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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 18

Here you see the topics that will be covered by this exam--

twice as much material as last time.

And, of course, the material is also more difficult.

I will touch upon most of these topics today--

not all--

and I can't go into great depth, for obvious reasons.

We don't have the time.

And I want to emphasize that what is not covered today does not mean at all that it will not be on the exam.

I want to ask your attention for these two key concepts: the work-energy theorem, which tells you that if an object moves from A to B that the work done on that object is the kinetic energy at point B minus the kinetic energy at point A.

This always applies both for conservative forces, like gravitational and spring forces, but it also holds for nonconservative forces such as friction.

Friction can remove kinetic energy.

It turns kinetic energy into heat, and that is perfectly fine in the work-energy theorem.

However, the conservation of mechanical energy only holds for conservative forces.

There you have the conservation of the sum of potential energy and kinetic energy, and now, of course, you cannot afford to lose kinetic energy through heat, because then the conservation of mechanical energy would not hold, and so that can only be used exclusively in the case of conservative forces.

Let's start with a simple example of an incline at an angle theta.

I have here an object, mass m, friction coefficient kinetic is mu k, and the static friction coefficient equals mu s.

This point here, let that be point A, and let the bottom of the incline be B, and let the distance between them along the slope be l.

And the first thing that you want to do with a problem like this, you want to make what we call a free-body diagram.

That means you want to draw all the forces on that object.

Clearly there is gravity, which is mg.

And then there is the normal force perpendicular to the surface.

The surface pushes up, and we call this N, the normal force.

The object wants to slide down.

Friction holds it up.

So there is also a frictional force.

And these are the only three forces that are on it.

So with a free-body diagram, you won't know more.

However, I want you to appreciate the fact that the force from the surface onto this object is, of course, the vectorial sum between these two, and that is what's called normally the contact force.

And that contact force better be exactly the same as mg, but in opposite direction, otherwise there could never be equilibrium.

Of course, we always split it into perpendicular directions because of the way that we analyze this.

There is no acceleration in the y direction, only in x direction when it starts moving, and that's why we split it.

But, of course, the vectorial sum is really the force with which the surface pushes onto that object.

All right, let's suppose now that we increase the angle theta until the object starts to slide.

So now I'm going to decompose these forces.

In the x direction I have here mg sine theta, and the component of gravity in the y direction equals mg cosine theta.

Since there is no acceleration in the y direction, the normal force must be also mg cosine theta.

I lifted up the incline to the point that it is just about to start sliding.

And so that means that the frictional force is a maximum value possible, which is the static friction coefficient times N, and therefore that is mu of s times mg times the cosine of theta.

And when will that happen? When the frictional force and mg sine theta are exactly equal.

If I go then one hair further, it will start to slide, and so that's the case when this equals mg sine theta, and so you lose your mg, and you find that mu s equals tangent theta.

That's when it will happen.

This is a way that you can measure the static friction coefficient.

Alternatively, if you know the static friction coefficient, you can predict at what angle that will occur.

Now, at that situation that it's just hanging by its thumbs, so to speak, by its fingernails, we touch it very lightly, we blow on it--

[imitates blowing]--

and it starts to slide.

And now I'm interested in knowing what the acceleration is downhill.

The reason why it will be accelerated, that this friction coefficient goes down now to mu k, and so now there is a t force along the slope in the plus x direction, and so I can write down now Newton's Second Law: ma, which is the force...

the net force in the positive x direction, would be equal mg times the sine of theta minus the frictional force, but now this becomes mu k mg cosine theta.

I lose my m, and so the acceleration in the positive x direction--

that's the way I defined positive x direction--

equals g times the sine of theta minus mu of k times the cosine of theta.

That's acceleration, and now a natural question would be, what is the speed at which it reaches point B? Well, what we would have done early on in the course, we would have said, "Well, if I know the acceleration, "and I know the distance, "and I know the initial speed is zero, "then clearly that distance l must be one-half a t squared, t being the time that it takes to go from A to B." And so t would be the square root of 2l divided by a.

What speed would it have at B? Well, the speed at B would be a t--

early on 801.

And so that would become, if we multiply this by a, the square root of 2al.

And if you want to see that in all glorious detail, then of course you'll have to take that a and you have to multiply that by 2l and take the square root.

So we get 2gl times the sine of theta minus mu of k times the cosine of theta and the whole thing to the power one-half.

That is the speed at point B.

I, however, would not do it that way.

I don't find it very elegant.

I would say, "Hey, why not apply the work-energy theorem?" Why not say that the work done when that object moves from A to B must be the kinetic energy at point B minus the kinetic energy at point A? Now, I know that I release it at zero speed, so this is zero.

And so the kinetic energy at B is clearly one-half m vB squared, so it comes down now to applying the work-energy theorem and calculating how much work was done by all the forces at stake--

conservative and nonconservative, it makes no difference--

and that puts it equal to one-half m vB squared.

Well, first of all, when it goes from point A to B, gravity is doing positive work.

It's going down, and the amount of work that gravity is doing is plus mgh, but h divided by l is the sine of theta, so this is also mgl times the sine of theta.

That's positive work done by gravity.

So we have mgl times the sine of theta--

that's the positive contribution by gravity--

and now we get a negative contribution by friction, because the frictional force is uphill, but the motion is downhill.

They're 180 degrees opposite, so I don't have to worry about the dot product, because the cosine of the angle is minus 1, and so I can ignore the dot and simply take that force--

the frictional force--

and multiply it by that distance l, but I have to, of course, take the minus sign, because the cosine of that angle equals minus 1.

And so I let l times the frictional force, which is mu k mg cosine of theta.

And that's the negative work done by the friction, and this equals one-half m vB squared.

And I lose an m, and if you look very carefully--

if you're willing to do that in your head--

you bring the 2 to the left, so you get 2gl, and then you get the g sine theta mu k cosine theta to the power one-half.

You get exactly the same result, and I find this somehow more pleasing to use the work-energy theorem.

So friction does negative work--

it's converted into heat.

But that's no problem for the work-energy theorem.

We know in life that friction takes kinetic energy out.

Whenever something is moving, it will come to a halt.

If you spin a top, that we know that the top will not last very long because of friction.

And I have here a top.

I hope you can see it shortly--

there it is.

It's a teeny-weeny little top.

It's very cute--

I play with it in my office.

I'm rotating it, and you see that friction is taking out kinetic energy and it's converting it to heat.

Please stay on the desk, will you? Will you please not fall on the floor? Ah, it does--

great.

So you see, that's what friction does.

Kinetic energy has been removed, and now it has been converted to heat.

This surface is a little smoother than the desk, and so I will give it a spin here, so it may last a little longer, but obviously it can't last very long, so very shortly, it, too, will fall over and come to rest, and that kinetic energy will have been converted into heat.

All right, so let's continue now on another subject, and the one that I want to talk to you about now is pendulum.

I have a pendulum, and the pendulum at time t equals zero.

This angle is theta zero, and I know what that angle is.

It's five degrees, which is approximately 0.09 radians.

At the time t equals zero, I give this pendulum a tangential speed.

I call it v of B, because I call this point B.

It's going to arc.

I call this A.

So this is a circle, and it comes to a halt, let's say here at point C.

And then this angle is the maximum angle possible, is theta maximum.

Let the length of the pendulum, for simplicity, simply be one meter.

It is small-angle approximation.

The angles will never be very large, as you will see later when we calculate theta maximum, and so we know that we're going to get a simple harmonic oscillation to a very good approximation.

So theta is going to be that angle theta maximum times the cosine--

or if you want to, be my guest, you can make this a sine; I always work with cosines--

omega t plus phi.

And omega equals the square root of g over l.

I will give you some equations during your exam, but this one I will not give you--

I just assume that you remember this--

and that the period of the pendulum equals 2 pi times the square root of l over g.

So we know omega, because we know g and we know l, and now a reasonable question is, what would be theta maximum? Well, if we knew this, then we would know what theta maximum is, because this part here equals l times the cosine of theta max, and so this is l minus l cosine theta max.

So it comes down to calculating how high this object comes above point A.

Well, I will split this into two heights, this one, which I call h1, and this one, which I call h2.

And so the one that we really want to know is what is h1 plus h2? Because h1 plus h2 will be l times 1 minus the cosine of theta max.

And so the moment we have h1 plus h2, we immediately have the maximum angle.

Now, h1 is a piece of cake, because we know that point B...

that this object is at B when theta zero is five degrees.

So h1 equals l times 1 minus the cosine of theta zero.

And you know theta zero.

And so you'll find, then, if you were interested in numbers--

but don't bother if you don't like numbers--

for the numbers that I gave you, this is 0.0038 meters.

It's only 3.8 millimeters.

It's a teeny-weeny little bit.

It's a large displacement in this direction but very little in this direction.

So we know already what h1 is.

So now what is h2? Well, we could now use the conservation of mechanical energy, and we could say, let us call arbitrarily the gravitational potential energy at point A, let's call that zero.

And going to apply the conservation of mechanical energy, which means that the kinetic energy at A plus the potential energy at A must be the kinetic energy at B plus the potential energy at B equals the kinetic energy at C plus the potential energy at C.

Now, the potential energy at A is zero because we define it that way.

The kinetic energy at C is zero because it comes to a halt.

So now we can write down that one-half m vA squared equals one-half m vB squared plus mg times h1--

that is the potential energy at point B--

it's higher than this point A--

vertical separation h1--

and that equals the potential energy at point C, which is mg times h1 plus h2.

And if you compare this part of the equation, you see that you lose mgh1, and so you'll find that m vB squared--

want to carry the one-half for now, we can still do that--

equals mg times h2.

And so you will find immediately h2.

h2 equals v of B squared, which is known.

In our case it must be a known number--

I will give you in a minute what that is--

and that is divided by 2g.

And this v of B that I had in mind--

numbers are really not that important; you don't have a calculator anyhow at your exam--

but I was going to give this a speed of 0.3 meters per second in this direction in order to keep the maximum angle quite modest.

So I know h2, and now that I know h2 and I know h1, I can go to this equation and I can ask, what is the cosine of theta maximum? h2, by the way, in case you're interested in numbers, is about 0.0045.

So it is 4.5 millimeters.

So it was originally 3.8 millimeters, h1, and now h2 is 4.5 millimeters higher.

So you can calculate the cosine of theta max.

However, I said to myself, if we use the conservation of mechanical energy, can we not also use the work-energy theorem? Why not? That should work as well.

The work-energy theorem tells me that the work done on that object when the object moves from A to B equals the kinetic energy at B minus the kinetic energy at A.

That is one-half m vB squared minus one-half m vA squared.

And how much work was done by gravity in going from A to B? Because gravity is the only force that does work.

The tension is not doing any work, because the tension, which is in this direction, is always perpendicular to the direction of motion.

And work is a dot product between force and the direction of motion, perpendicular to each other.

So that's zero.

So it's only mg that matters--

the gravitational force.

Well, when it goes from A to B, the work done by gravity is minus mgh1.

Object goes up, it does negative work, equals minus mgh1.

Now, look at that equation, and look, if you want to, here.

Compare this one with this one.

Completely identical.

So the work-energy theorem is, of course, in a way the same as the conservation of mechanical energy.

The work done when the object goes from B to C is the kinetic energy at C minus the kinetic energy at B.

That equals one-half m vC squared minus one-half m vB squared, and that is the work that gravity is doing when the object goes from B to C.

And that work is minus mgh2.

So this equals minus mgh2.

This is zero.

And so you see, what you see here is exactly what you see here.

The two are identical.

And so you could have used the work-energy theorem, or you can use the conservation of mechanical energy.

That makes no difference.

And so you now can calculate theta maximum, and for those of you who want some numbers, I found that theta maximum was plus or minus 7.4 degrees.

You always get two angles.

And in radians, that would be plus or minus 0.13 radians.

You know what the cosine of the angle is, so you always get two angles.

There's nothing you can do about it.

And so now you can ask yourself the question, what is phi? Phi is always a bit of a pain in the neck, and there is really not all that much physics in phi.

But I was sort of curious for these initial conditions what phi would be.

And so let's just take a quick look at that.

And so if we want to know what phi is, we have to look at the initial condition that at t equals zero the velocity at point B is plus 0.3 in this direction, and we know that theta equals theta zero, and we know what theta zero is--

that was the five degrees, that is the .09 radians.

And so I'm going to substitute that in my solution.

So I know when t equals zero, I know that theta equals theta zero.

So theta zero--

I know what theta zero is; we just calculated what theta max is; times the cosine of phi at t equals zero.

And so out pop two angles of phi, plus and minus phi 1.

You always find a plus and a minus sign because the cosine of plus the angle is the same as the cosine of minus the angle.

So now we have to find out which of the two it is.

By the way, when you find that theta maximum equals plus or minus 0.13 radians, you could have picked either plus or minus.

I picked the plus.

If you would have picked the minus, you would have found a different phase angle, but you can't pick minus.

I just want to remind you that I picked the plus in whatever follows.

And so now I have to take into account the fact that at t equals zero that I know the velocity vB.

And how does that come in? Well, I take the derivative of that equation there, and so I get that d theta/dt--

which is the angular velocity in radians per second at any moment in time, but we're going to evaluate it at t equals zero--

equals minus omega theta max times the sine of omega t plus phi, but we will evaluate it at t equals zero.

What is d theta/dt? Well, d theta/dt is v divided by l.

This is v divided by l and is therefore plus 0.3--

this plus because at t equals zero, the angle is increasing.

So it is plus.

How do we know it's v over l? Well, remember, we discussed that before.

If the pendulum changes the angle by an amount d theta, and if this arc here--

I call that ds--

and if the length is l then the definition of d theta--

the angle in radians--

is ds divided by l.

If you divide both sides by dt, which mathematicians cannot do, but physicists can, then you get d theta/dt equals ds/dt, and that is the velocity divided by l.

So you see that d theta/dt is v divided by l.

And so we now have a second equation.

We now can solve for sine phi.

That gives you again two angles.

That gives you an angle phi 2 and that gives you 180 degrees minus phi 2.

They have the same value for sine phi.

But only one of this will be the same as one of these, and that's the one that you pick.

And in my case, where for my value of plus 0.13, I find then that phi equals minus 0.82 radians.

It is about minus 47 degrees.

There's not much physics in the phase angle.

What is interesting perhaps is to mention that if we had chosen the speed at times t equals zero, if we had given the speed in this direction of 0.3 meters per second, the theta maximum would not have changed--

of course not--

but the phi would have changed.

In fact, the phi that you would have found would have been plus 0.82.

And for those of you who had preferred to use the theta maximum to take minus 0.13, they would have found a different phase angle altogether.

All right.

By now this top, of course, must be dead like a doornail.

So let's just take a look at that.

And I can't believe--

what's going on? Do you understand why it's still rotating? Why is friction not...?
Holy smoke! The basic foundations of physics are at stake again.

And it seems that every time at lectures, we seem to have a way to overthrow them.

Friction must take kinetic energy out.

I'm extremely puzzled.

Let's not look at it.

I hate this.

Things that I cannot explain, I hate them.

Let's just not look at it.

Let's go on.

We can always take a look at the end of the lecture and see what it's doing.

By that time it's got to be dead, right? It can't go on forever.

So, let's now talk about a spring.

What I just did with the pendulum, I can do something similar with a spring, similar in the sense that at time t equals zero I stretch the spring a little and I give it a kick and then I let it oscillate.

Very similar to what I just did, but now it is a simpler problem, because a spring is exactly one-dimensional.

Let this be the relaxed length l of the spring.

This is point A.

I stretch it to point B.

I give it a kick.

I will not put in any numbers now.

I'll give it a speed vB and it comes to a halt here at point C.

Let the spring constant be k, and we know we're going to get a...

so this is a t equals zero--

we're going to get a simple harmonic oscillation, x equals x max.

This is going to be x max.

This is going to be xB.

And this is x of A, which is zero.

So we're going to get x max times the cosine of omega t plus phi--

same old story, it's getting pretty boring.

Omega equals the square root of k over m, and T equals two pi divided by omega.

These equations you will not see on your exam.

What is now x max? How would you approach it? You've just seen how I did it with the pendulum.

I used first the conservation of mechanical energy--

there was no friction--

and then I showed you that it's completely consistent with the work-energy theorem, so what would you do now? How do we get x max? Any suggestions? Any proposals? Come on! You must have learned something from the last ten minutes.

Any idea? It's the same concept.

We know the initial conditions, and we know it's a simple harmonic oscillation, and we want to know what the maximum displacement is going to be.

Anyone with the courage to speak out? In the worst case, you're wrong.

Believe me, I am wrong so often, if only you knew.

[student responds]

LEWIN: So speak up.

Shall we try the conservation of mechanical energy? Is there anything wrong with that? Nothing wrong with it.

Let's try the conservation of mechanical energy.

The conservation of mechanical energy tells me that the kinetic energy at A plus the potential energy at A must be the kinetic energy at B plus the potential energy at B, and that must be equal to the kinetic energy at C plus the potential energy at C.

Right? There is no friction, there are no nonconservative forces, so this has got to work.

Well, the potential energy at A is zero, because remember, the potential energy of a spring is one-half k x squared, if x is the displacement from its relaxed position.

So this is zero.

When it comes to a halt here, this is zero.

When I watched this lecture, I noticed a slip of the tongue.

Clearly the potential energy at A is zero That's fine.

But at C, the object is standing still, so at C it is the kinetic energy that is zero and not the potential energy.

And I continued the problem correctly, because I wrote down here one-half k x max squared, which is exactly this term.

That is the potential energy at C, which reaches a maximum.

It is the kinetic energy that is zero here.

And I accidentally mentioned that it was the potential energy that is zero, which is obviously not so.

So we get that one-half m vA squared equals one-half m vB squared minus one-half k x of B squared--

that is the potential energy at this location--

and that equals one-half k x of C, which we have called x max, squared, and you are in business.

If I tell you where the object is--

with the pendulum, I gave you the initial angle, my theta zero--

I tell you where the object is, so you know x of B, I tell you what speed I gave it, so you know this one, you can calculate what the maximum displacement is.

Exactly the same idea, and if you want to apply the work-energy theorem, of course, you will get exactly the same result.

This is, in fact, the same thing.

So let's now turn to Newton's universal law of gravity, and believe it or not, I think I even give you on the exam the gravitational force of Newton's universal law of gravity.

I don't think you need it, but you will find it on the exam.

Let's take the Earth going around the sun, and let's approximate the orbit by a circle.

So here is the Earth, here is the sun, mass sun, and the orbital radius is capital R.

The Earth has an orbital velocity, I call it v orbital, and there is on the object the angular velocity, say, is omega.

It goes around with a certain angular velocity.

And there has to be here a force on the system, the gravitational force, which is the same as the centripetal force.

That's the only way the object can go around in a circle.

So this gravitational force equals mass of the sun mass of the Earth times G divided by R squared--

the force of gravity.

But it also must equal to the centripetal acceleration, and this is here the mass of the Earth, and so it also is the mass of the Earth times the orbital speed v squared divided by the orbital radius, and if you prefer to write that in terms of omega squared R, that's fine, too.

Look at this here.

You lose your Earth mass, if you want to know what your orbital speed is, you lose one R, and so you find that the orbital speed equals the mass of the sun times g divided by R to the power one-half, and that is about 30 kilometers per second.

I give it in kilometers per second, but of course when you calculate it, make sure you always stay in MKS before you make a conversion to other units.

So you know what the period is to go around the sun.

That is 2 pi R divided by the orbital, and that is about 365Ω days--

that's one year.

That's how long it takes us to go around the sun.

Now I wonder, what is the kinetic energy of the Earth in orbit around the sun? Well, that's completely trivial, you would say, because it is one-half of the mass of the Earth times the orbital speed squared.

Indeed, that is not very difficult.

So the kinetic energy of the Earth in orbit equals one-half mass of the Earth times v orbit squared.

So I take this equation and I square it.

I lose the half there, and so I get M earth M sun times G divided by R.

That is the kinetic energy in orbit.

But that...

oh, there's a half here.

Whenever I make a slip, you should always interrupt.

So there is the half here.

This is also minus one-half the potential energy.

And remember, the potential energy equals minus M sun M earth G divided by R.

So here you see plus half, here you see minus 1, so this is indeed minus half the potential energy.

What is the total energy of the Earth around the sun? That means the sum of potential energy and kinetic energy.

Well, the kinetic energy is minus one-half U, and the potential energy is U, and so this is...

this is plus one-half U.

That means it's negative, because U itself is negative.

It is also equal to minus K.

I just calculated for fun how large that number is--

it's a negative number--

how large that is, and it turns out to be approximately minus 2.7 times ten to the 33 joules.

Deeply negative.

Now, if we get bored and we want to leave the solar system, then we can ask ourselves the question, if we mount a rocket to the Earth and we fire that rocket, how much speed should we give it to get away from the sun once and for all and never come back? Well, all we would have to do is make the total energy of the Earth zero, so it can cruise all the way to infinity and have zero speed when it gets there.

How can I make this zero? Well, I have to add to the kinetic energy exactly K, because minus K plus K is zero.

If I have to add the kinetic energy K, then the new kinetic energy after the rocket burn would be exactly 2K.

The potential energy is not changing, because you fire the rocket when you're there, and so you're still at the same position after the rocket fire, and so your potential energy hasn't changed, but your kinetic energy must have doubled.

If it hasn't doubled, this doesn't become zero.

Aha! But if the kinetic energy has doubled, then the speed must have gone up by the square root of two, because kinetic energy is proportional to v squared, so the escape velocity that you need must be the square root of two times the orbital velocity, and that would be for the Earth the square root of two times 30, which is about 42 kilometers per second.

So by looking at the energy considerations, you can find the connection between total energy, escape velocity, and make the whole picture internally consistent.

We talked for one whole lecture about resistive forces.

I still remember the number of that lecture--

it was number 12--

because it's the one lecture that I worked on it more than on any other with the help of my graduate student Dave Pooley, who did these wonderful calculations, these numerical solutions.

And so let's talk a little bit about the resistive forces.

If an object moves through a medium--

air or a liquid--

there is a resistive force which always opposes the velocity.

It has two terms: k1 times v--

this is the speed, this is always positive--

plus k2 times v squared, and this is the unit vector of the velocity.

It's always opposing the velocity, so this is the magnitude of the resistive force.

We dealt exclusively with spheres--

remember? And we did some huge experiments.

For spheres, k1 equals c1 times r and k2 equals c2 times r squared.

And we did some experiments with corn syrup.

And this was a syrup for which c1 was approximately 160 and c2 was about 1.2 times ten to the third kilograms per cubic meters.

I always remember the units of this because that's always density.

In fact, it's always a little smaller than the actual density of the liquid, of the syrup itself.

This is a more complicated unit, but you can figure that one out for yourself.

And so we had ball bearings from an eighth-of-an-inch diameter to a quarter of an inch, and we dropped them in the Karo syrup.

If we take the one for now, which is a quarter-of-an-inch diameter, then the mass of that one is about 0.1 grams.

We knew the density of steel, and so you can figure out the mass, just take my word for it.

And so the question now is, if this ball bearing starts to fall, what is the speed that it will achieve? Well, in the beginning, there is only mg on that ball and a teeny-weeny little resistive force.

As the speed builds up, the resistive force will grow and there comes a time that it's equal to mg, and then the object will not be accelerated anymore and it will have what we call the terminal velocity.

And so what are the conditions for terminal velocity? That this term equals mg.

In other words, that c1 r v plus c2 r squared v squared becomes equal to mg, and when that's the case, then we have reached the terminal velocity.

Well, this is a quadratic equation in v of t.

You know c1, you know c2, you know the radius of these ball bearings, you know the mass of the ball bearings, so you can solve for the terminal velocity.

You will get two solutions; it's a quadratic equation.

One will be nonphysical, you will see, so you keep the one that is physical.

But if you did that, that would be a silly thing to do.

And the reason why that would be silly is that I claim that this term...

which by the way we call the pressure term, that this term can be completely ignored compared to this term, which we call the viscous term.

How do we know that that pressure term can be ignored? Well, let's first ask the question, when is the viscous term and the pressure term the same for what velocity? So forget for now that here are t's.

We just want to know that this term is the same as this term.

That will be the case when the velocity, which we call the critical velocity, equals c1 divided by c2 times r.

It's very easy to show that this term is then exactly identical to that one.

There's nothing critical about that number.

Really, it's a wrong name, critical velocity, but we give it that name.

It simply means that the two terms are equal--

that's all it means, nothing else.

If you calculate for a quarter-inch-diameter steel ball bearing, you can calculate r, you will find that the critical velocity is about 100 miles per hour, and we know very well there is no way when you drop a quarter-inch ball bearing into this syrup that it's going to come anywhere near that, so the speed will remain way below 100 miles per hour, and therefore this is a term that will dominate and this term can be completely ignored.

And if that term can be ignored, you can see that immediately the terminal velocity is going to be mg divided by c1r, and that, by the way, for our quarter-inch ball bearing, turns out to be something like two centimeters per second--

many, many orders of magnitude lower than the 100 miles per hour, which was the critical velocity.

What we did is we measured the times for these ball bearings to fall over a distance of four centimeters.

And if you know the time to go a distance of four centimeters, you know the terminal velocity.

And we know the mass of the ball bearing, we know the radii of the ball bearing, we know c1...

no, we didn't know c1.

I just wanted to point out to you that you can measure c1 that way.

You measure the speed, you know r, you know m, you measure c1, and that's what I did before the lecture that morning.

That's how I came up with that number 160, otherwise I wouldn't have known.

c1 is extremely temperature-dependent.

In fact, it even changed between classes.

If the temperature goes up, the value for c1 goes down.

So that's the way you can measure the value for c1.

Okay, conservation of momentum.

We dealt with the conservation of momentum in one lecture--

your parents were here--

and we only dealt exclusively with completely inelastic collisions.

Whenever we have a system of objects with zero net external force, the momentum must be conserved.

They can collide, they can cause fireworks, they can explode, they can break up in pieces, but momentum will be conserved in the absence of any net external force.

It's a very nonintuitive concept.

Kinetic energy can be destroyed, but momentum cannot be destroyed in the absence of a net external force on the system as a whole.

We have an object m1 here, speed v1, object m2, speed v2.

Let's make it very simple--

m1 is one kilogram, v1 is three meters per second, m2 is two kilograms, and v2 equals...

I make this five meters per second, and I make this three meters per second.

Momentum is conserved, so I can write down that m1 times v1 plus m2 times v2 must be m1 plus m2--

it is a completely inelastic collision--

times v prime.

They come together, they stick together, the total mass is going to be m1 plus m2, and they're going to fly either in this direction or in this direction, but whatever it is, I call it v prime--

it's after the collision.

You may say, "Hey, you forgot your arrows here," because this is a vector equation.

Yeah, I purposely forgot them, because this is a one-dimensional case.

You can always leave the arrows off, because the signs take care of it.

And you, of course, have to be generous, namely you have to put the minus signs in when needed.

mv is one times five, that is plus five.

m2 v2 is going to be negative, because I must now observe this as a minus three, so I get minus six, so the momentum before and after the collision must be minus one and that must be equal to three times v prime.

And v prime is therefore minus one-third meters per second.

And so this object will stick, and after the collision, they will move with a speed of one-third of a meter per second.

Kinetic energy was almost exclusively lost.

There was almost nothing left, remember? Clearly that's because of friction--

not external friction; there is no external friction, because if there were any external friction, momentum would not be conserved.

Think of them as the two car wrecks--

they go into each other.

Fireworks! Man, you hear them scratch, metal over metal.

A lot of heat is produced, kinetic energy is removed.

But momentum is conserved, because those frictional forces are all internal.

And that's fine.

So that is why it is such a very nonintuitive concept.

Let's now make sure that this crazy object has come to a halt.

If not, I... oh, I better not say what I was going to do.

My goodness! It's still rotating! Now, I realize that it may cause you sleepless nights, but on the other hand, with an exam coming up, I would suggest you just forget about this.

Pretend you haven't seen it.

Pretend this never happened.

That may make you happier.

Okay, see you Monday, and good luck.