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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 9

The exam on Wednesday will cover our first five lectures and the first two homework assignments.

And so I list here the topics the way we discussed them.

Of course, it is not possible to discuss all of them today but I will make a selection.

I recall that we discussed scaling and we used the interesting example of Galileo Galilei--

an animal, and the animal has legs.

And we defined the overall size of the animal as yea big--

we called that "s." And then we said, well, there is here the femur and the femur has length l and thickness d.

It was completely reasonable to say well... that l will have to be proportional to S.

If an animal is ten times larger than another its legs will be typically ten times longer.

Since the mass of the animal must be proportional to its size to the power three it will also be proportional to the length of the femur to the power three, and then came in this key argument--

namely, you don't want the bones to be crushed.

Which is called "yielding" in physics.

If I take a piece of concrete, a block of concrete, and I put too much pressure on it, it starts to crumble.

And that's what Galileo Galilei may have had in mind.

And in order to protect animals who get bigger and bigger and bigger against this crushing, we argued--

and I will not go through that argument now anymore--

that the mass will have to be proportional to d squared, which is the cross-section of the femur.

And so, you see immediately that d squared has to be proportional to l to the third so d must be proportional to the length of the femur to the power one and a half.

So this would mean that if you compare an elephant with a mouse the elephant's overall size is about 100 times larger than a mouse.

You would expect the femur to be about 100 times larger, which is true.

But you would then expect the femur to be about 1,000 times thicker and that turns out to be not true, as we have seen.

In fact, the femur of the elephant is only 100 times thicker, so it scales just as the size.

And the answer lies in the fact that nature doesn't have to protect against crumbling of the bones.

There is a much larger danger, which we call "buckling." And buckling is the phenomenon that the bones do this and if now you put too much pressure on it the bones will break.

And if that's the case, you remember that, in fact, all you have to do is you have to scale d proportional to l, which is not intuitive--

that's not so easy to derive--

but that's the case.

And so the danger, then, that nature protects animals against is this buckling, and when the buckling becomes too much then, I would imagine, the bones, at some point in time--

well, these are tough bones, aren't they?--

[snaps]

will break, and that's what nature tries to prevent.

So that was a scaling argument.

And let's now talk about dot products.

If I look there...

I scan it a little bit in a random way over my topics, so let's now talk about dot products.

I have a vector A...

Ax times x roof, which is the unit vector in the x direction, plus Ay y roof plus Az Z roof.

So these are the three unit vectors in the x, y and z direction.

And these are the x components, y and the z component of the vector A.

I have another vector, B.

B of x, x roof, B of y, y roof, B of z, z roof.

Now, the dot product...

A dot B--

also called the scalar product--

is the same as B dot A and it is defined as Ax Bx plus Ay By plus Az Bz.

And it's a number.

It is a scalar, it is a simple number.

And so this number can be larger than zero--

it can be positive--

it can be equal to zero, it can also be smaller than zero.

They're just dumb numbers.

There is another way that you can define...

You can call this method number one, if you prefer that.

There is another way that you can find the dot product.

It would give you exactly the same result.

If you have a vector A and you have the vector B and the angle between them is theta, then you can project B on A--

or A on B, for that matter, it makes no difference--

and that projection...

the length of this projection is then, of course, B cosine theta.

And so A dot B...

and that is exactly the same.

You may want to go through a proof of that.

It is the length of A times the length of B times cosine of theta.

And that will give you precisely the same result.

What is interesting about this formulation, which this lacks, that you can immediately see that if the two are at 90-degree angles or 270 degrees, for that matter, then the dot product is zero.

So that's an insight that you get through this one which you lack through that other method.

Let us take a down-to-earth example of a dot product.

Suppose A equals 3x and B equals 2x plus 2y, and I am asking you, what is the dot product? Well, you could use method number one, which, in this case is by far the fastest, believe me.

Ax is 3 and Bx is 2, so that gives me a 6.

There is no Ay, there is no Az, so that's the answer.

It's just 6--

that's the dot product.

You could have done it that way.

It's a little bit more complicated but I certainly want to show you that it works.

If this is the x direction and this is the y direction--

we don't have to look into the z direction because there is no z component--

then this would be vector A and this point would be at 3.

B... this would be 2, and this would be 2 and so this would be the vector B.

And it's immediately clear now that this angle... 45 degrees.

That follows from the 2 and the 2.

So if we now apply method number two, A dot B.

First the length of A, that's 3, times the length of B, that is 2, times the square root of 2--

this is 2, this is 2, this is 2... square root 2--

times the cosine of 45 degrees, which is one-half square root 2, and the answer is 6.

Notice that this square root of 2 and this square root of 2 equal just 2, and you get 6.

You get the same answer, of course.

But it would be a dumb thing to do it since it can be done so much easier.

On cross products...

I don't want to go through the formalism of cross products the way we did that with the determinant.

I just want to remind you that if you have a cross product of two vectors, that is minus B cross A, and that the magnitude of C is the length of A times the length of B times the sine of the angle between them.

The vector C, the dot product...

the cross product is always perpendicular to both A and perpendicular to B.

In other words, it's perpendicular to the plane of the two vectors.

Now, if it's perpendicular to the plane, then in that case, it's perpendicular to the blackboard.

You have two choices: it's either coming at you perpendicular or it's coming right straight into the blackboard.

And now everyone has his own way of doing it.

I taught you what's called "the right-hand corkscrew" rule.

You take the first one that is mentioned--

in this case, A--

and you rotate it over the shortest angle to B.

When you do that, you rotate your corkscrew--

seen from where you're sitting--

counterclockwise.

Then the corkscrew comes to you.

And so the direction of the vector is such that you will see the tip of the vector as though it's coming straight out of the blackboard.

And so that gives you, then, the direction.

Now I will give you the position x of an object as a function of time and then we're going to ask ourselves a lot of questions about velocities, accelerations, sort of everything you can think of, everything we have covered--

speeds...

And I will cover here four seconds of time.

So this is the time axis in seconds and we will cover four seconds.

So let this be one, two, three, four.

And let the object be at position plus six.

This is my x-axis, this is where the object is actually moving, and this is three, and here is minus three and this, let's say, is in meters.

Let's make a little grid so that's easier for me to put in the curve.

All right, so now comes x as a function of t.

The time t equals t seconds.

The object is here and it came from there.

And this part is a parabola and this parabola here is horizontal.

It's important, you have to know that, so this is a parabola and this, here, is horizontal.

So the object goes from plus six to three, then it goes to minus three, then it stays there for one second and then it goes back in one second to plus six.

It's a one-dimensional problem.

The motion is only in the x-axis, along the x direction.

Well, let's analyze all these different seconds that occur.

Let's first take the first second, during the first second.

Since this is a parabola, you know that the acceleration is constant so I hope you will conclude immediately that a must be a constant.

If a is a constant, the position x as a function of time should change as follows: x zero plus v zero t plus one-half a t squared.

I expect you to know this equation.

Very often do I give you equations at the exam and that may well happen during the second and the third exam, but it will not be the case this time.

The equations are all very fundamental and you have to make them part of your world.

So this is an equation that you will have to remember.

All right, what is the velocity here? The velocity starts out to be zero and the velocity here is not zero anymore.

If I look at time t equals one, then I have here x zero or six.

It starts out with velocity zero--

that's a given.

And I get plus one-half times a t squared but this is only one second, and so when it is at three, I have six plus one-half a times one squared, and so you find that a equals minus six meters per second squared.

So during this first second the acceleration is minus six meters per second squared.

And the velocity, v, as a function of time, is the derivative of this one, is v zero plus at.

V zero was zero, and so that is minus six times t.

So the velocity is changing in a linear fashion.

What do I know about the end of the first second? Well, I can say that x is three.

What do I know about the velocity? Well, the velocity is minus six.

What do I know about a? I don't... I don't know about a.

It's true that during this first second a is minus six meters per second squared, but it changes abruptly at this point so it's ill-defined at this point.

In fact, it's actually nonphysical.

So I really don't know exactly at the end what the acceleration is.

Let's now go to the second second and let's see what happens there.

The second second.

And first let's look during, and then we'll look at the situation at the end.

During the second second, it is clear--

since this is a straight line--

that the velocity remains constant and it remains minus six meters per second.

That is exactly what it was at this point at the end.

You can see it go six meters--

from plus three to minus three--

in one second so the velocity is minus six meters per second.

The acceleration is therefore zero.

You see that the acceleration changes abruptly from minus six meters per second squared to zero so I can't tell you what it is exactly at this moment in time.

So that's the situation during the second second.

And what is the situation at the end of the second section... second second? At the end, I know that x equals minus three.

What is the velocity? I don't know, because it changes abruptly here from minus six to zero, so I don't know exactly what it is at that point.

It's a nonphysical thing, it's a very abrupt change.

And the acceleration, yeah, that's also a very tricky thing, because if the velocity is minus six on this side of the two seconds and here becomes zero, and if that happens in a split second, there must be ahuge acceleration just at that point which is nonphysical.

So I would also put a question mark at the a...

I don't know what the a is.

So we'll go to the third second... this part.

Let's first look during the third second.

Well, the object isn't going anywhere, it's just sitting there.

x remains minus three and the velocity is zero and a is zero--

We can agree on that.

What is the situation at the end of the third second? That means t equals three.

Well, all I know is that x is minus three.

That's nonnegotiable.

What the velocity is, I don't know, because it's changing abruptly from zero to a positive value.

So that's ill-defined and the same is true for the acceleration.

There is a sudden change in velocity.

That means there must be a huge acceleration.

It's unknown, ill-defined because this curve is, of course, not very physical.

Let's now look at the last second.

This is the fourth second.

First, during.

Well, it's going from minus three to plus six and it's a straight line, so the velocity is constant.

If the velocity is constant then you can immediately conclude that a is zero--

there is no acceleration--

and it goes nine meters in a time span of one second.

But it's now plus--

plus nine meters per second.

So the object first went from positive values of x to zero and to negative values for x.

During all that time, the velocity was negative by our sign convention and now the velocity, it goes back to plus six.

The velocity becomes plus nine meters per second.

What is the story at the end of the fourth second? Well, all I can say is that x equals plus six.

I don't know much more.

I don't know what the velocity is.

Neither do I know what the acceleration is.

The plot stops there, anyhow.

Now, I would think that it is reasonable to ask the following question: What is the average velocity, for instance, between time zero and time four? Average velocity.

We define average velocity as the position at time four seconds minus the position at time zero, divided by four.

That is our definition.

At zero, it is at plus six, at four, it is at plus six.

So the upstairs is zero, so the average velocity during this four-second trip is zero.

You may not like that, it may go against your intuition.

Of course! I couldn't agree more with you but that's the way we define velocity.

Speed is defined differently.

Speed is the magnitude of the velocity vector and the speed, therefore, always has a positive value.

And I will show you now what is the average speed between time zero and four.

That is the distance that it has traveled in these four seconds.

Well, let's first go through the first second.

It goes from plus six to plus three.

So it already travels three meters.

Then in the second second it goes from plus three to minus three so it travels another six meters.

And then in the third second it's lazy, it doesn't do anything, so the distance traveled is zero.

And then in the last one second it gets very active and it travels nine meters.

Notice you only see plus signs here.

There are no minus signs, it would make no sense.

And this occurs in four seconds, so that is 4.5 meters per second.

So the average speed is 4.5 meters per second, but the average velocity is zero.

We could now make a plot of the velocity as a function of time.

Let me put here the 4.5.

I just have enough room here to make the velocity plot as a function of time.

I'll make a new one.

This is my time axis, and this is the velocity.

This is zero.

One second, two seconds, three seconds, four seconds.

And this velocity is in meters per second.

I go up here to plus ten and here is minus five, here is minus six.

So, what do I do now? I know that the velocity during the first second is minus six t so it's linear.

And so during the first second this is the velocity as a function of time.

It starts at zero, you can see that, and when it is here it has a velocity of minus six meters per second.

During the second second it remains minus six meters per second.

So during the second second, the velocity is not changing.

It stays there.

During the third second the velocity jumps all of a sudden to zero--

you see how nonphysical that is.

And so all of a sudden, during the third second it becomes zero.

So there has to be somehow a connection, of course, between the two to make this physical.

So in a very small amount of time that will have to occur.

That's why you get ahuge acceleration here at that point.

Of course, you also get an acceleration here at this point, because there's also a change in velocity.

And then, during the fourth second, the velocity is plus nine meters per second, and so we jump up.

Let's make this plus nine.

And so we have here during the last second...

And again, this is nonphysical, so there has to be somehow a transition. And so here you see the velocity as a function of time.

Now comes an interesting question.

Is it possible, if I gave you this--

so this is a given, you can't see that--

could you convert this back to that? And the answer is yes, provided that I tell you what the position is at t zero.

At t equals zero, x equals plus six and that is sufficient for you to use this information and to reconstruct that.

It's an interesting thing to do, and if you feel like it I would say, give it a shot.

All right, so far, about speeds and average velocities and accelerations.

Let's now go to trajectories, three-dimensional trajectories.

Trajectories, thank goodness, are almost never three-dimensional.

They're always two-dimensional, because the trajectory itself is in a vertical plane and so we normally...

When we throw up an object in a gravitational field, you have the trajectory in a plane.

So we're going to have one trajectory.

Let this be the x direction and let this be the y direction.

Increasing values of y, increasing values of x.

I take an object and I throw it up with an initial velocity v zero.

And what is the object going to do? You're going to get a parabola under the influence of gravity and it comes down here again.

And where we have this kind of a problem we will decompose it in two one-dimensional motions, one in the x direction and one in the y direction.

We already decompose right away the velocity at time t equals zero into a component which I call v zero x and that, of course, is v zero times the cosine of alpha if the angle is alpha.

And the velocity in the y direction at time t equals zero--

I will call that v zero in the y direction and that is v zero times the sine of alpha.

And now I have to know how the object moves in the x direction as a function of time and how it behaves as a function of time in the y direction.

So here come the equations for the x direction.

x as a function of time equals x zero plus v zero x times t.

That's all--

there is no acceleration.

The velocity in the x direction as a function of time is simply v zero x--

it never changes.

So that's the x direction.

Now we take the y direction.

y as a function of time equals y zero plus v zero y t plus one-half at squared.

My g value that I'm going to use is always positive--

either 9.8 meters per second squared or sometimes I make it easy to use it, 10--

but mine is always positive.

And since in this case I have chosen this to be the increasing value of y, that's the only reason why I would now have to put in minus one-half gt squared--

not, as some of you think, because the acceleration is down.

That's not a reason.

Because I could have called this direction increasing y.

Then it would have been plus one-half gt squared.

So the consequence of my choosing this the direction in which y increases...

Therefore, the plus one-half at squared that you would normally see, I'm going to replace that now by minus one-half gt squared.

Then the velocity in the y direction as a function of time would be this derivative, that is, v zero y minus gt and the acceleration equals minus g.

So these are the three equations that govern the motion in the y direction.

This only holds if there is no air drag, no friction of any kind.

That is very unrealistic if we are near Earth, but when we are far away from Earth, as we were with the KC-135--

which was flying at an altitude of about 30,000 feet--

that, of course, is a little bit more realistic.

And therefore the example that I have picked to throw up an object is the one whereby the KC-135, at an altitude somewhere around 25,000 or 30,000 feet, comes in at a speed of 425 miles per hour, turns the engines off and then, for the remaining whatever it was--

about 30 seconds--

everyone, including the airplane, has no weight.

That's the case that I now want to work out quantitatively with you.

In the case of the KC-135, we will take an angle for alpha of 45 degrees and we will take v zero, which was about 425 miles per hour.

You may remember that from that lecture.

425 miles per hour translates into about 189 meters per second.

And so that means that the velocity v zero y and v zero x are both the same because of the 45-degree angle, and that is, of course, the 189 divided by the square root of 2.

And that is about 133 meters per second.

Both are positive--

keep that in mind because this is what I call the increasing value for y and this is the increasing value of x.

They are both positive values.

Signs do matter.

This is a given now.

And now comes the first question that I could ask you on an exam.

When is the plane at its highest point of its trajectory and how high is it above the point where it started when it turned the engines off when it went into free fall? So when is it here and what is this distance? Well, when is it there? That's when the velocity in the y direction becomes zero.

It is positive.

It gets smaller and smaller because of the gravitational acceleration, comes to a halt and becomes zero.

So I ask this equation, when are you zero? This is the one I pick and so I say, zero equals plus 133 minus 10 times t.

You may think that the gravitational acceleration at an altitude of 30,000 feet could be substantially less than the canonical number of 10.

It is a little less because you're a little bit further away from the Earth, but it's only 0.3 percent less, and so we'll just accept the 10.

It's easy to work with.

And so when is it at the highest point? That is when t equals 13.3 seconds.

So that's about how long it takes to get there.

When I gave the lecture last time, I said it's about 15 seconds, because I made the numbers...

I rounded them off.

It's about 30.3 seconds.

And what is this distance h now? Ah! Now I have to go to this equation.

I say h equals zero, because I'm going to define the point where the plane starts its trajectory.

I call that y zero zero, I'm free to do that.

h equals zero plus 133--

that is the speed--

times 13.3 seconds minus one-half times g--

that is 5--

times 13.3 squared.

That is what h must be.

And that turns out to be about 885 meters.

I think I told you last time it's about 900, close enough.

So we know now how long it takes to reach p and we know what the vertical distance is.

And the whole trip back to this starting point--

if we call this sort of a starting point, starting altitude--

this whole trip will take twice the amount of time.

To get back to this point when the engines are restarted is about 26.5, 27 seconds.

How far has the plane traveled, then, in horizontal direction? Well, now I go back to this equation.

So now I say, aha! x then, when it is back at this point, must be x zero--

which I conveniently choose zero--

plus 133 meters per second, which is the velocity in the x direction, which never changes.

When the plane is here, that velocity in the x direction is the same 133 meters per second as it was here, which, by the way, is about 300 miles per hour.

That never changes if there is no air drag or air friction of any kind.

So we get plus 133 times the time and the whole trip takes 26.6 seconds, and that, if you convert that to kilometers is about 3.5 kilometers.

Now, you could ask yourself the question: What is the velocity of that plane when it is at that point s?

And now... you may want to abandon now this one-dimensional idea of x and y.

You may say, "Well, look.

"This is a parabola and it is completely symmetric.

"If the plane comes up here "with 425 miles per hour at an angle of 45 degrees, "then obviously it comes down here at an angle of 45 degrees and the speed must again be 425 miles per hour." And you would score 100 percent, of course--

it's clear.

I want you to appreciate, however, that I could continue to think of this as two one-dimensional motions.

And I can therefore calculate what the velocity in the x direction is at s and what the velocity in the y direction is at s.

So what is the velocity in the x direction at point s? I go to equation...

the second equation there.

That is v zero x, that is plus 133 meters per second.

What is the velocity in the y direction? Ah, I have to go to this equation now.

v zero y minus gt.

So I get plus 133 minus 10 times the 26.6 seconds to reach that point s.

And what do I find? Minus 133 meters per second.

The velocity in the y direction started off plus 133, but now it is minus 133.

You see, this is sign-sensitive.

This is wonderful.

That's the great thing about treating it that way.

So you now know that it comes in with a velocity of 133 in the x direction--

positive--

133 in the minus y direction, and so what is the net, the sum of the two vectors? That, of course, is this vector and no surprise, this angle is 45 degrees and this one is the square root of 2 times 133 and that, of course, gives you back your 189 meters per second.

189 meters per second, and that is 425 miles per hour.

I'm not recommending that you would do this, of course.

It is perfectly reasonable to immediately come to that conclusion because of the symmetry of the parabola.

Let's now turn to uniform circular motion.

Uniform circular motion occurs when an object goes around in a circle and when the speed never changes.

If the speed doesn't change, then the velocity, of course, does change because the direction changes all the time, but the speed does not.

So here we have our circle.

Let this be radius r, and at this moment in time, the object is here.

It has a certain velocity.

This is 90 degrees.

And later in time, the object is here, the speed is the same, but the direction has changed, 90 degrees.

So these vectors, they have the same length.

In a situation like this that we have uniform circular motion--

so it's uniform...

circular motion--

we first identify what we call the period T in seconds.

That's the time to go around.

Then we identify what we call the frequency, that is, how many times it goes around per second.

I prefer the letter f, but our book uses the Greek letter nu.

I find the nu often very confusing with the v of velocity.

That's why I prefer the f.

It is one over T, and so the units are seconds minus one but most physicists would call that "hertz." Ten hertz means to go ten times around per second.

And then we identify omega, the angular velocity.

Omega, which is in radians per second.

Since it takes T seconds to go around two pi radians, omega is two pi divided by T.

Now, then we have the speed, which we can also think of as a linear velocity.

How many meters per second is linear, as opposed to how many radians per second, which is angular velocity.

So this is a linear velocity, this is an angular velocity.

And that linear velocity, which, in this case, is really your speed, is of course the circumference of the circle divided by how many seconds it takes to go around.

And that is also omega r and that is now in meters per second.

All this is only possible if there is an acceleration, and the acceleration is called the centripetal acceleration.

It is always pointed towards the center: "a" centripetal, "a" centripetal.

And the centripetal acceleration--

the magnitude--

is v squared divided by r, which is therefore also omega squared r, and that, of course, is in meters per second squared.

I want to work out a specific example, and the example that I have chosen is the human centrifuge that is used by NASA in Houston for experiments on humans to see how they deal with strong accelerations.

This is that centrifuge.

The radius from the axis of rotation--

the axis of rotation is here--

and the distance from here to here, though you may not think so, is about 15 meters.

So the astronauts go in here and then the thing goes around.

And so I would like to work out this with some numbers.

The radius r--

I'll give your light back because it may be nicer for you...

The radius is 15 meters.

It depends, of course, a little bit on where the person is located in that sphere.

It goes around 24 revolutions per minute and that translates into 0.4 hertz.

So the period to go around for one rotation is 2.5 seconds.

The thing goes around once in 2.5 seconds.

So the angular velocity omega, which is two pi divided by T...

If you take two pi and divide it by 2.5, it just comes out to be roughly 2.5.

[chuckling): It's a purely accident, that's the way it is.

Don't ever think that that has to be the same, of course.

It just happens to come out that way for these dimensions.

So omega is about 2.5 radians per second.

And the speed, linear speed--

tangential speed, if you want to call it--

is omega r.

That comes out to be about 35...

37.7 meters per second, and that translates into about 85 miles per hour, so it's a sizable speed.

What, of course, the goal is for NASA: What is the centripetal acceleration--

that is omega squared r--

or, if you prefer to take v squared divided by r, you'll find, of course, exactly the same answer if you haven't made a slip, and that is 95 meters per second squared.

And that is about ten times the gravitational acceleration on Earth,
which is really phenomenal, if you add, too, the fact that the direction is changing all the time when you go around, so you feel the 10 g in this direction and then you feel it in a different direction.

I can't imagine how people can actually survive that--

I mean, not faint.

Most people, like you and me, if we were to be accelerated along a straight line, not even a circle, where the direction changed, but along a straight line, most of us faint when we get close to 6 g.

And there is a reason for that.

You get problems with your blood circulation and not enough oxygen goes to your brains, and that's why you faint.

How these astronauts can do it at 10 g and the direction changing all the time, it beats me.

If you take a Boeing 747, it takes 30 seconds from the moment that it starts on the runway until it takes off.

You should time that, when you get a chance.

It's very close to 30 seconds, and by that time the plane has reached a speed of about 150 miles per hour.

And if you calculate, if you assume that the acceleration is constant--

it's an easy calculation--

it turns out that the acceleration is only two meters per second squared.

That is only one-fifth of the gravitational acceleration.

Feels sort of good, right? It's very comfortable, when you're taking off.

It's only 2 meters per second squared.

These poor people, men and women, 95 meters per second squared.

I would like to address something that is not part of the exam, but that is something that I want you to think about, something that is fun, and it's always nice to do something that is fun.

It has to do with my last lecture.

I have to clean my hands first for it to work quite well.

I have a yardstick here, and I am going to put the yardstick on my hands, on my two fingers, which I hold in front of me.

Here it is.

It's resting on my two fingers, and I'm going to move my two fingers towards each other.

One of them begins to slide first, of course.

I can't tell you which one.

But something very strange will happen.

If this one starts to slide first, it comes to a stop and then the other one starts to slide and it comes to a stop.

And then this one starts to slide and so on.

And that is very strange.

This is something you should be able to explain, certainly after the lecture we had last time.

Look at this.

Did you see the alternation? I'll do it a little faster.

Left is going, right is going, left is going, right is going, left is going.

Once more... look at it.
Left is going, right is going, left is going, right is going.

They alternate.

Give this some thought, and you know, PIVoT has an option that you can discuss problems with other students, so make use of this discussion button and see whether you can come to an explanation.

Good luck on your exam.

See you next Friday.