MIT OpenCourseWare
http://ocw.mit.edu

8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 17

Remember, earlier in the course we measured the average speed of a bullet which we fired from a rifle.

That was because we had the ability of very fast timing.

In the old days, fast timing was not possible and people measured the speed of bullets in a very delicate way and all the tools that we have learned we can apply now to this device which we call the ballistic pendulum.

We have a pendulum with a very heavy object hanging here at the end--

I call it the block.

You see it here.

And this pendulum has length L.

Ours is about one meter.

I will give you the exact numbers later.

And we have a bullet of mass little m and the bullet comes in with velocity v.

It gets completely absorbed, sticks in there.

It's a completely inelastic collision, and the pendulum will then pick up the velocity v prime with the bullet inside.

The bullet is somewhere here.

Momentum is conserved, so we clearly have that, m v equals (m plus M) times v prime.

So if you could measure v prime, then you could measure the speed of the bullet, which is v.

How do we measure v prime? Well, we wait for the pendulum to come to a halt, let's say here, when the speed is zero.

When it was here, it had a speed v prime.

And we know that there was kinetic energy here--

no gravitational potential energy.

I can call this level U = 0, but right here, if this difference in height is h, then all the kinetic energy has been converted to gravitational potential energy.

So we apply the theorem, the work-energy theorem, or you could say... and it's equally valid, you could say we applied the conservation of mechanical energy.

And so this kinetic energy, which is one-half (m plus M) times v prime squared is now converted exclusively to gravitational potential energy, which equals (m plus M) times g times that h.

And we lose our (m plus M), so v prime would be the square root of 2gh.

And so all you would have to measure h, and then you know v prime, and if you know v prime, you know the speed of the bullet.

But life is not that simple.

It's very difficult to measure h, and I can make you see that.

Suppose this angle--

angle theta--

when it comes to a halt is only two degrees.

Then h, which is L times (1 minus cosine theta), is only 0.6 millimeters for the dimensions that I have chosen here for a length of one meter.

And you can't even see it--

it's invisible--

let alone that you can measure it to any degree of accuracy.

So what are we going to do now? Well, we are going to not measure h, but we are going to measure x.

I call this x = 0.

And here, when the pendulum comes to a halt, I call that x.

For a two-degree angle, x is approximately 3Ω centimeters.

It can easily be checked by you, of course.

So you get a huge displacement in this direction compared to h.

If you use small angle approximation--

and you better believe that two degrees is very small--

then you can prove, which is purely geometrical mathematics--

and I leave you with that proof--

that this is approximately x squared divided by 2L.

I want you to prove that.

You take the expansion of the cosine, the theories of the Taylor series, and you cut it off somewhere, and this is not so difficult to prove.

In other words, v prime squared, which is 2gh, can now be replaced by approximately 2g times x squared divided by 2L which is g times x squared divided by L.

And so the velocity of the bullet, v, which is (m plus M) divided by m--

I bring the m down there--

times v prime, but v prime is now the square root of this, so I get an x here times the square root of g over L.

And what you see now, but in a very clever way, by getting x in here, we can now do a quite accurate measurement of the speed of this bullet, because we can measure x with a fair accuracy--

maybe an uncertainty of only one or two millimeters in x out of the 3Ω or four or five centimeters.

We're going to measure the speed of such a bullet--

we did that before--

to get a number which is not too different.

I think we got something like 200 to 250 meters per second.

I will give you the input for this experiment.

The bullet mass is 2 plus or minus 0.2 grams.

I apologize that I don't give you all the numbers in mks units.

You will have to convert them, of course, to mks units.

The length of the pendulum is 1.13 meters plus or minus two centimeters--

we're not certain--

with an accuracy of about two centimeters.

And the mass of that block, which is huge, I believe is 3,200 grams, 3.2 kilograms, with an uncertainty of about two grams.

So we have a ten percent uncertainty in the mass, we have a two percent uncertainty in the length, and we have a negligible uncertainty in the mass of the block--

that's negligibly small.

So if I want to know now what the velocity is of the speed of the bullet, I can calculate what (m plus M) divided by m is and I can calculate the square root of g over L, and with those numbers I find 4.7 times ten to the third times x.

We have to measure x.

Now if you look at the uncertainties in the whole thing, we're going to measure x, which may not be 3Ω centimeters, it may be four or five--

but the uncertainty in our measurement will probably be one or two millimeters.

Let's say it's 0.2 centimeters.

So this is not our real number yet.

So that would be an uncertainty of about two out of 50, is four out of a hundred, is four percent.

This is four percent, roughly four percent.

So I would say we get the final speed of the bullet to an accuracy of about 15% if we combine all the uncertainties.

So let's give it a shot, no pun implied.

And we'll... we can make you see the pendulum right there very shortly.

And it is very cleverly designed.

When the pendulum starts to swing, it's going to move a small object.

There you have it.

I think I can turn this on again.

I think that's fine.

There is a very small wiper, which is this black wiper, and as the pendulum swings--

this is five centimeters, this is ten centimeters--

the wiper will stay at the largest extension when the pendulum swings back.

Okay.

When we fire bullets, it's always a little risky.

I have here the... the bolt.

Put the bolt in.

The bullet's in my pocket--

there's one.

That's right.

I take the bullet.

And I cock the gun.

Everything done right, yes? So you can look there and then see the swing of this very massive block, and the bullet will get absorbed in that block.

You ready for this? Three, two, one

[fires], zero.

I would say 5.2 centimeters seems to be about right, so x observed is about 5.2 centimeters, and we know the uncertainty of the 15% already, so I have to calculate now the 4.7 times ten to the third.

4.7 exponent third, multiplied by...

I go mks, of course, so that this .052, and that is 244 meters per second.

I remember last time we had something very similar.

So the speed of the bullet is about 244 meters per second.

It's a little under the speed of sound, which is 340 meters per second, and we came to the same conclusion last time.

All right.

We have kinetic energy in the bullet before the bullet hit the block, and you can calculate how much that is, because you know the speed now and you know the mass--

one-half mv squared.

You can also calculate how much kinetic energy there is when this bullet is absorbed in here.

That's very easy--

that's one-half times the total mass, (m plus M), times v prime squared, which you also know now.

And so you will see then, perhaps to your surprise, that if you compare the two, that 99.94% of all available kinetic energy before the collision was destroyed, and therefore was converted to heat.

That happened, of course... the heat was produced in that block.

Now I'm going to change to the concept of impulse.

It's not completely unrelated to what we just did.

An impulse is giving someone a kick, that's what an impulse is.

Our bullet gave an impulse to this block, it gave it a kick.

Impulse.

Impulse is a vector, and it is defined as the integral of F dt during a certain amount of time--

let's say from zero to delta t.

Now, F equals ma, which is also dp/dt--

we have seen this now several times--

the rate of change of momentum--

and so I can substitute that in here, and so I find then the integral from zero to delta t of dp/dt dt, and that makes me move to the domain of momenta, so I have now simply the integral over dp from some initial momentum, pi, to some final momentum, pf.

And so that is simply the final momentum minus the initial momentum.

So what an impulse does, it changes the momentum.

There is a force that acts on something for a short amount of time--

could be a little longer, as you will see with rockets--

and that gives it a change in momentum.

If we have an object that we drop on the floor, so we have an object, mass m, and we drop it on the floor and we let it fall over a distance h, then it's going to hit the floor with a certain speed--

we know it's down, the velocity, and that equals the square root of 2gh.

If this were a completely elastic collision, which depends, of course, on the quality of the object, and it depends on the quality of the floor--

maybe a super ball on marble would be almost completely elastic, then the ball would bounce back with that same velocity.

And if that were indeed a completely elastic collision, then you can see that the impulse that the ball is given to--

that is given to the ball as the ball hits the floor--

the floor is giving an impulse to the ball, and that impulse equals 2mv.

The ball changes its momentum.

It was first mv in this direction, and now it's mv in this direction, so the change is 2mv.

So an impulse is given to the ball.

Now, if the collision were completely inelastic, then the ball would just...

say, like a tomato, I throw a tomato on the floor, it goes

[splat].

No speed anymore when it hits the ground, then, of course, the impulse would only be mv, because then it doesn't come back up, so there is no... the change in momentum is then smaller.

We have here two balls that look alike.

They have a mass of 0.1 kilogram, so m equals 0.1 kilogram, and I will drop them from a height of about 1Ω meters, and that gives them a speed when they hit the floor of about 5Ω meters per second.

And so the momentum change is 2mv, so the impulse equals 2mv, is about 1.1, and that would be kilograms-meters per second.

And that means that if the collision time is delta t seconds, that the average force acting upon this ball during the collision with the floor equals the impulse divided by delta t, because remember, that was our definition of impulse.

So if we know the impulse, we get a feeling for the average force.

And for the ball that I will drop on the floor, we have done fast photography.

I will show you some results of the fast photography with a different ball, but nevertheless, we did it with the ball that I will drop on the floor very shortly--

which is this one--

that impact time is only two milliseconds.

It's hard to believe that in two milliseconds the entire collision occurs.

And so if you substitute in here now two milliseconds, then you get for the average force 550 newtons.

Just imagine, this ball has a mass of 0.1 kilogram, the weight is one newton, and during the impact, it weighs 550 times more.

What an incredible weight increase! And the average acceleration that it experiences during the impact is 550 times g.

People play tennis, and they have speeds of hundreds of miles per hour--

speeds are way higher than we have here, ten times higher--

and so the weight increase is even more.

Now, if the collision were completely inelastic, so that if it were a tomato or an egg, and this one wouldn't come up, the average force would still be approximately the same, the reason being that the impulse will be half.

But if the impact time were also half and the impulse were half, then, of course, the force...

the average force will be the same--

very high, but for a shorter amount of time.

So I want to show...

oh, I first want to show you now the... these two balls.

One is almost complete elastic collision with the floor.

Whether this is a complete elastic collision depends not only on this ball--

whether it is a super ball--

it also depends on the condition of the floor.

This is not a very good floor, this is not marble.

So when I drop this one, it doesn't come up to this point here.

So it's not a completely elastic collision, but it bounces pretty much.

So it is somewhere in completely inelastic and completely elastic.

It's not bad, right? It's not bad.

Now this one.

Watch it.

[splat]

Looks alike, but it ain't.

This one is completely inelastic.

It goes to the floor and it goes clunk.

You see a small bounce, but that's it.

And so the impact times are very short--

two milliseconds in the case of the one that bounces back, one millisecond in the case of the one that went clunk, and their average weight is about 550 times their normal weight.

I'd like to show you fast photography on not this very same ball, but on another one that we did.

Let me take this... this out.

And that is a ball that comes down with a speed of four meters per second.

And each frame is one millisecond, so you will see a ruler, and the ruler indicates...

has marks in centimeters, and so you will see it go...

in four milliseconds it will go one centimeter, so it has a speed of 2Ω meters per second.

It will hit the floor, and then we can count the number of milliseconds that it takes contact and going back up again.

It's not going to be two milliseconds, it's a little longer, but, again, impressively short.

All right, we're going to make it rather dark in order to get decent quality.

I'm going to turn five of these off and I'm going to set the TV at two, and now I will start this.

There we go.

Let's hope that that will go.

Okay, there comes the ball down.

These marks are in centimeters.

So there's a ruler in centimeters.

This is one centimeter.

I'll rewind a little, because we were a little too late.

Okay, let's start again.

So watch when it passes this mark--

one, two, three--

you see four milliseconds for about one centimeter.

So that's 2Ω meters per second.

And now we'll count the number of milliseconds at impact.

One, two, three, four, five, six, and it's off.

About six, maybe seven milliseconds.

And this is no special ball.

These impact times are amazingly short.

Now I have something very special for you--

something really special, something that has kept me awake--

a lot of things keep me awake in physics, and not only in physics...

[scattered laughter]

but this... but this is very special.

This is very special.

I have here a basketball.

[ball bouncing]

Not completely elastic, but not bad.

A tennis ball--

not completely elastic, but not bad.

Now I'm going to drop them together vertically down, and then this ball will bounce up somehow.

And so the question that I have for you is, do you think if I drop it from this height that this tennis ball will sort of come up at most to this height, or do you think it will be lower, or do you think it will be higher? So use your intuition.

[chuckling]: In the worst case it can be...

it can be wrong--

he is already pushing his finger up--

so what do you think? Will the tennis ball reach about the same height? Who is in favor of that? Who is in favor of higher?
Wow.

Who is in favor of a lot higher? Okay.

Okay, I'll try it.

Now, I cannot guarantee you that this ball will go straight up after the impact, because clearly that's impossible.

That has zero chance, so it probably go up in some direction.

But you will see the effect that I had in mind.

So there we go.

[balls bounce]

And you see that, indeed, that tennis ball...

goes way higher.

I'll try it once more to see whether I can get it to go up a little bit more vertically, but that is very difficult.

It goes way higher, and this is something that you should be able to calculate, and you can, and you will.

Believe me, it's part of assignment number six.

You haven't seen it yet.

There we go.

[ball bounces]

Oh, boy, that was better.

Well... oh! Do any one of you know approximately how much higher it goes, if this ball has way higher mass than this one? Of course, the mass ratio comes into it.

Any idea? Twice as high? You'll be surprised when you do assignment six.

Much higher.

Okay, in fact, you could even see it here all right that it was quite a bit higher than twice.

Great... a great experiment, and it's something you can do yourself in your dormitory.

Now I want to discuss in the remaining time about rockets.

A rocket experience an impulse from the engine, and that changes the momentum of the rocket.

But before we go into the details of the rocket, think about, again, this idea of throwing objects on the floor.

And let's turn to tomatoes.

I want tomatoes, because I want a complete inelastic collision, so tomatoes hit the floor, and it's not that I...

I'm not only going to throw one tomato on the floor, but I'm very angry today, I'm going to throw a lot of these tomatoes on the floor--

n, as in Nancy, tomatoes on the floor.

If one tomato hits the floor, the change of momentum is mv, if m is the mass of the tomato.

But I'm going to throw n on the floor, so the change of momentum is n, as in Nancy, times the mass of the tomato times v.

And this is the number of kilograms per second of tomatoes that I throw on the floor.

So this is a change of momentum.

And so this equals delta p divided by delta t, and that's an average force.

So the floor will experience a force in down direction.

Of course that was also the case here when the ball experienced a force up, which we calculated here.

The floor experienced, of course, the same force in down direction--

action equals minus reaction--

Newton's third law.

So the force experiences...

the floor experiences a force in down direction.

And I can write it down in a somewhat more civilized form: F equals dm/dt times the velocity, if the velocity of the tomatoes that hit the floor is constant.

And we're going to apply this to rockets, whereby the exhaust out of rockets has, relative to the rocket, a constant speed.

And this is then the number of kilograms per second that I throw on the floor.

This is very real.

I can throw these tomatoes on a bathroom scale, and if I threw four kilograms per second on a bathroom scale, and they hit the bathroom scale with five meters per second, you better believe it that you will see that the bathroom scale will reach an average force of about 20 newtons--

four times five.

If it weren't tomatoes, but if they were super balls which would bounce up, then the momentum change would be double, and so the bathroom scale would indicate 40 newtons.

So this is a real thing, it's a real force that you can record.

Now I'm going to be a little bit unpleasant to you.

I'm going to throw rotten tomatoes at you.

Here you are, and I have here a tomato, and this tomato has zero speed to start with.

Let me make you a little bigger, otherwise I won't even...

I won't even hit you.

So this is you.

And so I give this tomato a certain velocity, v of x.

The tomato hits you,

[splat].

Maybe it stays there, it's possible.

Maybe

[squish], it will drip down.

But in any case, the velocity in the x direction is gone.

So it hits you with a velocity vx and then v of x equals zero.

And it may make a mess.

You will experience a force.

If I keep throwing these tomatoes at you all the time...

and this is the force that you will experience--

and that force is, of course, in this direction.

You've got all these tomatoes, and you feel that as a force.

But now look at the symmetry of the problem.

Here the velocity goes from vx to zero.

But I, throwing the tomatoes, have to increase the velocity from zero to vx.

So for obvious reasons, I must then feel a force in this direction--

think of it as a recoil when you fire a bullet.

So I experience exactly the same force, but in this direction, and that now is the idea behind a rocket.

A rocket is spewing out tomatoes--

well, not quite tomatoes--

it's spewing out hot gas in this direction, and then the rocket will experience a force in that direction.

That is the basic concept behind a rocket.

And the higher the speed of the gas that it throws out--

the higher that velocity--

the more kilograms per second it spits out, the higher dm/dt, the higher will be the force on the rocket, and this force on the rocket is called the thrust of the rocket.

So if we have a rocket in space--

here's the rocket--

and the rocket is spewing out gas with a velocity, u, which is fixed relative to the rocket--

it's a burning of chemical energy.

Chemicals are burned, it comes out with a certain speed, and the rocket will then experience a force, which we call the thrust, and that is given by this equation.

If you know how many kilograms per second are spewed out, and you know what the velocity is, which I've called u here, this will tell you what the thrust is of that rocket.

If we take the case of the Saturn rockets that were used for the landing on the Moon...

Saturn.

For the Saturn rockets, the speed u was about 2Ω kilometers per second.

So the gas came out with 2Ω kilometers per second relative to the rocket, and the rate at which this gas was coming out is phenomenal--

15 tons of material per second.

dm/dt is about 15,000 kilograms per second--

it's almost unimaginable.

And that would give it then a thrust of about 35 million newtons, which, of course, was higher than the weight of the rocket, otherwise the rocket would never go up.

An incredible thrust.

I have also a nice problem for you in assignment six with the Saturn rockets.

You'll see these numbers again.

These are rounded off.

So rockets obtain an impulse from their engines, a force acts upon the rocket for a certain amount of time--

we call that the burn time--

but as they burn the fuel, the mass of the rocket goes down, because the fuel leaves.

And therefore the acceleration during the burn goes up, because the mass goes down.

And this makes it somewhat complicated to derive the velocity change that you get during the burn, during this impulse.

It's done in your book--

Ohanian.

I find that derivation a little bit complicated.

I looked in other books, and I found one in the book by Tipler on physics, and his derivation--

and I will go through that in general terms...

I put that on the Web for you.

It should be there tonight, so you may want to take very few notes.

The entire derivation, of which I will just highlight the important issues, is on the Web.

And the way that Tipler is doing it is exclusively from your frame of reference.

You're sitting in 26.100, and you are seeing this rocket go up.

Let's keep that equation, which is the thrust of the rocket, except that in the case of a rocket we would call this u, which is the speed of the exhaust relative to the rocket, and this is what we call the thrust.

So let's now take a rocket, which at time t as seen from your frame of reference--

where you are sitting.

I use v--

it's your frame of reference.

It's going up with a velocity v.

The mass of the rocket is m.

And now we're going to look at time t plus delta t.

The rocket has increased its speed, v plus delta v.

The mass is m--

let me do that in... not in color.

The mass is now m minus delta m, and here is a little bit of exhaust that was spewed out with velocity u relative to the rocket--

u relative to the rocket.

So you in 26.100 will see the velocity of this little piece delta m--

you will see this to be v minus u.

If the velocity of the rocket is larger than u--

could be--

you will see the exhaust going up from your frame of reference.

If the velocity of the exhaust is larger than the velocity of the rocket, from you frame of reference, you will see it go down.

That's taken care of in the signs.

Now I'm going to compare the momentum here with the momentum there, and we take the situation that there are no external forces acting upon it--

somewhere in outer space, this rocket burns.

Momentum must be conserved.

The momentum at time t equals m times v.

That is very simple.

That is time t.

The momentum at time t plus delta t of the entire system including the exhaust--

if I'm telling you that momentum is conserved, you can't ignore the exhaust.

It's momentum of the system that is conserved.

The momentum of the rocket is going to change and the momentum of the exhaust is going to change, but not of the system.

So we're going to get mass times velocity--

(m minus delta m) times (v plus delta v) plus delta m, which has a velocity v minus u.

And how large is this? Well, there is mv plus m delta v.

Here you see the m delta v, here you see the mv, and then you get minus u delta m, and the delta mv here cancels minus delta mv here, and this term, delta m delta v, is the product of two incredibly small numbers--

I ignore that.

So this is the momentum at t plus delta t and this is the momentum at time t, and so the change in momentum, delta p--

which must be zero because momentum is conserved--

is m delta v minus u delta m.

m delta v minus u delta m.

I can take the derivative of this equation, so I get on the left side dp/dt.

dp/dt is going to be zero, so I get zero equals m times dv/dt, but dv/dt is the acceleration of the rocket minus u dm/dt.

And that is the thrust on the rocket.

So what you see here is something that is very easy to digest--

that ma, which is the... this is the acceleration of the rocket, this is the mass of the rocket at time t--

equals the thrust of the rocket, and that equals u dm/dt.

And some people call this the... the "rocket equation." Now, this is true if there is no external force on the system.

It is interesting to include a real launch from Earth, and if you have a real launch from Earth, then the rocket is going up in this direction, but then gravity is exactly in the opposite direction.

In other words, only when you launch vertically from Earth would you have a thrust like this, and you would have mg like this.

In that case, this equation has to be adjusted, and then you get ma equals m thrust minus mg.

Only if you have a launch from Earth vertically up.

Now you have to do a little bit of massaging, and I will leave you with that massaging.

A couple of integrals are necessary to convert this into the final velocity of the rocket after the burn compared to the initial velocity.

And that part I will leave you with, but you will see that worked out in all detail on the notes that I left on the Web.

And then you come up with a very famous equation that the final velocity of the rocket minus the initial velocity of the rocket equals minus u times the logarithm of the final mass of the rocket divided by the initial mass of the rocket.

This is if there were no gravity at all.

Just in case, and only in case of a vertical launch from Earth, there is also here a term, minus gt--

only if you have a vertical launch from Earth.

When I watched my lecture, I noticed that in my enthusiasm I put brackets around the minus gt.

This is quite misleading, because it may give you the wrong impression that there's a product at stake here, which is not so.

So the brackets really should not be around the minus gt.

It is this term, minus u logarithm of mf divided by m of i minus gt.

Let's now look at this equation in a little bit more detail, so that we get a little bit of feeling for it.

Suppose we had a vertical launch from Earth, but we had no rocket.

It's possible.

So this term does not exist.

What do you see? That the velocity equals v initial--

we have called that before in 801 v zero--

that's the initial speed--

minus gt.

Ha! We had that result during our first lecture--

completely consistent with this equation.

If you have no rocket, you get that v equals v zero minus gt, if you throw the object vertically up and we had a vertical launch.

So that looks good.

We launch from the Earth, and we now have an initial speed which is zero.

The rocket is standing there, and we fire the rocket.

t, by the way, is the burn time of the rocket.

Let me write that down.

t is the burn time.

So initial speed is zero.

So now, this final velocity...

if we want this final velocity to be anything physically meaningful--

a positive number--

this thing has to come out positive.

And you will say, "But how can that be?" Because we have a minus sign here and we have a minus sign there.

How can that ever become positive? Well, don't forget that the final mass is smaller always than the initial mass, because you burn fuel, and so this logarithm is always going to be negative, and so this term is always going to be positive, if you burn any fuel.

Now, of course for this velocity to be a real value--

physically meaningful--

this term has to kill this one, has to be larger than this one, otherwise the rocket won't even go up.

You could be spewing out fuel all the time, and the rocket would just be sitting there, because the thrust up effectively is not enough to lift it off.

So the first term has to win from the second term in case of a vertical launch.

Let's take a example with some numbers--

it always gives a little bit of insight.

We have a burn time of about 100 seconds, and the initial speed is zero, and let us take a case whereby u is 1,000 meters per second.

That is less than the Saturn rocket, but it's still a sizable...

it's a kilometer per second.

So the exhaust is coming out relative to the rocket with one kilometer per second.

And the final mass divided by the initial mass of the rocket is 0.1, so 90% was burned away, the fuel that is gone.

You have only ten percent left when the burn is over.

So now we can calculate--

if this was a launch from Earth, a vertical launch up--

we can calculate how large this term is, take the logarithm of this value, multiply this by minus u, and then we find that the final velocity...

The first term is 2,300, and the second term, 100 seconds, and this is ten, is minus 1000.

So you pay a price for the gravitational field.

And so you have about 1.3 kilometers per second is the final speed of that rocket.

These are meters per second, and that is in kilometers per second.

Now, if there were no gravity, then of course the gain in speed--

this difference--

would be 2,300 meters per second.

Also, if there is no vertical launch but if, for instance, the rocket were in orbit around the Earth--

here we have the rocket and going in orbit around the Earth--

then, of course, even though there is gravity, gravity is now not doing any work.

And therefore if you fire this rocket for 100 seconds, the change in velocity, the tangential change, will also be 2,300 meters per second.

You cannot say now there is gravity and therefore we have to take the gt term into account.

It will be the same for this equation.

If you had an object going in this direction, this minus gt term wouldn't be there either.

It's only, of course, when you deal with a vertical motion.

It is very important to realize, and very non-intuitive, that when you burn a certain amount of fuel for a given amount of time you obtain a fixed change in the velocity.

This is fixed for a given amount of fuel.

The change in kinetic energy is not fixed, and I'll give you some numbers so that you can immediately check that for yourself.

And it is very non-intuitive.

It is, in fact, a mistake that is made by many physicists who think that if you burn the same amount of fuel of the same rocket for the same amount of time that the increase in kinetic energy is a given.

That is not true.

It's the change in velocity that is a given.

But suppose that v final minus v initial is 100 meters per second.

That's just a given.

I have a rocket, I have a certain amount of fuel, I burn it, and that is my change in velocity.

I start off with v initial zero, so my kinetic energy increase is one-half m times 100 squared, which is ten to the fourth.

That's what I got out of this burn.

Now I use the very same rocket, the same amount of fuel, the same burn time, so I get again that the final velocity minus the initial velocity is still 100.

That's exactly the same.

But before the burn, this rocket had an initial velocity of 1,000 meters per second.

What is now the gain of kinetic energy? The final velocity is now 1,100.

That's non-negotiable, because the rocket changes the momentum changes the velocity by a fixed amount.

So now the gain of kinetic energy, the increase of kinetic energy equals one-half m times 1,100 squared minus 1,000 squared.

This is the new velocity and this was the old one.

And this number is one-half m times 200,000.

This could be in joules, and this is also in joules.

This number is 20 times higher.

So you see that the change in velocity was exactly the same.

The rocket burned the same amount of time, the same amount of fuel, but the kinetic energy increase is way more if the rocket has a higher speed to start with.

We here in 26.100 made our own rocket.

It's a very down-to-earth model, no pun implied.

But it is quite powerful, and I would like to show it to you, because we are quite proud of it.

As a rocket, we use a fire extinguisher--

carbon dioxide--

and this fantastic invention I have here.

And this powerful rocket is enough to reach the escape velocity of...

[loud whooshing]

...of 26.100.

[students laughing]

I almost reached the escape velocity, but I crashed.

See you next Friday.