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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 13

All right...

long weekend ahead of us.

One more lecture to go.

If I have an object, mass m, in gravitational field, gravitational force is in this direction, if this is my increasing value of y, then this force, vectorially written, equals minus mg y roof.

Since this is a one-dimensional problem, we will often simply write F equals minus mg.

This minus sign is important because that's the increasing value of y.

If this level here is y equals zero, then I could call this gravitational potential energy zero.

And this is y...

Then the gravitational potential energy here equals plus mg y.

This is u.

So if I make a plot of the gravitational potential energy as a function of y, then I would get a straight line.

This is zero.

So this equals u...

equals mg y, plus sign.

If I'm here at point A and I move that object to point B, I, Walter Lewin, move it, I have to do positive work.

Notice that the gravitational potential energy increases.

If I do positive work, the gravity is doing negative work.

If I go from A to some other point--

call it B prime--

then I do negative work.

Notice the gravitational potential energy goes down.

If I do negative work, then gravity is doing positive work.

I could have chosen my zero point of potential energy anywhere I please.

I could have chosen it right here and nothing would change other than that I offset the zero point of my potential energy.

But again, if I go from A to B, the gravitational potential energy increases by exactly the same amount--

I have to do exactly the same work.

So you are free to choose, when you are near Earth, where you choose your zero.

Now we take the situation whereby we are not so close to the Earth.

Here is the Earth itself.

Of course you can also replace that by the sun if you want to.

And this is increasing value of r.

The distance between here and this object m equals r.

I now know that there is a gravitational force on this object--

Newton's Universal Law of Gravity--

and that gravitational force equals minus m M-Earth G divided by this r squared, r roof so this is a vectorial notation.

Since it is really one-dimensional, we would...

Just like we did there, we would delete the arrow and we would delete the unit vector in the positive r direction and so we would simply write it this way.

The gravitational potential energy we derived last time equals minus m M-Earth G divided by r--

and notice, here is r and here is r squared--

and if you plot that, then the plot goes sort of like this.

This is r, this is increasing potential energy--

all these values here are negative--

and you get a curve which is sort of like this.

This is proportional to one over r.

Now, of course, if the Earth had a radius which is this big, then, of course, this curve does not exist, it stops right here.

If I move from point A to point B, with a mass m in my hand, notice that the gravitational potential energy increases.

I have to do positive work, there is no difference.

If I go from A to another point, B prime, which is closer to the Earth, notice that the gravitational potential energy decreases.

I do negative work.

If I do positive work, gravity is doing negative work.

If I do negative work, gravity is doing positive work.

Right here near Earth, where this one-over-r curve hits the Earth, that is, of course, exactly that line.

That dependence on y is exactly the same as the dependence on r and then you can simplify matters when you are near Earth.

When the gravitational acceleration doesn't change you get a linear relation.

But that's only an exceptional case when you don't move very far.

The gravitational force is in the direction opposite the increasing potential energy.

Notice that when I'm here, the gravitational force is in this direction.

Increasing potential energy is this way.

The force is in this way.

When I'm here, gravitational potential energy increases in this way; the gravitational force is in this direction.

When I'm here, the gravitational potential energy increases in this direction.

The gravitational force is in this direction.

When I'm here, the gravitational potential energy increases in this direction.

The gravitational force is in this direction.

The force is always in the opposite direction than the increasing value of the potential energy.

If I release an object at zero speed, it therefore will always move towards a lower potential energy because the force will drive it to lower potential energy.

Now I change from gravity to a spring.

I have a spring which is relaxed length l--

I call this x equals zero--

and I extend it over a distance x and there's a mass m at the end and there will be a spring force.

And that spring force...

F equals minus kx.

It's a one-dimensional situation so I can write it without having to worry about the arrows.

There is no friction here.

It's clear that if I hold this in my hand, that the force Walter Lewin equals plus kx.

It's in the direction of increasing x.

If I call this point A at x equals zero and I call this point B at x equals x, then I can calculate the work that I have to do to bring it from A to B.

So the work that Walter Lewin has to do to bring it from A to B is the integral in going from A to B of my force, dx.

It's a dot product, but since the angle between the two is zero degrees, the cosine of the angle is one, so I can forget about the fact that there's a dot product.

I move it in this direction.

So that becomes the integral in going from zero to a position x of plus x plus kx dx, and that is one-half k x squared.

And this is what we call the potential energy of the spring.

This is potential energy.

It means, then, that we...

At x equals zero, we define potential energy to be zero.

You don't have to do that, but it would be ridiculous to do it any other way.

So in the case that we have the near-Earth situation of gravity, we had a choice where we put our zero potential energy.

In the case that we deal with very large distances, we do not have a choice anymore.

We defined it in such a way that the potential energy at infinity is zero.

As a result of that, all potential energies are negative.

And here, with the spring, you don't have a choice, either.

You choose...

at x equals zero, you choose potential energy to be zero.

So if you now make a plot of the potential energy as a function of x, you get a parabola, and if you are here, if the object is here, then the force is always in the direction opposing the increasing potential energy.

If you go this way, potential energy increases.

So it's clear that the force is going to be in this direction.

If you are here, the force is always in the direction opposing increasing potential energy.

Increasing potential energy is in this direction, so the force is in this direction.

You see, it's a restoring force.

The force is always in the direction opposite to increasing potential energy.

If you release an object here at zero speed, it will therefore go towards lower potential energy.

The force will drive it to lower potential energy.

If we know the force--

in this case, the spring force or in those cases, the gravitational force--

we were able to calculate the potential energy.

Now, the question is, can we also go back? Suppose we knew the potential energy.

Can we then find the force again? And the answer is yes, we can.

Let's take the situation of the spring first.

We have that the potential energy u of the spring equals plus one-half k x squared and if I take the derivative of that versus x then I get plus kx, but the force, the spring force itself, is minus kx, so this equals minus the spring force.

So we have that du/dx equals minus F, and I put the x there because it's a one-dimensional problem--

it is only in the x direction.

The minus sign is telling you that the force is always pointing in the direction which is opposite to increasing values of the potential energy.

That is what the minus sign is telling you.

It's staring you in the face what I have been telling you for the past five minutes.

If we have a three-dimensional situation that we know the potential energy as a function of x, y and z, then we can go back and find the forces as a function of x, y and z.

It doesn't matter whether these are springs or whether it is gravity or whether it's electric forces or nuclear forces, you then find that du/dx equals minus F of x, du/dy equals minus Fy and du/dz equals minus Fz.

What does this mean? It means that if you're in three-dimensional space, you move only in the x direction.

You keep y and z constant, and the change equals minus Fx.

That gives you the component of the force in the x direction.

You move only in the y direction, you keep x and z constant, and then you find the component of the force in the y direction.

We call these partial derivatives, so we don't give them a "d" but we give them a little curled delta.

If we go back to the situation where we had gravity, we had there the situation near Earth.

We had u... was plus mg y, so what is du/dy? This is a one-dimensional situation so I don't have to use the partial derivatives.

I can simply say du/dy.

That is plus mg, and notice that the gravitational force was minus mg.

Remember? The minus sign is still there.

It's still there.

And so you see that here, indeed, du/dy is minus the gravitational force.

Now we take the situation that we are not near Earth--

we have there--

so we have u equals minus m M-Earth G divided by r--

there's only an r here--

so du/dr...

The derivative of one over r is minus one over r squared.

The minus sign eats up this minus sign, so I get plus m M-Earth G divided by r squared so the gravitational force...

the gravitational force equals minus that.

It is minus du/dr and, indeed, that's exactly what we have there--

minus that value.

So whenever you know the potential as a function of space, you can always find the three components of the forces in the three orthogonal directions.

Suppose I have a curved surface--

literally, a surface here in 26.100, which sort of looks like this...

something like this.

I call this, arbitrarily, y equals zero and I could call this u gravitational potential energy zero, for that matter.

So this is a function y as a function of x and the curve itself represents effectively the gravitational potential energy.

This is y and this is x.

So the gravitational potential energy u equals mg y, but y is a function of x, so that is also u times m...

excuse me, that is m times g times that function of x.

There are points here where du/dx equals zero.

I'll get a nice mg in here...

Where's zero...

and where are those points? Those points are here, here, here, here and here.

If du/dx is zero, it means that the force--

the component of the force in the x direction--

is zero, because du/dx is minus the force in the x direction.

So if we visit those points, for instance here, then there is, of course, gravity, mg, if there is an object there in the y direction...

in the minus y direction and there is a normal force in the plus y direction and these two exactly cancel each other.

So the net result is that here, here, here, here and there there is no force on the object at all so the object is not going to move, it's going to stay put.

Well, yes, it's going to stay put.

However, there is a huge difference between this point here and that point there and you sense immediately that difference.

If I put a marble here, I will have a hell of a time to keep the marble in place, because if there is a fly there in the corner of 26.100 which does something then the slightest amount of force on this one and it will start to roll off.

In fact, what will happen is it will go to a lower potential energy.

Here, however, if this one is offset, then it will want to go to a lower potential energy.

The force is always opposing the direction of increasing potential energy, so the force will drive it back and so that's why we call this a stable equilibrium.

It will always go back to that point.

And this is an unstable equilibrium.

We have a setup here, and I would like to show you how that will work.

So we do have something that is a curved object.

It's a track.

Let me give you a little bit better light condition.

So you see there, there is that object, a little ball.

And no surprise, if I offset it from the lowest point that it will be driven back to that point--

that's trivial.

What is less trivial is that there is a point here whereby, indeed, the net forces are zero and it is not easy to achieve that, but I will try to put it there so that it, indeed, stays put.

I'm not too fortunate, it is very difficult.

I'm trying... no...

Yeah! Did it.

It's there, it's very unstable.

I blow...

oh, it's not so unstable.

And there it goes.

So you see, that's the difference between stable equilibrium and unstable equilibrium.

At the stable point, the second derivative of the potential energy versus x is positive.

At the unstable point, the second derivative is negative.

I'm going to return now to my spring and I'm going to show you that if you use the potential energy of the spring alone that you can show that an object that oscillates on a spring follows a simple harmonic motion.

So here...

is u as a function of x, and this is the parabola that we already had which equals one-half kx squared.

Let the object be at a position x maximum here.

It's going to oscillate between plus x max and minus x max.

When it is at a random position x, there is a force on it and the force is always in the direction opposing the increasing potential energy so the force is clearly in this direction.

It's being driven back to equilibrium.

When it is there, it will have a certain velocity.

The velocity could either be in this direction or it could be in that direction.

It has a certain speed and since spring forces are conservative forces, I can now apply the conservation of mechanical energy.

We call this a potential well.

The object is going to oscillate in a potential well.

Of course, it doesn't oscillate like that.

It really oscillates like this, of course.

It's a one-dimensional problem.

The total energy that I started with if I release it here at zero speed equals one-half k x max squared.

That is e total.

That will always be the same if there is no friction of any kind and I have to assume that there is no friction.

That must be, now, one-half m v squared at a random position x, plus one-half k x squared, which is the potential energy at position x.

So this is the kinetic energy and this is the potential energy.

v is the first derivative of position versus time, so I can write for this an x dot.

And now what I'm going to do, I'm going to rewrite it slightly differently.

I'll bring the x dot squared to one side, my halfs go away and I divide by m, so I get plus k over m times x squared, and then I get minus k x max squared equals zero.

Did I do that right? Yes, I divide...

Oh, there's an m here, and the m has to be here.

And now what I'm going to do, I'm going to take the time derivative of this equation.

Now you will see something remarkable falling in place.

Just for free.

I take the derivative versus time.

That gives me a two x dot, but I have to apply the chain rule so I also get x double dot, the second derivative--

it's the acceleration--

plus I get a two k over m times x, with the chain rule gives me an x dot.

This is a constant, that's the total energy when I started, so the whole thing equals zero.

I lose my two, I lose my x dot because it's zero and what do I find? x double dot plus k over m equals zero.

And this makes my day because I know this is a simple harmonic oscillation.

You've seen this equation before.

We derived it in a different way.

We didn't use forces today.

We only used the concept of mechanical energy, which is conserved.

We know the solution to this equation...

There is an x here.

I heard someone mention the x--

thank you very much.

The solution is: x equals x max times the cosine omega t plus phi.

This is the amplitude.

And omega equals the square root of k over m, and the period for one oscillation equals two pi divided by omega.

We were able to do this.

We were able to apply the conservation of mechanical energy because spring forces are conservative forces.

So you've seen, in a completely different way, how you arrive at the same result.

Now I'm going to try something similar to another potential well and that potential well is a track and the track is a perfect circle.

And I'm going to slide down that track an object mass m, and I'm going to evaluate the oscillation along a perfect circular track.

And to make it as perfect as I can, I even have here a pair of compasses...

make it a...

that's the track.

And the track has a radius R, and at this moment in time the angle equals theta, and here is the object.

I call this x equals zero.

That is also, of course, where theta equals zero.

This is increasing value of y, and I choose this y equals zero.

And so the gravitational potential energy of this object is its own mg y, so I have to know what this y is, and therefore I have to know what this distance is.

That's very easy.

This one here equals R cosine theta and so this one is R minus R cosine theta.

So the potential energy equals mg times R one minus cosine theta, if I choose zero there.

I'm free to change that but that's, of course, a logical thing to do here.

Notice if theta equals zero and the cosine theta equals one, then you find u equals zero.

That's, of course...

I have defined it.

That's the way I defined my y equals zero.

So u equals zero.

Notice that when theta equals pi over two--

if the object were here--

that you find that the potential energy u equals mg R.

That's exactly right, because then the distance between here and the y zero is R, so this is the potential energy as a function of angle theta.

The velocity of that object as a function of theta is given by R d theta/dt.

And I can make you see that very easily.

Let this be the angle d theta so it moves in a short amount of time over an angle d theta and the arc here is dS, and the radius is R.

The definition of theta--

that's the definition of theta which is in radians--

is that dS divided by R equals d theta.

That's our definition of radians.

So I take the derivative, the time derivative, left and right, so I get the dS/dt--

which, of course, is the tangential velocity along that arc--

equals R times d theta/dt, for which you can write R theta dot.

d theta/dt... d theta/dt is sometimes called omega, which is the angular velocity, but keep in mind that in this case the angular velocity omega--

if you want to call this omega, which is the angular velocity--

is changing with time.

The angular velocity is zero when you release it and is a maximum when it goes through the lowest point.

So I can now apply the conservation of mechanical energy, because I know what the velocity is at any angle of theta and I know what the kinetic...

what the potential energy is.

So, let the total energy be just the mechanical energy which depends on my initial conditions wherever I start.

Maybe it's just that I release it here with zero speed; maybe I give it a little speed.

It is a number, it is a constant.

So that is going to be one-half mv squared at a random angle of theta.

And that means this is v, so that is R squared times theta dot squared.

This is simply one-half mv squared, nothing else, so this is the kinetic energy.

Plus the potential energy, which is mg times R times one minus cosine theta.

And this is always the same, independent of the angle of theta, because gravity is a conservative force.

So this is the conservation of mechanical energy.

This angle cosine theta is really a pain in the neck, and therefore what we're going to do is something we have seen before--

we are going to make a small angle approximation...

small angle approximation.

And we're going to write for cosine theta one minus theta squared divided by two.

That is a very, very good approximation.

That approximation is way better than the one we did before when we simply said the cosine of theta equals one.

Remember we did that once? We said, "Oh... for theta is very small.

The cosine of theta is about one." If we did that now, we would be dead in the waters, because if we said the cosine of theta is one, this becomes zero and you end up with nonsense, because it would say that the mechanical energy is changing all the time because this velocity is changing all the time.

So we cannot do that.

We would kill ourselves if we did that.

The approximation is really amazingly good.

If I give you here theta in radians and I give you here the cosine of theta and here I give you one minus theta squared over two, then if I take 1/60 of a radian--

and I pick 1/60 since that is approximately one degree.

But I pick exactly 1/60--

and I ask what the cosine is, that is 0.999.

And then I have an 861114.

I just used my calculator.

Then I calculate what one minus theta squared over two is and I find 0.999861111.

That is very, very close.

That is only... it only differs by three parts in a billion. That is very close.

That means the difference between the two is only one-third of a millionth of a percent.

Suppose now I go a little rougher and I go to one-fifth of a radian, which is about 12 degrees, so this is very roughly 12 degrees.

Then the cosine of theta equals 0.98007, and one minus theta squared over two equals 0.98000.

So that still is amazingly close--

that is, only differs by seven parts in 100,000, so the difference is less than 1/100 of a percent.

So with this in mind, I feel comfortable to pursue my conservation of mechanical energy.

And I'm going to replace this cosine theta by one minus theta squared divided by two.

So I will continue here--

the center blackboard is always nice, you can see it best--

and I will massage that equation a little further and, of course, you can already guess what I am going to do when I massage it a little further.

I'm going to take the time derivative just as I did in the case of the spring.

So we are going to get that the mechanical energy--

which is not changing--

equals one-half m R squared theta dot squared plus mg R.

Cosine squared becomes one minus theta squared over two so we have a minus times minus becomes plus so I get simply theta squared over two.

And now I take the time derivative...

for this becomes zero...

equals...

Now, I get a two out of here, which eats up this one-half, so I get m R squared, then I get theta dot, but the chain rule gives me theta double dot.

Excuse me? Anything wrong? I don't think so, thank you.

So I have to take the derivative of this one.

The two flips out, which eats up this two, so I get mg R and then I get a theta.

With the chain rule, gives me a theta dot.

I lose my m, I lose one R, I lose my theta dot--

I picked the wrong one; I lose my theta dot, not the theta--

and what do I find? That theta double dot plus g over R times theta equals zero.

And I couldn't be happier, because this tells me that the motion is that of a simple harmonic oscillation.

And the solution is x...

excuse me, not x.

Theta equals some maximum angle for theta.

It's the amplitude in angle times the cosine of omega t plus phi.

This is the angle of frequency.

It has nothing, nothing to do with that omega there, which is the angular velocity, which is changing in time.

This is a constant, this is angular frequency and this omega equals the square root of g over R, and so the period of the oscillation is two pi times the square root of R over g.

And when you see that, you say, "Hey! I have seen that before." Where have we seen this before?
Almost a carbon copy of something that we have seen before.

What is it?

[class murmurs]

LEWIN: Excuse me, speak louder.

[echoing class]: Pendulum! We had a pendulum whereby we had length l of a massless string.

We had an object m hanging on the end and what was it doing? It was going along a perfect arc which is exactly identical.

The problem is the same, it's not a surprise, because now we have a surface which is an exact, perfect arc.

It's a circle, we have no friction.

We assumed with the pendulum that there was no friction either.

So it shouldn't surprise us that you get exactly the same period that you had with the pendulum and...

except that, of course with the pendulum, what we called l is now R.

Gravity is the only force that does work and so it is justified to use the conservation of mechanical energy because gravity is a conservative force.

We used the small angle approximation to make it work.

In the case of the spring, we had that the potential energy was proportional to x squared and out came a perfect simple harmonic oscillation, no approximation necessary.

Now we forced this potential energy...

we forced it into being dependent on theta squared.

That's really what we did.

You see, that is the term of the potential energy that you have there.

And by the approximation of cosine theta being one minus theta squared over two, we forced this term to become quadratic in theta and therefore now, with that approximation it becomes a perfect simple harmonic oscillation.

Now comes a key question.

I said, "Gravity is really the only force that does work." Is that true? There's no friction for now.

Is that really true? When we had the pendulum, it's true there is gravity.

That's clear.

There's a gravitational force, which is mg, but there is also tension.

We never mentioned that.

We didn't even talk about it when we did the conservation of mechanical energy.

When the object is here, sure, there is gravity and sure, there is no friction.

So there is no force along the arc, but there must be a normal force.

Is the tension not doing any work? Is the normal force not doing any work? Did we, perhaps, forget something? Remember last week, I put my life on the line.

I was so convinced that the conservation of mechanical energy was going to work that I almost killed myself--

not quite--

with this huge, 15Ω kilogram pendulum that I was swinging.

I believed in the conservation of mechanical energy and I overlooked the tension.

Is it possible that the tension does, perhaps, positive work? If that's the case, I could have died. What is the answer? Is the tension doing any work and in the case of my circular track is, perhaps, the normal force doing any work? What is the answer?

[class murmurs]

LEWIN: I want to hear it loud and clear! CLASS: No.

LEWIN: No! Why is it no? Why is it not doing any work? Because what?

[student answers]

Exactly. You got it, man! That's it.

The force is always perpendicular to the direction of motion.

And since work is a dot product between force and the direction that it travels, neither the tension nor the normal force does any work.

So don't overlook the force, but do appreciate the fact that they don't do any work.

Great! So now I'm going to show you a demonstration which I find one of the most mind-boggling demonstrations that I have ever seen.

We do have a circular track.

You have it right in front of you.

That is a circle, although you may not think it is, but it is.

And that circle has a radius which, according to the manufacturer, is 115 meters with an uncertainty of about...

I think it's about five meters.

It is extremely difficult to measure and even during transport, you think it could change.

Let me try to clean this a little better.

And so the radius of this... the radius of curvature of our arc, which is also an air track, equals 115 plus or minus five meters.

So we can calculate now what the period of oscillations is.

The whole track is five meters long.

So half the track is about 2Ω meters, so the angle theta maximum is approximately 2Ω meters--

which is half the length of the track--

divided by 115 and that is an extremely small angle.

That is about 1.2 degrees, because this is in radians and this is in degrees.

So the angle is very small, so we should be able to make a perfect prediction about the period.

And I am going to do that.

I take two pi times the square root of R over G and R is 115, 115...

I divide it by G.

I take the square root, I multiply by two.

I multiply by pi and I get 21.5.

T--

and this is a prediction...

equals 21.5.

The uncertainty in R is about 4.3%.

Since we have the square root of R, that becomes 2.2%.

So if I multiply that by .022, I get an uncertainty of about 0.47.

Let's call this 0.5 seconds.

So this is a hard prediction what the period of an oscillation should be--

21.5 plus or minus a half second.

Now I'm going to observe it and we're going to see what we're going to...

how this compares.

I don't want to... I don't want to oscillate it ten times.

That will take three, four, five minutes--

that's too long.

It is not really necessary because my reaction time is 0.1 second, so even if I did only one oscillation, that would be enough to see whether it is coincident with that...

consistent with that number.

However, it is such a beautiful experiment.

It's so much fun to see that object go back and forth in 21 seconds, that I will go...

For your pleasure and for my own pleasure, I will go three oscillations.

Not that it is necessary, but I will do it.

3T is going to be something plus or minus...

and this is my reaction time, which is 0.1 second, and then we can all divide that by three and then, of course, the error will go down by a factor of three, and we will see whether this number agrees with this one.

All right, can you imagine someone making a track like this...

air track with a radius of 115 meters? I mean, what is this? This may be eight meters.

115 meters! That is something like ten times higher...

more--

15 times higher than this ceiling.

Amazing that people were able to do that.

In fact, nowadays, you can't even buy this anymore.

This is probably some 50 years old, if not older.

I have to get the air flowing out of all these holes.

There are many, many small holes in here that you cannot see.

The air is now blowing.

And this object is going to be put on here and just because of gravity, it will go.

That's all it is--

only gravity will do work.

Here's the timer and we're going to time it.

I will start it off first and then when it comes back to a stop, I will start to time because that's, for me, a very sharp criterion.

When the object comes back and comes to a halt here, it's very easy for me to start the timing.

You may notice, as you watch, that some of the amplitude will decrease because there is...

hold it, hold it, hold it! Because there is, of course, a little bit of friction.

It's very little, but it is not zero.

Enjoy this, just look at it.

Isn't this incredible? It just goes simply by gravity.

It's like a pendulum which has a length of 115 meters.

It's about to complete its first oscillation.

It goes back...

Actually, some of you may be able to see the curvature.

You can really see that it is not straight.

So we're coming up to the second.

I better get back in position.

So when it stops here, it has made three complete oscillations.

Sixty-four point zero five.

Let me turn this off.

So 3T equals 64.05.

I'm lazy--

64.05, I divide that by three.

That is 21.35, plus or minus .03.

That's exactly in agreement with the prediction, with the uncertainty of the prediction.

I have something very similar, and that is, again, a curved track.

It's not...

Oop, I hope I can retrieve that ball.

It would be nice.

Hmm, what happened? Boy! You have to be...

Gee, what's happening here? Oh, yeah, I got it, got it, got it.

Phew! Tricky to make a hole in here.

This is an arc, not unlike this one.

There's more, a little bit more friction and, in this case, the radius is 85 centimeters.

So we can calculate what the maximum angle is.

The radius is 85 centimeters and the arc to the edge is about 20 centimeters.

So we have now a situation like this.

R equals 85 centimeters and this here is approximately 20 centimeters, so theta maximum is roughly 20 divided by 85 and that is something like 13 degrees.

13 degrees is not a bad situation because the difference between the cosine theta and one minus theta squared over two is less than 1/100 of a percent, it is that small.

So I can make a prediction of the period of this oscillation, predict and you can go through exactly the same exercise.

You take two pi times the square root of R over d and you find 1.85.

The uncertainty of this radius is, of course, not very large but we are not certain about the radius to about one centimeter, so it's 85 plus or minus one centimeters.

So that's about a 1.2 percent error and so the error, then, in the prediction will be 0.6 percent; it's about .01 seconds.

So I expect...

this is my prediction.

Now, I really want to challenge this .01 and so now I'm going to make the observations and surely I'm going to do it now 10 times, because then the uncertainty will be 0.1 seconds--

that's my reaction time--

and so I have the final period to an accuracy of .01 seconds and so we can compare these numbers directly and that is what I will do now.

I have here the timer and I'm going to oscillate that back and forth--

and that would only take 20 seconds--

zero it, we started here.

We have great confidence in physics, right? We believe in physics.

We believe in the conservation of mechanical energy.

Starts... are you counting? Is this two? Yeah? Is this three? Four? CLASS: Four.

LEWIN: I don't believe you.

Okay, we start again.

Now! One, two, three, four, five, six, seven...

I'm getting nervous.

Eight, nine, ten.

Holy smoke! 22.7 seconds! It should have been 18! 22.7 seconds.

There must be something fundamentally wrong with the conservation of mechanical energy.

Or is there something else? And what is the difference between the two experiments? STUDENT: Friction.

LEWIN: Excuse me? STUDENT: Friction.

LEWIN: Oh, no, the friction is so low, that is not the reason.

There's a huge difference.

Think about it when you take your shower this weekend.

There is a huge difference between this object moving and that object moving and when you find out, that is the reason why that is way slower, not friction.

See you next Wednesday.