Massachusetts Institute of Technology - Physics Department
Physics-8.01 Fall 1999

Solutions for Assignment # 6

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November 1, 1999 10:14 am

Problem 6.1 (Ohanian, page 291, problem 13)

The masses of the two automobiles are $m_1 = 540$ kg and $m_2 = 1400$ kg and their velocities, measured along the original direction of m_1, are $v_1 = 80$ km/h ≈ 22.22 m/s and $v_2 = -80$ km/h ≈ -22.22 m/s.

(a) We are told that the two automobiles remain locked together after the collision. After the collision both automobiles move as one, hence they both must have the same velocity which must also be the velocity of the center of mass (v_{CM}). There are no outside forces, so the velocity of the center of mass remains unchanged during the collision. Therefore

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{540 - 1400}{540 + 1400} \cdot 80 \text{ km/h} \approx -35 \text{ km/h} \approx -9.9 \text{ m/s}$$

The velocity of the wreck is given by v_{CM}. Note, it was unnecessary to convert from the non-metric unit "km/h" if we wanted the value of v_{CM} in units of "km/h".

(b) The total kinetic energy before the collision is

$$K_{before} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{1}{2}(540)(22.22)^2 + \frac{1}{2}(1400)(22.22)^2 \approx 4.8 \times 10^5 \text{ J}$$

The total kinetic energy after the collision is

$$K_{after} = \frac{(m_1 + m_2) v_{CM}^2}{2} \approx 8.2 \times 10^4 \text{ J}$$

Note, it was necessary to convert from "km/h" to "m/s" to achieve an answer in units of "J".

(c) This calculation is best done in the center of mass frame. Since there are no outside forces, this frame is inertial; hence the acceleration in this frame is the same as the acceleration relative to the ground. We will make use of the kinematic relation

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

For the automobile of mass m_1, its speed in the center of mass frame is

$$v_1' = v_1 - v_{CM} = 22.2 - (-9.9) = 32.1 \text{ m/s}$$

The acceleration the passenger compartment experiences is given by

$$a_1 = \frac{v_1'^2}{2d} \approx 8.6 \times 10^2 \text{ m/s}^2$$

For the automobile of mass m_2, its speed in the center of mass frame is

$$v_2' = v_2 - v_{CM} = -22.2 - (-9.9) = -12.3 \text{ m/s}$$

The acceleration the passenger compartment experiences is given by

$$a_2 = \frac{v_2'^2}{2d} \approx 1.3 \times 10^2 \text{ m/s}^2$$

Problem 6.2 (Ohanian, page 292, problem 23)

This problem is illustrated in Figure 11.12 on page 292. Let m be the mass of each steel ball. The first steel ball swings and gains kinetic energy until it collides with the second steel ball. Initially the first steel ball is at rest at an angle θ relative to vertical, so its height relative to the bottom of the swing is

$$h = l(1 - \cos\theta)$$

Its mechanical energy is

$$E = mgh = mgl(1 - \cos\theta)$$

At the bottom of its swing, just before the collision, its mechanical energy is

$$E = \frac{mv^2}{2}$$

Conservation of mechanical energy gives

$$mgl(1 - \cos\theta) = \frac{mv^2}{2} \qquad \Longrightarrow \qquad v = \sqrt{2gl(1 - \cos\theta)}$$

Also in terms of just h, conservation of mechanical energy gives

$$mgh = \frac{mv^2}{2} \qquad \Longrightarrow \qquad h = \frac{v^2}{2g} \tag{1}$$

(a) Because we are told the collision is elastic, we know that kinetic energy is conserved. Momentum is also conserved. If we label the velocity, before the collision, of the first steel ball v_1, and we label the velocity, after the collision, of the first and second steel balls v_1' and v_2' respectively, then

$$\frac{mv_1^2}{2} = \frac{mv_1'^2}{2} + \frac{mv_2'^2}{2}$$

and

$$mv_1 = mv_1' + mv_2'$$

These equations are precisely the same equations for one-dimensional collisions. The solution is given in Equations (11) and (12) on page 277.

$$v_1' = \frac{m-m}{m+m} \cdot v_1 = 0$$

$$v_2' = \frac{2m}{m+m} \cdot v_1 = v_1$$

All of the mechanical energy is transferred to the second steel ball (since the first steel ball comes to rest), so the second steel ball must reach the same height as the first one started from.

$$h' = \frac{v_2'^2}{2g} = \frac{v_1^2}{2g} = h$$

(b) We are told the putty balls remain stuck together after the collision. If we label the velocity of the joined putty balls after the collision v', then momentum conservation gives

$$mv_1 = 2mv' \qquad \Longrightarrow \qquad v' = \frac{v_1}{2}$$

$$h' = \frac{v'^2}{2g} = \frac{\left(\frac{v_1}{2}\right)^2}{2g} = \frac{1}{4} \cdot \frac{v_1^2}{2g} = \frac{h}{4}$$

Problem 6.3

Let $m_J = 100$ kg, $m_N = 80$ kg, and $m_B = 10$ kg. There are no outside forces acting on John, Nancy, or the block, so momentum is conserved. Velocity will be considered positive when it is directed from John to Nancy. Note that kinetic energy is not necessarily conserved: John and Nancy can interchange internal energy with kinetic energy by sliding and catching the block.

(a) Let v_{J1} be the velocity of John's sled and v_{B1} be the velocity of the block after John releases the block. The speed of the block relative to John's sled is said to be 3 m/s, thus

$$v_{B1} = v_{J1} + 3$$

Momentum conservation gives

$$\begin{aligned} 0 = m_J v_{J1} + m_B v_{B1} &= m_J v_{J1} + m_B(v_{J1} + 3) \qquad \Longrightarrow \\ v_{J1} &= -\frac{3m_B}{m_J + m_B} \approx -0.27 \text{ m/s} \qquad \Longrightarrow \\ v_{B1} &= v_{J1} + 3 = \frac{3m_J}{m_J + m_B} \approx 2.7 \text{ m/s} \end{aligned}$$

(b) Let v_{N1} be the velocity of Nancy's sled after she catches the block. Momentum conservation gives

$$m_B v_{B1} = (m_B + m_N) v_{N1} \qquad \Longrightarrow$$

$$v_{N1} = \frac{m_B}{m_B + m_N} \cdot v_{B1} = \frac{m_B}{m_B + m_N} \cdot \frac{3m_J}{m_J + m_B} \approx 0.30 \text{ m/s}$$

(c) Let v_{N2} be the velocity of Nancy's sled and v_{B2} be the velocity of the block after Nancy slides the block to John. The speed of the block relative to Nancy's sled is said to be 3 m/s, thus

$$v_{B2} = v_{N2} - 3$$

(Note that here it is "-3" and above it is "$+3$". This is due to our choice for positive velocity.) Momentum conservation gives

$$\begin{aligned} (m_B + m_N) v_{N1} = m_B v_{B2} + m_N v_{N2} &= m_B(v_{N2} - 3) + m_N v_{N2} \qquad \Longrightarrow \\ v_{N2} &= \frac{(m_B + m_N) v_{N1} + 3m_B}{m_B + m_N} \approx 0.63 \text{ m/s} \qquad \Longrightarrow \\ v_{B2} &= v_{N2} - 3 \approx -2.4 \text{ m/s} \end{aligned}$$

(d) Let v_{J2} be the velocity of John's sled after he catches the block. Momentum conservation gives

$$m_J v_{J1} + m_B v_{B2} = (m_J + m_B) v_{J2} \qquad \Longrightarrow \qquad v_{J2} = \frac{m_J v_{J1} + m_B v_{B2}}{(m_J + m_B)} \approx -0.46 \text{ m/s}$$

(e) The total kinetic energy of the block and sled after John releases the block is

$$K = \frac{m_J v_{J1}^2}{2} + \frac{m_B v_{B1}^2}{2} \approx 41 \text{ J}$$

Initially there was no kinetic energy, but John converts chemical energy (stored in his muscles) to kinetic energy by sliding the block to Nancy.

(f) The total kinetic energy of the two sleds and the block just after Nancy catches the block is

$$K = \frac{m_J v_{J1}^2}{2} + \frac{(m_B + m_N) v_{N1}^2}{2} \approx 7.9 \text{ J}$$

There are internal forces involved when Nancy catches the block. These forces convert kinetic energy into other forms of energy.

(g) The total kinetic energy of the two sleds and the block just after Nancy releases the block is

$$K = \frac{m_J v_{J1}^2}{2} + \frac{m_B v_{B2}^2}{2} + \frac{m_N v_{N2}^2}{2} \approx 48 \text{ J}$$

Nancy converts chemical energy (stored in her muscles) to kinetic energy by sliding the block back to John.

(h) The total kinetic energy of the sleds after John catches the block is

$$K = \frac{(m_B + m_J) v_{J2}^2}{2} + \frac{m_N v_{N2}^2}{2} \approx 28 \text{ J}$$

There are internal forces involved when John catches the block. These forces again convert kinetic energy into other forms of energy.

Problem 6.4

The center of mass frame is defined as the frame in which the center of mass is at rest, so the two pucks must travel in opposite directions in that frame. If v_1 and v_2 are the velocities respectively of masses m_1 and m_2, then

$$m_1 v_1 + m_2 v_2 = 0 \qquad \Longrightarrow \qquad v_1 = -\frac{m_2}{m_1} \cdot v_2$$

Clearly the total momentum is zero, but the kinetic energy is

$$K = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$$

We will find it convenient to use the relationship between v_1 and v_2 to eliminate v_1 from the kinetic energy.

$$K = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1}{2}\left(\frac{m_2}{m_1} \cdot v_2\right)^2 + \frac{m_2 v_2^2}{2} = \frac{m_2 v_2^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right)$$

(a) We are told that the collision is elastic, so kinetic energy is conserved. Also, there are no relevant outside forces acting on the pucks, so momentum is conserved. If we let $\vec{v}_1'$ and $\vec{v}_2'$ label the velocities after the collision then momentum conservation gives

$$0 = m_1 \vec{v}_1' + m_2 \vec{v}_2' \qquad \Longrightarrow \qquad \vec{v}_1' = -\frac{m_2}{m_1}\vec{v}_2' \qquad \Longrightarrow \qquad |v_1'| = \frac{m_2}{m_1} \cdot |v_2'|$$

This is the same relation as v_1 and v_2 above, so the kinetic energy is given similarly by

$$K = \frac{m_1 |v_1'|^2}{2} + \frac{m_2 |v_2'|^2}{2} = \frac{m_2 |v_2'|^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right)$$

Then conservation of kinetic energy gives

$$\frac{m_2 v_2^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right) = \frac{m_2 |v_2'|^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right) \qquad \Longrightarrow$$

$$|v_2'| = v_2 \qquad \Longrightarrow \qquad v_1 = \frac{m_2}{m_1} \cdot v_2 = \frac{m_2}{m_1} \cdot |v_2'| = |v_1'|$$

Thus the general conclusion using only kinetic energy conservation and momentum conservation is

$$|v_2'| = v_2 \ \text{ and } \ |v_1'| = v_1 \ \text{ and } \ \vec{v}_1' = -\frac{m_2}{m_1}\vec{v}_2'$$

This indicates that the directions are exactly opposite.

(b) Any two vectors $\vec{v}_1'$ and $\vec{v}_2'$ that satisfy the above equation will also satisfy kinetic energy and momentum conservation. If we let $\hat{n}$ represent a unit vector in the plane, then

$$\vec{v}_1' = v_1 \hat{n} \ \text{ and } \ \vec{v}_2' = -v_2 \hat{n}$$

are all solutions, thus any direction of motion is consistent with kinetic energy and momentum conservation.

(c) The total kinetic energy in the center of mass frame before the collision was given above as

$$K = \frac{m_2 v_2^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right)$$

(d) We assumed that the collision was elastic, so kinetic energy is conserved. Therefore the total kinetic energy after the collision is given as

$$K = \frac{m_2 |v_2'|^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right) = \frac{m_2 v_2^2}{2} \cdot \left(\frac{m_2}{m_1} + 1\right)$$

Problem 6.5

The tennis ball and basketball do not collide until after the basketball strikes the ground. Both balls fall the same distance, call it h; hence both have the same velocity, $v = \sqrt{2gh}$ just before the basketball strikes the ground. The mass of the basketball is insignificant compared to the mass of the planet; thus the basketball will bounce back up with the same speed, v. (We assume that the collision is elastic, i.e. kinetic energy is conserved.)

Now the tennis ball and basketball collide. This collision is best viewed from the frame moving with the basketball. In this frame the tennis ball travels at a speed $2v$ and strikes the basketball at rest. The basketball is much heavier than the tennis ball, thus the tennis ball will bounce back up with the same speed, $2v$. (Again, we assume that the collision is elastic.) In the original frame, the tennis ball has the speed $2v + v = 3v$. Therefore, the height it will reach is $h' = \frac{(3v)^2}{2g} = 9\frac{v^2}{g} = 9h$.

Problem 6.6

Let $m = 6$ kg, $v = 350$ m/s, $m_1 = 2$ kg, $v_1 = 250$ m/s, $m_2 = 4$ kg, and $v_2 = 400$ m/s.

(a) The total momentum before the collision is

$$P_{before} = mv = 2.1 \times 10^3 \text{ kg} \cdot \text{m/s}$$

The total momentum after the collision is

$$P_{after} = m_1 v_1 + m_2 v_2 = 2.1 \times 10^3 \text{ kg} \cdot \text{m/s}$$

Therefore, $P_{before} = P_{after}$, and momentum is conserved.

(b) The total kinetic energy before the collision is

$$K_{before} = \frac{mv^2}{2} \approx 3.68 \times 10^5 \text{ J}$$

The total kinetic energy after the collision is

$$K_{after} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \approx 3.83 \times 10^5 \text{ J}$$

Therefore, $K_{before} \neq K_{after}$, and kinetic energy is not conserved.

(c) The center of mass velocity before the collision is

$$V_{before} = \frac{mv}{m} = v = 350 \text{ m/s}$$

The center of mass velocity after the collision is

$$V_{after} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 350 \text{ m/s}$$

Therefore, $V_{before} = V_{after}$, and center of mass velocity does not change.

(d) First we must transform the velocities v, v_1, and v_2 into the appropriate center of mass. Let v', v_1', and v_2' denote the corresponding velocities in the appropriate center of mass.

$$v' = v - V_{before} = v - v = 0$$

$$v_1' = v_1 - V_{after} = 250 - 350 = -100 \text{ m/s}$$

$$v_2' = v_2 - V_{after} = 400 - 350 = 50 \text{ m/s}$$

The total momentum before the collision in the center of mass frame is

$$P_{before} = mv' = 0$$

The total momentum after the collision in the center of mass frame is

$$P_{after} = m_1 v_1' + m_2 v_2' = 0$$

Therefore, $P_{before} = P_{after}$, and momentum is conserved in the center of mass frame. The total kinetic energy before the collision in the center of mass frame is

$$K_{before} = \frac{mv'^2}{2} = 0$$

The total kinetic energy after the collision in the center of mass frame is

$$K_{after} = \frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2} = 1.5 \times 10^4 \text{ J}$$

Therefore, $K_{before} \neq K_{after}$, and kinetic energy is not conserved in the center of mass frame. None of the previous conclusions change.

Problem 6.7

Let $m_A = m$ and $m_B = 3m$.

(a) The table is assumed to be frictionless, so the only other force is the spring force which is conservative; hence mechanical energy is conserved. (Gravity and the normal force are present, but if the table is level then these forces precisely balance and thus are irrelevant.) Therefore, the total mechanical energy of the system is constant, and we can evaluate it at $t = 0$, at which time both masses are at rest and the spring has potential energy U_0.

$$E = U_0$$

(b) In general if K_A and K_B are the kinetic energies of mass A and mass B and U_S is the potential energy of the spring, then the mechanical energy is

$$E = K_A + K_B + U_S$$

Mechanical energy conservation gives

$$U_0 = K_A + K_B + U_S \qquad \Longrightarrow \qquad U_S = U_0 - K_A - K_B$$

(c) There are no outside forces acting on this system, so momentum is conserved. (Again, gravity and the normal force are irrelevant since they cancel.) Therefore, the total momentum of the system is constant, and we can evaluate it at $t = 0$, at which time both masses are at rest.

$$P = 0$$

Now suppose at some time the velocities are $\vec{v_A}$ and $\vec{v_B}$. Then

$$P = m_A\vec{v_A} + m_B\vec{v_B} = m\vec{v_A} + 3m\vec{v_B}$$

Momentum conservation gives

$$0 = m\vec{v_A} + 3m\vec{v_B} \qquad \Longrightarrow \qquad \vec{v_B} = -\frac{1}{3}\vec{v_A} \qquad \Longrightarrow \qquad |v_B| = \frac{1}{3}|\vec{v_A}|$$

(d) The result above demonstrates that the particles are always moving in opposite directions. Thus if A is moving in the $+x$ direction then B is moving in the $-x$ direction.

(e) It is convient if we choose the center of our coordinate system to coincide with the center of mass of the system. Let x_A and x_B label the position of mass A and B respectively. Then by definition of the center of mass

$$0 = m_A x_A + m_B x_B = m x_A + 3m x_B \qquad \Longrightarrow \qquad x_B = -\frac{1}{3} x_A$$

Then the force acting on A is

$$m\ddot{x}_A = F = -k(x_A + x_B - l_0) = -k(x_A + \frac{1}{3}x_A - l_0) = -\frac{4k}{3} \cdot (x_A - \frac{3}{4} l_0)$$

Thus the equation of motion is

$$\ddot{x}_A = -\frac{4k}{3m} \cdot (x_A - \frac{3}{4} l_0)$$

This indicates that the motion is simple harmonic with frequency

$$\omega_A = \sqrt{\frac{4k}{3m}}$$

A similar argument for B yields

$$3m\ddot{x}_B = F = -k(x_A + x_B - l_0) = -k(3x_B + x_B - l_0) = -4k \cdot (x_A - l_0) \qquad \Longrightarrow$$

$$\omega_B = \sqrt{4k \cdot \frac{1}{3m}} = \sqrt{\frac{4k}{3m}}$$

Thus the motion is also simple harmonic in x_B with the same frequency ω given by

$$\omega = \sqrt{\frac{4k}{3m}}$$

Problem 6.8 (Ohanian, page 294, problem 38)

Collisions in two dimensions are discussed in Section 11.3 beginning on page 281. The relevant results, using conservation of momentum and kinetic energy, are

$$m_1 v_1 = m_1 v_1' \cos\theta_1' + m_2 v_2' \cos\theta_2'$$

$$0 = m_1 v_1' \sin\theta_1' - m_2 v_2' \sin\theta_2'$$

$$\frac{m_1 v_1^2}{2} = \frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2}$$

where the various quantities are illustrated in Figure 11.6 on page 281. For the problem at hand, $m_1 = m_2 = m$ where $m = 1.673 \times 10^{-27}$ kg is the mass of the proton; this allows us to cancel the masses on each side of the equation. The equations above become

$$v_1 = v_1' \cos\theta_1' + v_2' \cos\theta_2' \tag{2}$$

$$0 = v_1' \sin\theta_1' - v_2' \sin\theta_2' \tag{3}$$

$$v_1^2 = v_1'^2 + v_2'^2 \tag{4}$$

We are given the energy for the initial proton, which will give us v_1, and we can measure θ_1' and θ_2' from the picture; thus the above equations will suffice to determine v_1' and v_2', which will then determine the energy of each outgoing proton. To solve the above equations, multiply Equation (**??**) by $\sin\theta_2'$, and multiply Equation (**??**) by $\cos\theta_2'$. The resulting equations are

$$\sin\theta_2' v_1 = v_1' \sin\theta_2' \cos\theta_1' + v_2' \sin\theta_2' \cos\theta_2' \tag{5}$$

$$0 = v_1' \sin\theta_1' \cos\theta_2' - v_2' \sin\theta_2' \cos\theta_2' \tag{6}$$

If we add Equation (**??**) and Equation (**??**) then we get

$$\sin\theta_2' v_1 = v_1'(\sin\theta_2' \cos\theta_1' + \sin\theta_1' \cos\theta_2')$$

This gives us an equation for v_1'

$$v_1' = \frac{\sin\theta_2'}{\sin\theta_2' \cos\theta_1' + \sin\theta_1' \cos\theta_2'} \cdot v_1$$

We can measure the angles from the picture: $\theta_1' \approx 45^\circ$ is the angle of $\overline{PB}$ relative to $\overline{AP}$ and $\theta_2' \approx 44^\circ$ is the angle of $\overline{PC}$ relative to $\overline{AP}$. This gives

$$v_1' \approx 0.695 \cdot v_1$$

Using Equation (**??**) gives

$$v_1^2 = v_1'^2 + v_2'^2 \qquad \Longrightarrow \qquad v_2' = \sqrt{v_1^2 - v_1'^2} \approx 0.719 \cdot v_1$$

The energy for the proton initially traveling along $\overline{AP}$ is

$$E_1 = \frac{m v_1^2}{2} = 8.0 \times 10^{-13} \text{ J}$$

The energy for the proton traveling along $\overline{PB}$ is

$$E_1' = \frac{mv_1'^2}{2} = (0.695)^2 \cdot \frac{mv_1^2}{2} = 0.483 \cdot E_1 \approx 3.9 \times 10^{-13} \text{ J}$$

The energy for the proton traveling along $\overline{PC}$ is

$$E_2' = \frac{mv_2'^2}{2} = (0.719)^2 \cdot \frac{mv_1^2}{2} = 0.516 \cdot E_1 \approx 4.1 \times 10^{-13} \text{ J}$$

Problem 6.9

We can use Equation (51) on page 263 which relates the initial velocity and mass (v_i and M_i) to the velocity and mass at a later time (v_f and M_f) given the exhaust gas speed u_{ex}.

$$v_f - v_i = u_{ex} \ln\left(\frac{M_i}{M_f}\right)$$

We are told that $v_i = 8 \times 10^3$ m/s, $v_f = 8.5 \times 10^3$ m/s, and $u_{ex} = 2.5 \times 10^3$ m/s. If we assume that the shuttle uses all its fuel, then $M_f = 10^5$ kg and $M_i = M_F + M_f$ where M_F is the mass of the fuel. We can solve the above equation for M_F by the following sequence of steps.

$$v_f - v_i = u_{ex} \ln\left(\frac{M_F + M_f}{M_f}\right)$$

$$\frac{v_f - v_i}{u_{ex}} = \ln\left(\frac{M_F + M_f}{M_f}\right)$$

$$e^{\frac{v_f - v_i}{u_{ex}}} = \frac{M_F + M_f}{M_f}$$

$$M_F = M_f e^{\frac{v_f - v_i}{u_{ex}}} - M_f \approx 2.2 \times 10^4 \text{ kg}$$

Problem 6.10

Rocket motion is discussed in Lecture Supplement "Rocket Equations - supplements for 10/20/99" on the 801 Home Page.

(a) If the exhaust speed is u_{ex} and the burn rate is R then the thrust F_{th} is given by

$$F_{th} = u_{ex} R \qquad \Longrightarrow \qquad u_{ex} = \frac{F_{th}}{R}$$

We are told that $F_{th} = 34 \times 10^6$ N and $R = 13.8 \times 10^3$ kg/s, thus the exhaust speed is

$$u_{ex} = \frac{F_{th}}{R} = \frac{34 \times 10^6}{13.8 \times 10^3} \approx 2.5 \times 10^3 \text{ m/s}$$

(b) The engines will burn until there is no more fuel. If the initial mass of the rocket and fuel is $M_i = 2.85 \times 10^6$ kg and the final mass of the rocket (with no fuel) is $M_f = 0.77 \times 10^6$ kg then the mass of fuel M_F is

$$M_F = M_i - M_f = 2.85 \times 10^6 - 0.77 \times 10^6 = 2.08 \times 10^6 \text{ kg}$$

If the burn rate is constant, then the burn time T is given by

$$R = \frac{M_F}{T} \qquad \Longrightarrow \qquad T = \frac{M_F}{R} = \frac{2.08 \times 10^6}{13.8 \times 10^3} \approx 151s$$

(c) The force equation for the rocket is

$$M(t) \cdot a = F_{th} - M(t) \cdot g \qquad \Longrightarrow \qquad a = \frac{F_{th}}{M(t)} - g$$

where $M(t)$ is the mass of the rocket and fuel inside the rocket at time t. The initial mass is $M(0) = M_i$, so the initial acceleration is

$$a = \frac{F_{th}}{M_i} - g = \frac{34 \times 10^6}{2.85 \times 10^6} - 10 \approx 1.9 \text{ m/s}^2$$

(d) The final mass is $M(T) = M_f$, so the acceleration just before the engines stop is

$$a = \frac{F_{th}}{M_f} - g = \frac{34 \times 10^6}{0.77 \times 10^6} - g \approx 34 \text{ m/s}^2$$

(e) The solution to the above differential equation is

$$v_f = -u \ln\left(\frac{M_f}{M_i}\right) - gt$$

At $t = T = \frac{M_F}{R}$, $RT = M_F$ and $M_i - M_F = M_f$, so the speed is

$$v_f = -2.5 \times 10^3 \cdot \ln\left(\frac{0.77 \times 10^6}{2.85 \times 10^6}\right) - 10 \cdot 151 \approx 1.8 \times 10^3 \text{ m/s}$$

Problem 6.11 (Ohanian, page 271, problem 54)

We are told that the reaction products have a mass of 4.4 kg and that these products have a kinetic energy of 4.2 $\times 10^7$ J. If the velocity of this mass is u then

$$\frac{4.4 \cdot u^2}{2} = 4.2 \times 10^7 \qquad \Longrightarrow \qquad u = 4.4 \times 10^3 \text{ m/s}$$