Massachusetts Institute of Technology - Physics Department
Physics-8.01 Fall 1999

Solutions for Assignment # 10

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December 8, 1999 9:54 am

Problem 10.1

(a) We pull with a force F (both ropes combined). At the point O the sidewalk exerts an unknown force F' on the cylinder. The condition for just lifting the cylinder off the street is $N = 0$ where N is the usual normal force the street exerts on the cylinder. We can eliminate F' by measuring all torques about O. (A torque is defined as positive if it would cause the cylinder to roll up the sidewalk.) The condition for just rotating about the corner is $\tau = 0$ where τ is the total torque about O.

$$0 = FR\sin(\pi - (\theta + \alpha)) - MgR\sin\left(\pi - \left(\frac{\pi}{2} - \theta\right)\right) = FR\sin(\theta + \alpha) - MgR\cos\theta$$

where we use the trigonometric identities $\sin(\pi - x) = \sin x$ and $\sin\left(\frac{\pi}{2} + x\right) = \cos x$. Solving the above equation for F gives

$$\frac{F}{Mg} = \frac{\cos\theta}{\sin(\theta + \alpha)}$$

(b) A local minimum or maximum occurs when

$$0 = \frac{dF}{d\alpha} = -Mg \cdot \frac{\cos\theta\cos(\theta + \alpha)}{\sin^2(\theta + \alpha)}$$

The only solution to the above equation is

$$\cos(\theta + \alpha) = 0 \qquad \Longrightarrow \qquad \alpha = 90^\circ - \theta$$

For $\theta = 30^\circ$ then $\alpha = 60^\circ$. By plotting $\frac{F}{Mg}$ we see that $\alpha = 60^\circ$ is a local minimum and the global minimum and $\alpha = 0$ is the global maximum (not local). It's not hard to show that if $\theta < 45^\circ$ then the maximum is at $\alpha = 0^\circ$, and if $\theta > 45^\circ$ then the maximum is at $\alpha = 45^\circ$.

Problem 10.2

Let $L = 5$ m, $r = \frac{1}{2} \cdot 0.01$ m, $A = \pi r^2$, $M = 400$ kg, and $Y = 0.36 \times 10^{10}$ N/m^2 (please see Table 14.1 on page 366 for nylon). Equation (27) on page 366 gives

$$\frac{\Delta L}{L} = \frac{1}{Y}\frac{F_{\text{APP}}}{A} \qquad \Longrightarrow \qquad F_{\text{APP}} = \frac{YA}{L} \cdot \Delta L$$

where F_{APP} is the total applied force and ΔL is the corresponding increase in length. It is convenient to define a constant k as

$$k = \frac{YA}{L} \approx 5.7 \times 10^4 \text{ N/m} \qquad \Longrightarrow \qquad F_{\text{APP}} = k\Delta L$$

First we need to calculate the amount the rope is stretched, Δ, due to the weight of 400 kg alone.

$$\Delta = \frac{Mg}{k} \approx 0.070 \text{ m}$$

(a) We need to find the additional force required to stretch the rope another 0.03 m.

$$F + Mg = k(\Delta + 0.03) \qquad \Longrightarrow \qquad F = k \cdot 0.03 \approx 1.78 \times 10^3 \text{ N}$$

(b) Let x denote the displacement down from equilibrium. (Equilibrium corresponds to the rope length $L + \Delta$.) The equation above gives the necessary applied force F_{APP} to stretch the rope by ΔL; therefore the rope must exert a restoring force of equal and opposite magnitude. Therefore for a displacement of x down from equilibrium, now with the applied force not present, the total force is

$$F_{\text{TOT}} = -k(\Delta + x) + Mg = -kx$$

where in the above, F is the total force down corresponding to the displacement x down from equilibrium.

The above equation is valid for x such that $\Delta + x > 0$ ($x > -\Delta \approx -0.07$ m) because if $\Delta + x < 0$ then the rope will develop slack and the restoring force will vanish and the form of the above equation would have to change. For the present, the initial displacement is 0.03 m with no initial velocity, so the motion will never create slack in the rope; hence the above equation remains valid for all the motion. The motion is, therefore, simple harmonic in x with period

$$T = 2\pi \cdot \sqrt{\frac{M}{k}} \approx 0.53 \text{ s}$$

(c) With the initial condition of $x = 0.10m$ with no initial velocity the motion will create a slack in the rope. The mass will start at $x = 0.10$ m proceed through $x = 0$ to

$x = -0.07$ m at which point the rope will start to go slack and there will be no restoring force, only the force of gravity. The motion will no longer be completely simple harmonic motion.

(d) The ultimate tensile strength is given by Table 14.1 on page 366 as $\left(\frac{F}{A}\right)_{\max} = 3.2 \times 10^8$ N/m^2. The maximum mass will then be

$$M_{\max} = \frac{A}{g}\left(\frac{F}{A}\right)_{\max} \approx 2.6 \times 10^3 \text{ kg}$$

You can only calculate a conservative lower limit to the amount by which the rope will stretch. As we have seen in lectures, Hooke's law (thus $\frac{\Delta L}{L} = \frac{F}{YA}$) no longer holds near the breaking point. The lower limit would be

$$\Delta L = \frac{L}{Y}\left(\frac{F}{A}\right)_{\max} \approx 0.44 \text{ m}$$

Thus a conservative lower limit for the length of the rope is 5.44 m.

Problem 10.3

Let $\rho = 1$ gm/cm^3 be the density of water, ρ_W be the density of the wood, ρ_O be the density of the oil, and V be the volume of wood. For the water,

$$\rho \cdot \frac{2}{3}V = \rho_W \cdot V \qquad \Longrightarrow \qquad \rho_W = \frac{2}{3} \cdot \rho \approx 0.67 \text{ gm/cm}^3$$

For the oil,

$$\rho_O \cdot 0.9 \cdot V = \rho_W \cdot V \qquad \Longrightarrow \qquad \rho_O = \frac{1}{0.9} \cdot \rho_W \approx 0.74 \text{ gm/cm}^3$$

Problem 10.4

Let p_0 be atmospheric pressure, $\rho = 10^3$ kg/m^3 the density of water, M the mass of the rod, A its cross-section area, $d = 3$ m its equilibrium depth, and x its displacement down from its equilibrium depth. The equilibrium depth is given by

$$Mg = \rho g d A$$

The force equation is

$$p_0 A + Mg - (p_0 + \rho g(d + x))A = M\ddot{x}$$

where $p_0 + \rho g(d + x)$ is the pressure at the bottom of the rod. If we use the expression $Mg = \rho g d A$ in the above equation we have

$$-\rho g A x = M\ddot{x}$$

If we use the equivalent expression $\rho g A = \frac{Mg}{d}$ in the above equation and cancel a factor of M we have

$$-\frac{g}{d}\cdot x = \ddot{x} \qquad \Longrightarrow \qquad T = 2\pi\sqrt{\frac{d}{g}} \approx 3.5 \text{ s}$$

Notice that it was unnecessary to specify the value of $\text{kg}\cdot\text{m/s}_0$ to evaluate the expression for T; hence a change in atmospheric pressure of 5% will not change the period of oscillation at all.

Problem 10.5 (Ohanian, page 378, problem 63)

Let $\rho = 7.8 \times 10^3$ kg be the density of this steel. Now focus on one half of the meter stick, and let x denote the distance from the axis of rotation to a particular bit of length dx. If the cross-section area is A then the corresponding mass is $dm = \rho A dx$. The centripetal force required to accelerate this bit of steel is $dF = dm\cdot\omega^2 x = \rho A\omega^2 x dx$ where ω is the angular velocity of the meter stick. The total tension at the center of the meter stick is then

$$T = \int dF = \int_0^{\frac{1}{2}} \rho A\omega^2 x dx = \rho A\omega^2 \left.\frac{1}{2}x^2\right|_0^{\frac{1}{2}} = \frac{\omega^2\rho A}{8} \qquad \Longrightarrow$$

$$\omega = \sqrt{\frac{8}{\rho}\frac{T}{A}}$$

where $\frac{1}{2}$ is the length of half the meter stick. The maximum angular velocity is

$$\omega_{\text{max}} = \sqrt{\frac{8}{\rho}\left(\frac{T}{A}\right)_{\text{max}}} \approx 624 \text{ radian/s}$$

where the ultimate tensile strength for steel, $\left(\frac{T}{A}\right)_{\text{max}} = 3.8 \times 10^8$ N/m^2, is given in Table 14.1 on page 366.

Problem 10.6 (Ohanian, page 537, problem 13)

(a) The rate of increase of heat is the amount of energy deposited by each electron (3.2×10^{-9} J) times the rate for electrons (3.0×10^{14} s^{-1}).

$$\frac{\Delta Q}{\Delta t} = 3.2 \times 10^{-9} \cdot 3.0 \times 10^{14} = 9.6 \times 10^5 \text{ J/s} \approx 230 \text{ kcal/s}$$

where the conversion used is 1 cal = 4.186 J.

(b) The mass of water in the beam dump is $m = 10^3 \times$ kg/m$^3 \cdot 12$ m$^3 = 1.2 \times 10^4$ kg. Equation (1) on page 517 is

$$\Delta Q = mc\Delta T$$

Therefore the rate of increase of temperature is

$$\frac{\Delta T}{\Delta t} = \frac{1}{mc}\frac{\Delta Q}{\Delta t} \approx 0.019°\text{C/s}$$

where the specific heat capacity for water is given by Table 20.1 on page 516 as $c =$ 1.00 kcal/kg $\cdot°$ C.

Problem 10.7 (Ohanian, page 512, problem 17)

The volume of one mole of gas at STP is

$$V = \frac{RT}{p} \approx \frac{8.31 \cdot 273}{10^5} \approx 0.0227 \text{ m}^3$$

The volume occupied by the helium atoms is

$$V_{\text{atom}} \approx 6.02 \times 10^{23} \cdot 3 \cdot 10^{-30} \approx 1.81 \times 10^{-6} \text{ m}^3$$

Therefore the fraction of volume occupied by the helium atoms is

$$f = \frac{V_{\text{atom}}}{V} \approx 8 \times 10^{-5} = 0.008\%$$

Problem 10.8 (Ohanian, page 513, problem 24)

(a) Let $p = 1$ atm $= 1.01 \times 10^5$ N/m^2, A be the cross-section area of the diving bell, $h = 2$ m, and h' be the height of air in the diving bell once it is immersed. Then p', the pressure of air in the diving bell once it is immersed, is given by the pressure of the water at a depth of 15 m.

$$p' = p + \rho g z$$

where $\rho = 10^3$ kg/m^3 is the density of water, and $z = 15$ m. This gives the value

$$p' = 2.48 \times 10^5 \text{ N/m}^2$$

Assuming the temperature is constant, then

$$phA = pV = p'V' = p'h'A \qquad \Longrightarrow \qquad h' = \left(\frac{p}{p'}\right) h \approx 0.81 \text{ m}$$

Therefore the water rises $2 - 0.81 \approx 1.2$ m.

(b) The air must be pumped in at a pressure equal to the pressure of the water at the bottom of the diving bell, which is at a depth of $15 + 1.2 = 16.2$ m. The pressure at that level is

$$p_f = p + \rho g z \approx 2.6 \times 10^5 \text{ N/m}^2$$

where $z = 16.2$ m. The amount of air required at that pressure is $\frac{p_f V}{RT}$ where $V \approx 3.5$ m^3 and $T = 15°C = 288$ K. The original amount of air is $\frac{pV}{RT}$. Therefore the number of moles which must be added is $\frac{(p_f - p)V}{RT}$. The mean molecular mass of air is 29 g. (Please see Example 3 on page 498.) Therefore the mass of air which must be added is

$$M = (0.029) \cdot \frac{(p_f - p)V}{RT} \approx 6.84 \text{ kg}$$

Problem 10.9 (Ohanian, page 512, problem 16)

We will assume that the air forming the bubble does satisfy the ideal gas law.

$$pV = nRT$$

If the temperature and the amount of air remains constant, then the right-hand side of the above equation is constant.

$$p_1 V_1 = p_2 V_2 \tag{1}$$

The pressure and volume at the depth of 15 m is

$$p_1 = 1.013 \times 10^5 \text{ N/m}^2 + (9.8 \text{ m/s}^2)(1.0 \times 10^3 \text{ kg/m}^3)(15 \text{ m}) = 2.483 \times 10^5 \text{ N/m}^2$$

$$V_1 = \frac{4}{3}\pi \left(\frac{1.0 \text{ cm}}{2}\right)^3 = 5.236 \times 10^{-7} \text{ m}^3$$

The pressure at the surface is

$$p_2 = 1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$$

Using Equation (1) gives the volume at the surface as

$$V_2 = \frac{p_1}{p_2} \cdot V_1 = 1.283 \times 10^{-6} \text{ m}^3$$

Assuming the bubble remains spherical, the diameter at the surface is given by

$$d = 2 \cdot \left(\frac{3V_2}{4\pi}\right)^{\frac{1}{3}} = 1.348 \times 10^{-2} \text{ m} = 1.348 \text{ cm}$$

Problem 10.10 (Ohanian, page 512, problem 21)

The density of the external air is $\rho_\mathrm{a} = 1.20$ kg/m^3. The mass of the balloon, etc. without the air is $M_\mathrm{b} = 730$ kg, and the volume of the balloon is $V_\mathrm{b} = 2200$ m^3. If the mass of the hot air inside the balloon is M and the equivalent mass of cold air (same volume but atmospheric temperature) is $\rho_\mathrm{a} V_\mathrm{b}$, then the condition for the balloon to just lift-off is

$$\rho_\mathrm{a} V_\mathrm{b} - M = M_\mathrm{b} \qquad \Longrightarrow \qquad M = \rho_\mathrm{a} V_\mathrm{b} - M_\mathrm{b} = 1.91 \times 10^3 \text{ kg}$$

The balloon is open at the bottom, so the pressure of the air within is the same as the pressure of the external air. The mass per mole of air is given in Example 3 on page 498 as $m = 29.0$ g/mole. The pressure of the external air at $T_\mathrm{a} = 20°\mathrm{C} = 293.15$ K is

$$p_\mathrm{a} = \frac{n_\mathrm{a} R T_\mathrm{a}}{V_\mathrm{a}} = \frac{n_\mathrm{a} m}{V_\mathrm{a}} \cdot \frac{R T_\mathrm{a}}{m} = \rho_\mathrm{a} \cdot \frac{R T_\mathrm{a}}{m} = 1.0086 \times 10^5 \text{ N/m}^2$$

The air within the balloon has the mass

$$M = m n_\mathrm{b} = m \cdot \frac{p_\mathrm{b} V_\mathrm{b}}{R T_\mathrm{b}} \qquad \Longrightarrow \qquad T_\mathrm{b} = m \cdot \frac{p_\mathrm{b} V_\mathrm{b}}{RM} = 405 \text{ K} = 132°\mathrm{C}$$

where $p_\mathrm{b} = p_\mathrm{a}$.

Problem 10.11 (Ohanian, page 513, problem 28)

Consider a slab of air with horizontal area A and height dh. The pressure difference between the bottom and the top of the slab must support the weight of the air within the slab.

$$pA - (p + dp)A - (A\ dh) \cdot \rho \cdot g = 0 \qquad \Longrightarrow \qquad dp = -\rho g\ dh \tag{2}$$

If we would assume that ρ is a constant, then we would derive the usual result for pressure, but if the temperature remains constant with height, then the density must vary with height. We can obtain a relationship between the pressure and density by using the ideal gas law.

$$p = \frac{N}{V} \cdot kT = \frac{mN}{V} \cdot \frac{kT}{m} = \rho \frac{kT}{m} \qquad \Longrightarrow \qquad \rho = \frac{mp}{kT}$$

where $m = \frac{29.0\text{ g}}{N_\mathrm{A}} = 4.82 \times 10^{-26}$ kg is the mass of one molecule of air. We can use the above to write Equation (2) as

$$dp = -p \cdot \frac{mg}{kT}\ dh$$

This is now a valid differential equation for the pressure, p, as a function of height, h. We can solve this equation by following these steps.

$$\frac{1}{p}\,dp = -\frac{mg}{kT}\,dh$$

$$\ln p - \ln p_0 = \int \frac{dp}{p} = -\frac{mg}{kT}\int dh = -\frac{mg}{kT}(h-0)$$

$$p = p_0 e^{-\frac{mgh}{kT}}$$

Problem 10.12 (Ohanian, page 535, problem 5)

The lateral loops act as gaps to allow for thermal expansion in the long oil pipelines. Even for a small change in temperature, the increase in length of the pipeline could be very large. This would prevent the pipeline from being attached rigidly at each end. The loops allow each shorter section of pipe to stretch but prevent the net effect from being cumulative. Since the oil must still flow loops must be used instead of simple gaps as in bridges.