Massachusetts Institute of Technology - Physics Department
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Solutions for Assignment # 5

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Through out these solutions, the following quantities will be used:

M_S	=	1.99×10^{30} kg	is the mass of the sun
M_E	=	5.98×10^{24} kg	is the mass of the earth
R_E	=	6.4×10^{6} m	is an approximate radius of the earth
G	=	$6.67 \times 10^{-11}\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$	is the gravitational constant
g	=	$9.8\ \mathrm{m/s}^2$	is an approximate acceleration of gravity at the surface

Problem 5.1 (Ohanian, page 205, problem 22)

The law of conservation of mechanical energy states that

$$E = K + U = \frac{mv^2}{2} + U$$

Please see the Figure 8.14 on page 205.

First consider $E = E_1$.

(a) The turning points occur when $v = 0 \Rightarrow E = U$. For E_1 there is only one turning point, a left turning point at $x \approx 0.2$.

(b) The speed is maximum when U is minimum. For E_1 the speed is maximum for $x \approx 1.0$. The speed is a local minimum when U is a local maximum. For E_1 the speed is local a minimum for $x \approx 1.6$. Also for E_1 there is a turning point at which $v = 0$; thus *the* minimum speed occurs for $x \approx 0.2$.

(c) The orbit is only bound if there are both left and right turning points. For E_1 there is only a left turning point, so the orbit is unbound.

Now consider $E = E_2$.

(a) For E_2 there is a left turning point at $x \approx 0.3$ and a right turning point at $x \approx 3.0$.

(b) For E_2 the speed is maximum for $x \approx 1.0$. For E_2 the speed is a local minimum for $x \approx 1.6$. Also for E_2 there are two turning points at which $v = 0$; thus *the* minimum speed occurs for $x \approx 0.3$ and $x \approx 3.0$.

(c) For E_2 there is both a left and right turning point, so the orbit is bound.

Now consider $E = E_3$.

(a) For E_3 there is a left turning point at $x \approx 0.5$ and a right turning point at $x \approx 1.3$.

(b) For E_3 the speed is maximum for $x \approx 1.0$. For E_3 the speed is a local minimum for $x \approx 1.6$. Also for E_3 there are two turning points at which $v = 0$; thus *the* minimum speed occurs for $x \approx 0.5$ and $x \approx 1.3$.

(c) For E_3 there is both a left and right turning point, so the orbit is bound.

Problem 5.2 (Ohanian, page 205, problem 25)

The Table 8.1 on page 195 lists the two quantities you need

"Yearly energy expenditure of the United States"	8×10^{19} J
"Combustion of 1 gal. of gasoline"	1.3×10^{8} J

We can use these quantities to make various conversion factors. The amount of gasoline required by the United States per *year* would be

$$8 \times 10^{19} \text{ J/per year} = \frac{8 \times 10^{19}}{1.3 \times 10^{8}} \text{ gal. of gasoline/per year}$$

$$\approx 6.2 \times 10^{11} \text{ gal. of gasoline/per year}$$

The amount of gasoline required by the United States per *day* would be

$$6.2\times10^{11} \text{ gal. of gasoline/per year} = 6.2\times10^{11}\times\frac{1}{365} \text{ gal. of gasoline/per day}$$

$$\approx 1.7 \times 10^{9} \text{ gal. of gasoline/per day}$$

Problem 5.3 (Ohanian, page 207, problem 43)

(a) The total force that 6000 Egyptians can move is

$$F = 6000 \times 360 \text{ N} = 2.16 \times 10^{6} \text{ N}$$

The maximum weight Mg they can move is given by

$$F = \mu_k Mg \qquad \Longrightarrow \qquad Mg = \frac{F}{\mu_k} = \frac{2.16 \times 10^{6}}{0.3} = 7.2 \times 10^{6} \text{ N}$$

The corresponding maximum mass M is

$$M = \frac{Mg}{g} = \frac{7.2 \times 10^{6}}{g} \approx 7.3 \times 10^{5} \text{ kg}$$

(b) The total power that 6000 Egyptians can deliver is

$$P = 6000 \times 0.20 \text{ hp} = 1.2 \times 10^{3} \text{ hp}$$

Power in terms of force and velocity is given by the relation

$$P = Fv \qquad \Longrightarrow \qquad v = \frac{P}{F} = \frac{1.2 \times 10^{3}}{2.16 \times 10^{6}} \times \frac{1 \text{ hp}}{1 \text{ N}}$$

We need to be careful because the unit “hp” is not a metric unit. The conversion is

$$1 \text{ hp} = 745.7 \text{ W}$$

Thus the velocity is given by

$$v = \frac{P}{F} = \frac{1.2 \times 10^{3}}{2.16 \times 10^{6}} \times 745.7 \times \frac{\text{W}}{\text{N}} \approx 4.1 \times 10^{-1} \text{ m/s}$$

Once we convert from “hp” to the metric unit “W,” we are assured the the answer has the units of “m/s.”

Problem 5.4

Let L be the length of the unstretched cord, $h = 100$ m, $k = 100$ N/m, $m = 50$ kg, and let x denote the distance the bungee jumper has fallen from the bridge.

(a) Mechanical energy is conserved if we ignore air drag and if we assume that no heat is dissipated in the string. The relevant forces are then only gravity and the spring force, and these are conservative forces; thus mechanical energy is conserved.

The cord will not stretch until she has fallen a distance L. Thus for $x < L$ her mechanical energy is given by

$$E_{x<L} = mg(h-x) + \frac{mv^2}{2}$$

For $x > L$ she begins to stretch the cord, and her mechanical energy is given by

$$E_{x>L} = mg(h-x) + \frac{mv^2}{2} + \frac{k(x-L)^2}{2}$$

She starts from rest on the bridge, so her mechanical energy is given by

$$E = mgh$$

We want to choose L so that she comes to a stop just above the water; thus her mechanical energy at the point is given by

$$E = \frac{k(h-L)^2}{2}$$

Mechanical energy conservation gives

$$mgh = \frac{k(h-L)^2}{2} \qquad \Longrightarrow$$

$$L = h - \sqrt{\frac{2mgh}{k}} \approx 69 \text{ m}$$

(b) The bungee jumper will hang freely at the distance d above the water at which gravity and the spring force balance. This is given by

$$k(h-L-d) = mg \qquad \Longrightarrow \qquad d = h - L - \frac{mg}{k} \approx 26 \text{ m}$$

Problem 5.5

If the rotation rate was too fast, i.e. the gravitational force was not strong enough to provide for the centripetal force needed, the material at the equator would begin to move outwards, hence the planet would not be stable.

(a) Section 9.1 beginning on page 212 calculates the acceleration at the surface of a planet (assuming a spherical planet). The result is given in Equation (6) on page 214. With the substitutions $M_E \mapsto M$ and $R_E \mapsto R$, the equation is

$$g = \frac{GM}{R^2}$$

where M and R are the mass and radius of the planet respectively. If the planet has a uniform density ρ then the mass is given by

$$M = \frac{4}{3}\pi R^3 \rho \qquad \Longrightarrow \qquad g = \frac{4}{3}\pi G R \rho$$

By the argument above, the maximum rotational rate of the planet is given when this acceleration is precisely the centripetal force for the motion of the material at the surface. This corresponds to a minimum period T. Circular motion gives the relation

$$g = a = \frac{v^2}{R} \qquad \Longrightarrow \qquad v = \sqrt{gR} = \sqrt{\frac{4}{3}\pi G\rho R \cdot R} = R\sqrt{\frac{4}{3}\pi G\rho}$$

The period of motion is then given by

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{R\sqrt{\frac{4}{3}\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

(b) For the value $\rho = 3.0$ g/cm^3 $= 3.0 \times 10^3$ kg/m^3, the minimum period is

$$T = \sqrt{\frac{3\pi}{6.67 \times 10^{-11} \times 3 \times 10^3}} \approx 6.9\times10^3 \text{ s} \approx 1.9 \text{ hour} \approx 7.9\times10^{-2} \text{ day}$$

Problem 5.6 (Ohanian, page 239, problem 15)

Please see Figure 9.42 on page 239.

The center of mass, located at $r_{CM} = 0$ in the figure, is given as

$$r_{CM} = 0 = m_2 r_2 - m_1 r_1 \qquad \Longrightarrow \qquad m_1 r_1 = m_2 r_2$$

Now we suppose that the orbit of each star is a circle centered on $r_{CM} = 0$. Then the centripetal acceleration for the motion of m_1 is given by

$$a_1 = \frac{1}{m_1} F = \frac{1}{m_1} \frac{G m_1 m_2}{(r_1 + r_2)^2} = \frac{G m_2}{(r_1 + r_2)^2}$$

Similarly,

$$a_2 = \frac{G m_1}{(r_1 + r_2)^2}$$

We can check that using $m_1 r_1 = m_2 r_2 \Rightarrow \omega_1 = \omega_2 = \omega$ where ω is the period of the binary system. We would like to make this equality, i.e. symmetry between m_1 and m_2, more apparent.

$$a_1 = \frac{Gm_2}{(r_1+r_2)^2} = \omega^2 r_1 \tag{1}$$

$$a_2 = \frac{Gm_1}{(r_1+r_2)^2} = \omega^2 r_2 \tag{2}$$

Adding equation (1) and (2) gives

$$\omega^2(r_1+r_2) = (m_1+m_2)\cdot\frac{G}{(r_1+r_2)^2} \qquad \Longrightarrow$$

$$\omega^2 = \frac{G(m_1+m_2)}{(r_1+r_2)^3}$$

The period is then given by

$$T^2 = \frac{4\pi^2}{\omega^2} = \frac{4\pi^2(r_1+r_2)^3}{G(m_1+m_2)}$$

Problem 5.7 (Ohanian, page 239, problem 16)

We need to use the result from problem 5.6, which is

$$T^2 = \frac{4\pi^2}{G(m_1+m_2)}(r_1+r_2)^3 \qquad \Longrightarrow \qquad r_1+r_2 = \left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{1/3}$$

Let M_{sun} be the mass of the sun and $m_1 = 10M_{sun}$ and $m_2 = 25M_{sun}$ be the masses of the black hole and supergiant respectively. The period is $T = 5.6$ days $\approx 4.84\times10^5$ s, and the distance between the stars is

$$d = r_1+r_2 = \left(\frac{G(m_1+m_2)T^2}{4\pi^2}\right)^{1/3} = \left(\frac{35\cdot GM_{sun}(4.84\times10^5)^2}{4\pi^2}\right)^{1/3} \approx 3.0\times10^{10} \text{ m}$$

Problem 5.8 (Ohanian, page 241, problem 28)

(a) The mechanical energy of the projectile just after the gun shot is

$$E = \frac{mv^2}{2} - \frac{GM_E m}{R_E}$$

where M_E and R_E are the mass and radius, respectively, of earth; m is the projectile mass; and v is the "muzzle speed" of the gun. If the projectile is to just barely reach the distance of the moon, then the mechanical energy at that point is

$$E = -\frac{GM_E m}{D}$$

where D is the distance between the moon and the *center* of earth. Using the previous two equations and the conservation of mechanical energy gives

$$\frac{mv^2}{2} - \frac{GM_E m}{R_E} = -\frac{GM_E m}{D} \qquad \Longrightarrow$$

$$v = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{D}\right)} \approx 1.1 \times 10^4 \text{ m/s}$$

(b) The gun must deliver the appropriate kinetic energy to the projectile.

$$K = \frac{mv^2}{2} = mGM_E\left(\frac{1}{R_E} - \frac{1}{D}\right) \approx 1.2 \times 10^{11} \text{ J}$$

$$1.2 \times 10^{11} \text{ J} = \frac{1.2 \times 10^{11}}{4.2 \times 10^9} \text{ tons of TNT} \approx 29 \text{ tons of TNT}$$

(c) The equations of motion for constant acceleration are

$$v = at \quad \text{and} \quad x = \frac{at^2}{2}$$

The time required for the projectile to traverse the length $L = 500$ m of the barrel is

$$T = \sqrt{\frac{2L}{a}}$$

For the projectile to achieve the velocity $v = 1.1 \times 10^4$ m/s at that point, the acceleration must be

$$a = \frac{v^2}{2L} \approx 1.2 \times 10^5 \text{ m/s}^2$$

Problem 5.9 (Ohanian, page 241, problem 30)

(a) The orbital mechanical energy is given in equation (27) on page 226. Using the replacement $M_S \mapsto M_E$ and the values $m = 3500$ kg and $r = 100 \text{ km} + R_E = 10^5 \text{ m} + R_E$ gives

$$E_1 = -\frac{GM_Em}{2r} \approx -1.07 \times 10^{11} \text{ J}$$

For the satellite at rest on the surface of the planet, the mechanical energy is

$$E_2 = -\frac{GM_Em}{R_E} \approx -2.18 \times 10^{11} \text{ J}$$

The mechanical energy change is

$$\Delta E = E_2 - E_1 = -1.11 \times 10^{11} \text{ J}$$

(b) Energy is required to raise the temperature of a material; and energy is required to cause a material to pass from one phase to another phase. (This latter process occurs at one temperature.) Energy is often expressed as a calorie for such considerations. The conversion to Joules is

$$1 \text{ cal} = 4.187 \text{ J}$$

The "heat of fusion" for aluminum is 95.3 kcal/kg $\approx 3.99 \times 10^2$ J/kg. This is the amount of energy required to cause one kilogram of aluminum to melt. Thus the energy required to melt the entire satellite is

$$E_{melt} = 3.99 \times 10^2 \times 3500 \approx 1.4 \times 10^6 \text{ J}$$

(Actually another smaller amount of energy is required to raise the temperature of aluminum to its melting temperature.) The "heat of vaporization" for aluminum is 2520 kcal/kg $\approx 1.06 \times 10^4$ J/kg. This is the amount of energy required to cause one kilogram of aluminum to vaporize (boil). Thus the energy required to vaporize the entire satellite is

$$E_{vapor} = 1.06 \times 10^4 \times 3500 \approx 3.71 \times 10^7 \text{ J}$$

The total energy for both processes is $\approx 3.85 \times 10^7$ J. The energy change above is sufficient to cause both melting and vaporization.

Problem 5.10 (Ohanian, page 267, problem 12)

Let $m_1 = 1500$ kg and $m_2 = 3500$ kg be the mass of the car and truck respectively. Label the north direction by the y-axis and the east direction by the x-axis. Then the car has velocity $v_1 = 80$ km/h $= 22.2$ m/s along the y direction, and the truck has velocity $v_2 = 50$ km/h $= 13.9$ m/s along the x direction.

The momentum for the each is given by

$$\vec{p}_1 = m_1 v_1 \hat{y} = 3.330 \times 10^4 \ \hat{y}$$

$$\vec{p}_2 = m_2 v_2 \hat{x} = 4.865 \times 10^4 \ \hat{x}$$

where the units for $\vec{p}$ are kg $\cdot$ m/s^2.

(a) The momentum after the collision is

$$\vec{P} = (m_1 + m_2)\vec{V} = 5000\ \vec{V}$$

Momentum conservation gives

$$\vec{P} = \vec{p_1} + \vec{p_2} = 4.865 \times 10^4\ \hat{x} + 3.330 \times 10^4\ \hat{y}$$

Thus the velocity of the two cars after the collision is

$$\vec{V} = \frac{1}{5000}(4.865 \times 10^4\ \hat{x} + 3.330 \times 10^4\ \hat{y}) = 9.73\ \hat{x} + 6.66\ \hat{y}$$

The magnitude is

$$|V| = \sqrt{(9.73)^2 + (6.68)^2} \approx 11.8 \text{ m/s}$$

The direction is given by

$$\theta = \tan^{-1}\left(\frac{6.66}{9.73}\right) \approx 34^\circ$$

This means that both cars move 34° north of east.

(b) The kinetic energy before the collision is

$$K_{before} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \approx 7.1 \times 10^5 \text{ J}$$

The kinetic energy after the collision is

$$K_{after} = \frac{(m_1 + m_2)V^2}{2} \approx 3.5 \times 10^5 \text{ J}$$

The change in kinetic energy is

$$\Delta K = K_{after} - K_{before} \approx -3.6 \times 10^5 \text{ J}$$

Thus 3.6×10^5 J of kinetic energy is lost during the collision.

Problem 5.11 (Ohanian, page 267, problem 13)

The force of impact of the hydrogen atoms on the satellite is, as always, just

$$\vec{F} = \frac{d\vec{p}}{dt}$$

For convience, label the area of the satellite $A = 1.0 \text{ m}^2$, the density of ions $\rho = 10^7 \text{ cm}^{-3}$, the mass of each ion $m = 1.7 \times 10^{-27}$ kg, and the speed of each ion $v = 4 \times 10^5$ m/s.

Consider a small time interval, Δt. Then the ions impart a small amount of momentum, $\Delta\vec{p}$, to the satellite. This gives the relationship

$$\vec{F} \approx \frac{\Delta\vec{p}}{\Delta t}$$

which will become exact in the limit $\Delta t \to \infty$. (It turns out that if the solar wind is assumed to be uniform in time, then the relationship written is exactly true even for finite Δt.)

The volume of solar wind that strikes the satellite during the time interval Δt is

$$V = Av\Delta t$$

The number of ions in this volume is

$$N = \rho V = \rho Av\Delta t$$

If all the ions stick to the satellite, then the momentum delivered to the satellite, along the direction of the solar wind, is

$$\Delta p = mvN = m\rho Av^2\Delta t$$

Therefore the force is

$$\vec{F} \approx \frac{\Delta\vec{p}}{\Delta t} = \frac{m\rho Av^2\Delta t}{\Delta t} = m\rho Av^2$$

which is exact in the limit $\Delta t \to 0$. Therefore the force is exactly

$$\vec{F} = m\rho A v^2 = 2.72 \times 10^{-9} \text{ N}$$

where the direction is given by the solar wind.

Problem 5.12 (Ohanian, page 268, problem 27)

As in Example 6 on page 254, we will use Equation (26) on page 254 which gives

$$z_{CM} = \frac{1}{M} \int z\rho \, dV = \frac{1}{M} \left[\int_{semi} z\rho \, dV + \int_{rod} z\rho \, dV \right]$$

where we have broken the integral into two pieces: one piece, "semi", includes all the mass along the semicircular section, the other piece, "rod", includes all the mass along the straight section.

The "semi" integral was worked out in Example 6. The integral is then given as

$$\int_{semi} z\rho \, dV = 2\rho A R^2$$

(Be careful. The M here is the mass of the entire section; the M for Example 6 is the mass for only the "semi" section.) Now we need to calculate the remaining "rod" integral.

$$\int_{rod} z\rho \, dV$$

But this integral is clearly proportional to the z component of the center of mass for the rod, which is 0. We can also see this because $z = 0$ along the straight section. Therefore

$$\int_{rod} z\rho \, dV = 0$$

Therefore

$$z_{CM} = \frac{1}{M} \left[2\rho A R^2 + 0 \right]$$

The mass is given by

$$M = M_{semi} + M_{rod} = \rho A \pi R + \rho A 2R = AR\rho(\pi + 2)$$

and then

$$z_{CM} = \frac{2R}{2+\pi}$$

As in Example 6, the symmetry of the masses indicates that

$$x_{CM} = 0$$

Problem 5.13 (Ohanian, page 270, problem 45)

Let $v = 5.0 \times 10^3$ m/s and $h = 2.5 \times 10^4$ m and m be the mass of the ballistic missile.

Consider the equations of motion for an object of mass m' with only horizontal speed v' at an altitude of h. The equations of motion give

$$z = -\frac{gt^2}{2} \quad \text{and} \quad x = v't$$

This object will strike the ground at the time

$$t_{land} = \sqrt{\frac{2h}{g}}$$

and a location

$$x_{land} = v'\sqrt{\frac{2h}{g}}$$

For the piece with mass $m' = \frac{m}{2}$ that falls immediately downward, $v' = 0$, so

$$x_{land} = 0$$

For the piece with mass $m' = \frac{m}{2}$ that does not fall immediately downward, momentum conservation gives

$$\frac{m}{2} \cdot 0 + \frac{m}{2} \cdot v' = mv \qquad \Longrightarrow \qquad v' = 2v$$

The subsequent motion gives

$$x_{land} = 2v\sqrt{\frac{2h}{g}} \approx 7.1 \times 10^5 \text{ m}$$

The center of mass moves as if the missile never exploded, i.e. $m' = m$ and $v' = v$. Then

$$x_{land} = v\sqrt{\frac{2h}{g}}$$

which is clearly halfway between the landing points for the two fragments.

Problem 5.14 (Ohanian, page 270, problem 50)

This problem refers to problem 5.10.

(a) The translational kinetic energy of the center of mass before the collision is

$$\frac{(m_1 + m_2)v_{CM}^2}{2} = \frac{\vec{P}^2}{2(m_1 + m_2)} = \frac{(m_1 + m_2)V^2}{2}$$

where $\vec{P}$ is the total momentum and V is the velocity of the center of mass, both calculated in problem 5.10. This form illustrates that the translational kinetic energy is conserved if momentum is conserved. Thus the value is given by

$$\frac{(m_1 + m_2)v_{CM}^2}{2} = \frac{5000 \cdot (11.8)^2}{2} \approx 3.5 \times 10^5 \text{ J}$$

The internal kinetic energy satisfies equation (34) on page 260

$$K = K_{int} + \frac{(m_1 + m_2)v_{CM}^2}{2} \qquad \Longrightarrow \qquad K_{int} = K - \frac{(m_1 + m_2)v_{CM}^2}{2}$$

So before the collision,

$$K_{int} = K_{before} - 3.5 \times 10^5 \approx 7.1 \times 10^5 - 3.5 \times 10^5 \approx 3.6 \times 10^5 \text{ J}$$

where the value of K_{before} was calculated in problem 5.10.

(b) The translational kinetic energy of the center of mass after the collision is the same as before by the argument above. The value is given by

$$\frac{(m_1 + m_2)v_{CM}^2}{2} = \frac{5000 \times (1.18 \times 10^{-2})^2}{2} = 3.5 \times 10^5 \text{ J}$$

After the collision, both the car and truck move as one object, so there is no internal kinetic energy.

$$K_{int} = 0$$