mass of the sun, we have the radius of the sun so you can calculate the moment of inertia of the sun.

I have used my two-fifths mR squared, which is really a crude approximation, because the two-fifths m R squared for a solid sphere only holds if the mass is uniformly distributed throughout that sphere.

With a star, that's not the case; not with the earth either,

because the density is higher at the center.

But this sort of gives you a crude idea.

So we have there the moment of inertia, which is easy to calculate with that two-fifths mR squared, and I get the same for the earth.

This is the radius of the earth, and you see the moment of inertia of the earth.

Now I want to know how much kinetic energy of rotation these objects have.

Well, the sun rotates about its axis in 26 days, the earth in one day, and so I finally convert everything to MKS units and I find these numbers for the rotational kinetic energy.

Now, look at the number of the sun--

1Ω times ten to the 36th joules.

Our great-grandfathers must have been puzzled about where the solar energy came from--

the heat and the light, where it came from.

And conceivably it came from rotation.

Maybe the sun is spinning down, is slowing down and maybe the energy that we get is nothing but rotational kinetic energy.

If that were the case, however, since the sun produces four times ten to the 26th watts--

four times ten to the 26th joules--

per second, it would only last 125 years.

So you can completely forget the idea that the energy from the sun that we now know, of course, is nuclear, but our great-grandparents didn't know that--

that the energy would be tapped from kinetic energy of rotation.

Let's look at the earth.

2Ω times ten to the 29th joules.

Well, let me try something...

some fantasy on you, some crazy, some ridiculous idea and I'm telling you first, it is ridiculous.

Remember that the world consumption...

Six billion people on earth consume about four times ten to the 20th joules every year.

So if somehow...
I thought if you could tap the rotational energy of the earth by slowing the earth down, maybe we could use it to satisfy the world energy consumption.

Um, I wouldn't know how to do it, and it is, of course, complete fantasy.

All you would have to do is slow the earth down by about... 2.4 seconds.

After one year...
So you slow it down.

After one year, the day wouldn't last...

Day and night wouldn't last 24 hours but only 2.4 seconds longer.

But, of course, after a billion years, then, you would have consumed up all the rotational kinetic energy and then the earth would no longer be rotating.

It is, of course, a crazy idea but sometimes it's cute to speculate about crazy ideas.

There is an object which we call the Crab Pulsar.

It is a neutron star and it is located in the Crab Nebula.

The Crab Nebula is the result of a supernova explosion that went off in the year 1054, and during my next lecture I will talk a lot more about that.

For now, I just want to concentrate on this neutron star alone.

And so here you have the data on the Crab Pulsar.

The mass of the Crab Pulsar is not too different from that of the sun.

It's about 1Ω times more.

The radius is ridiculously small--

it's only ten kilometers.

All that mass is compact in a ten-kilometer-radius sphere.

It has a horrendous density of ten to the 14th grams per cubic centimeter.

So, of course, the moment of inertia is extremely modest compared to the sun, because the radius is so small and the moment of inertia goes with the radius squared.

However, if you look at rotational kinetic energy, the situation is very different, because this neutron star rotates in 33 milliseconds about its axis.

So it has a phenomenal angular velocity.

And so if now you calculate one-half I omega squared, you get a fantastic amount of rotational kinetic energy.

You get an amount which is more than a million times more than you have in the sun.

And this object, this pulsar in the Crab Nebula is radiating copious amounts of x-rays, of gamma rays.

There are jets coming out of ionized gas, and we are certain that all that energy that this object is producing comes from rotational kinetic energy.

And I will give you convincing arguments why there is no doubt about that.

If you take the Crab Pulsar and you calculate how much energy comes out in x-rays and gamma rays and everything that you can observe in astronomy, then you find that it has a power roughly of about six times ten to the 31st watts.

It's a phenomenal amount if you compare that with the sun, by the way.

The sun is only four times ten to the 26th watts.

So the Crab Pulsar alone generates about 150,000 times more power than the sun.

We know the period of the pulsar to a very high degree of accuracy.

The period of rotation of the neutron star is 0.0335028583 seconds.

That's what it is today.

I called my radio astronomy friends yesterday and I asked them, "What is the rotation period of the neutron star in the Crab Nebula?" and this was the answer.

Tomorrow, however, it is longer by 36.4 nanoseconds.

So tomorrow, you have to add this.

That means it's slowing down.

The Crab Pulsar is slowing down.

That means omega is going down.

That means one-half I omega square is going down.

And when you do your homework, which you should be able to do--

to compare the rotational kinetic energy today with the rotational kinetic energy tomorrow--

you will see that the loss of energy is six times ten to the 31st joules per second, which is exactly the power that we record in terms of x-rays, gamma rays and other forms of energy.

So there's no question that in the case of this rotating neutron star, all the energy that it radiates is at the expense of rotational kinetic energy.

It's a mind-boggling concept when you think of it.

And if the neutron star in the Crab Nebula were to continue to lose rotational kinetic energy at exactly this rate, then it would come to a halt in about 1,000 years.

Now I would like to show you a few slides, and I might as well cover this up so that we get it very dark in this room.

I want to show you the Crab Nebula, and I think I will also show you the beautiful flywheels in the Magnet Lab.

Now I need a flashlight.

I need my laser pointer.

I need a lot of stuff.

Okay, there we go, so I'm going to make it dark.

You ready for that? Okay, if I can get the first slide.

What you see here are these flywheels at the Magnet Lab.

These are the wheels that have a mass of 85 tons and that have a radius of 2.5 meters--

an incredible, ingenious device, and you can store in there 200 million joules, and you can dump it into magnetic energy and in five seconds dump it back into kinetic energy of rotation.

It is an amazing accomplishment, by the way.

And here you see the Crab Nebula.

The Crab Nebula is at a distance from us of about 5,000 light-years.

It is the remnant of a supernova explosion in the year 1054--

much more about that during my next lecture--

and what you see here is not stuff that is generated at this moment in time by the pulsar.

This, by the way, is the pulsar, and the red filaments that you see here is material that was thrown off when the explosion occurred.

The explosion, the supernova explosion throws the outer layers of the star with a huge speed--

some 10,000 kilometers per second--

into space, and that is what you are seeing.

From here to here is about seven light-years to give you an idea of the size of this object.

This pulsar alone, however, generates the...
six times 31... watts.

And we do know that it is this star that is the pulsar and we know that it is not that star.

And the way that that was observed, that that was measured, is as follows.

A stroboscopic picture, a stroboscopic exposure was made of the center portion of the Crab Nebula.

And a stroboscopic picture means that you are using a shutter which opens and closes.

In this case, you have to open and close it with exactly the same frequency as the rotation of the neutron star.

This neutron star--

for reasons that is not well understood--

is blinking at us.

It blinks at us at exactly the frequency of its rotation, 33 milliseconds.

That means 30 hertz.

Roughly 30 times per second you see the star become bright and then go dim again.

If now you set your frequency of your shutter of your...

in front of your photographic plate at exactly that frequency and you expose the photographic plate only when the star is bright, then you will see a very bright star when you develop your picture.

If now you take another picture, expose it the same amount of time, but the shutter is open when the star is dim and you develop that picture, the star is dim.

But the beauty is that all other stars in the vicinity, of course, will show up on both photographic plates with exactly the same strength because they are not blinking at you, since they don't blink at us with a period of 33 milliseconds.

That is what you will see on the next slide, which is a stroboscopic exposure.

This star is clearly not the pulsar as it is about equally bright on both exposures.

This is not the pulsar, but this one is.

You see, this one is missing here.

And so this is beyond any question that we know exactly which the pulsar is.

A very new observatory was launched only recently, and that is called the Chandra X-ray Observatory.

And Chandra made a picture very recently of the Crab Nebula, of the pulsar, and that's what I want to show you now.

It's on the Web, and I show you a picture that many of you probably haven't seen yet, which is the center part of the Crab Nebula, and the pulsar is located here.

And all this is x-rays, nothing to do with optical light.

This is all x-rays, and you see there is a huge nebula here around this pulsar which is about two light-years across, and all that energy in x-rays is all at the expense of rotational kinetic energy of the pulsar, which is quite amazing.

And when this picture was made with Chandra X-ray Observatory, they discovered immediately that the pulsar also produces a jet.

Maybe you can see that from where you are sitting.

There is a jet coming out here, and with a little bit of imagination you can see this jet going out there.

All that energy is at the expense of rotational kinetic energy.

MIT has a big stake, by the way, in the Chandra Observatory, and not only MIT but Cambridge as a whole.

The Center for Astrophysics and MIT are running the Chandra Science Center, from which all radio commands are given, which is here just across the street, a few blocks away.

And many MIT scientists have dedicated the major part of their careers in this endeavor.

And these are one of the wonderful results that have come out.

All right, you now have five minutes left.

You have a little more--

you have seven minutes left.

I would appreciate it a lot if you fill out the questionnaire, because that's the only way we can get your feedback and we can make changes if you think these changes are necessary.

So, see you Friday.