Massachusetts Institute of Technology - Physics Department

Physics - 8.01 **Assignment #1** Fall 1999

SOLUTIONS

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Problem 1.1 (*Estimates and Uncertainties – Ohanian Question 1.1*)

For the first two objects, I estimated the error by using half the smallest division on my wooden ruler, 0.5 mm. For the last two objects, I used a metal tape measure which can expand and contract with changing temperature, just as the wooden ruler can with changing humidity. For large distances, such as a desk, it can produce a noticeable error, which I estimated at ± 2 mm.

Object	Estimate by Eye	Measured Value	Comments
Mug (height)	22 cm	15.75 ± 0.05 cm	I didn't start off too hot
CD Case	15 cm	14.20 ± 0.05 cm	a bit better
Desk	1.6 m	1.524 ± 0.002 m	not bad for something so big
Ruler	30.5 cm	30.48 ± 0.05 cm	I am the greatest!

Problem 1.2 (*Fundamental Units – Ohanian Question 1.14*)

As Ohanian says, "position, time, and mass give a *complete* description of the behavior and the attributes of an ideal particle." Therefore, we must have units of each of these ([L], [T], and [M]) in some combination in any system of units that we use.

If we take length, mass, and density as our fundamental units, we have

$$[\text{length}] \rightarrow [\text{L}]\ ,\ [\text{mass}] \rightarrow [\text{M}]\ ,\ [\text{density}] \rightarrow \frac{[\text{M}]}{[\text{L}]^3}.$$

As you can see, we have no units of time [T] so we **cannot** take these as the three fundamental units. If, however, we take length, mass, and speed, we have

$$[\text{length}] \rightarrow [\text{L}]\ ,\ [\text{mass}] \rightarrow [\text{M}]\ ,\ [\text{speed}] \rightarrow \frac{[\text{L}]}{[\text{T}]}.$$

Here, we have all the necessary units, and this **is** a valid choice for the three fundamental units. It is important to remember that we must be able to "separate" the fundamental units from each other. For example, mass and speed alone contain all the necessary units but there would be no way to separate [L] from [T] with just these two. By having length, mass, and speed, we can extract each fundamental unit on its own (for example, by dividing length by speed to get [T]).

Problem 1.3 (*Thickness of a Sheet of Paper*)

a) Thickness = **10.0 mm**

b) Absolute uncertainty = $\pm$**0.5 mm**

c) Relative uncertainty = $\frac{\text{absolute uncertainty}}{\text{measurement}} = \frac{0.5\ \text{mm}}{10.0\ \text{mm}} = \mathbf{5\%}$

d) There's 85 pages, so the thickness of one page is $\frac{10.0\pm0.5\ \text{mm}}{85}$ =$\mathbf{118 \pm 6\ \mu m}$.

e) Absolute uncertainty = $\mathbf{6\ \mu m}$

f) The relative uncertainty is unchanged – **5%**.

g) Different students will apply different amounts of pressure when measuring the thickness. Another factor is the humidity of the room in which the measurement is done, as this will affect the thickness of the paper. Also, there is a possibility that the paper is slightly different from book to book.

Problem 1.4 (*Relative Uncertainties*)

The relative uncertainty is the absolute uncertainty divided by the value of the measurement. Let's pick the antelope as our bone. It has a measured thickness of 18.3 mm with an uncertainty of 1.0 mm; therefore, its relative uncertainty is

$$\frac{1.0\text{ mm}}{18.3\text{ mm}} = \mathbf{5.5\%}$$

The student's length (using Zach from the 10 AM lecture) was 183.2 cm with an uncertainty of 0.1 cm. The absolute uncertainties have roughly the same value (in the case of the antelope, it's exactly the same) in both cases. However, the relative uncertainty in Zach's length is

$$\frac{0.1\text{ cm}}{183.2\text{ cm}} = \mathbf{0.055\%}$$

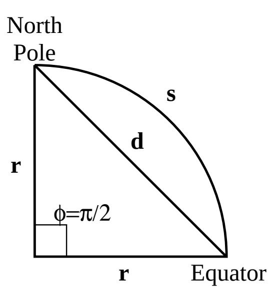
Why are the two relative uncertainties so very different? Because Zach is much longer than the antelope's femur is thick. They both have the same absolute uncertainty because they both come from the same source, namely, human error in "eye-balling" a ruler or meter stick. The relative uncertainties, however, are two orders of magnitude different!

Problem 1.5 (*Distant Quasar – Ohanian Problem 1.8*)

To figure out the distance on a map, all you need to do is multiply the actual distance (12.4×10^9 ly) by the scale of the map ($1 : 1.5 \times 10^{20}$).

$$\begin{aligned}
\text{(actual distance)} &= 12.4 \times 10^9\text{ ly} \cdot \frac{9.46 \times 10^{15}\text{ m}}{1\text{ ly}} = 1.17 \times 10^{26}\text{ m} \\
\text{(map distance)} &= \text{(actual distance)} \cdot \text{(scale)} \\
&= 1.17 \times 10^{26}\text{ m} \cdot \frac{1}{1.5 \times 10^{20}} \\
&= \mathbf{7.82 \times 10^5\ m} = 782\text{ km} = 486\text{ miles} \\
&= \text{roughly the distance between Cambridge, MA and} \\
&\quad\ \text{Richmond, VA}
\end{aligned}$$

Problem 1.6 (*Distances on Earth – Ohanian Problem 1.10*)



The distance from the pole to the equator measured along the surface of the earth is the length of the arc **s** in the figure. An easy way to remember how to compute arclength is to realize that the ratio of the arclength **s** to the whole circumference is the same as the ratio of the subtended angle ϕ to 2π.

$$\frac{\mathbf{s}}{2\pi\mathbf{r}} = \frac{\phi}{2\pi}$$

$$\mathbf{s} = \mathbf{r}\phi = 6.37 \times 10^6\text{ m} \cdot \frac{\pi}{2} = \mathbf{1.00 \times 10^7\ m} = 6220\text{ miles}$$

The distance **d** along a straight line is given by the Pythagorean theorem.

$$\mathrm{d} = \mathrm{r}\sqrt{2} = \mathbf{9.01 \times 10^6\ m} = 5600\text{ miles}$$

Problem 1.7 (*Atoms in your Body – Ohanian Problem 1.26*)

Let's use oxygen (O) as an example. First, figure out how many grams.

$$73 \text{ kg} \times 65\% = 47.45 \text{ kg} = 47450 \text{ g}$$

Then, we divide by the atomic weight (the number of grams per mole) and multiply by Avogadro's number (the number of atoms per mole).

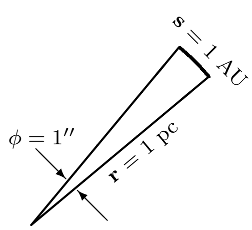
$$47450 \text{ g} \cdot \frac{1 \text{ mole}}{15.994 \text{ g}} \cdot \frac{6.02214 \times 10^{23} \text{ atoms}}{1 \text{ mole}} = 1.78661 \times 10^{27} \text{ atoms}$$

Repeat this for every element, and add to get the total.

Element	Percentage	$\rightarrow$	Grams	$\times$	$\frac{N_A}{\text{atomic weight}}$	=	# atoms
O	65%	$\rightarrow$	47450 g	$\times$	$\frac{6.022\times10^{23} \text{ atoms}}{15.994 \text{ g}}$	=	1.78661×10^{27} atoms
C	18.5%	$\rightarrow$	13050 g	$\times$	$\frac{6.022\times10^{23} \text{ atoms}}{12.011 \text{ g}}$	=	6.54308×10^{26} atoms
H	9.5%	$\rightarrow$	6935 g	$\times$	$\frac{6.022\times10^{23} \text{ atoms}}{1.00794 \text{ g}}$	=	4.14345×10^{27} atoms
N	3.3%	$\rightarrow$	2409 g	$\times$	$\frac{6.022\times10^{23} \text{ atoms}}{14.0067 \text{ g}}$	=	1.03574×10^{26} atoms
Ca	1.5%	$\rightarrow$	1095 g	$\times$	$\frac{6.022\times10^{23} \text{ atoms}}{40.08 \text{ g}}$	=	1.64527×10^{25} atoms
P	1%	$\rightarrow$	730 g	$\times$	$\frac{6.022\times10^{23} \text{ atoms}}{30.97376 \text{ g}}$	=	1.41932×10^{25} atoms
						TOTAL:	6.71859×10^{27} atoms

The answer should have the same precision as the original information – 2 significant digits. So the total number of atoms is $\mathbf{6.7 \times 10^{27}}$.

Problem 1.8 (*Astronomical Distances – Ohanian Problem 1.29*)



a) We can use the formula from Problem 1.5 (with ϕ in radians).

$$\mathbf{s} = \mathbf{r}\phi \rightarrow 1 \text{ AU} = 1 \text{ pc} \cdot 1'' \cdot \frac{1'}{60''} \cdot \frac{1^\circ}{60'} \cdot \frac{\pi}{180^\circ}$$
$$\rightarrow 1 \text{ pc} = \mathbf{206,265 \text{ AU}}$$

b) First we need to express ly in meters and then we can use the information given by Ohanian.

$$1 \text{ ly} = 3.00 \times 10^8 \text{ m/s} \cdot 86400 \text{ s/day} \cdot 365 \text{ days} = 9.46 \times 10^{15} \text{ m}$$

$$\rightarrow 1 \text{ pc} = 206,265\text{AU} \cdot \frac{1.496 \times 10^{11} \text{ m}}{1 \text{ AU}} \cdot \frac{1 \text{ ly}}{9.46 \times 10^{15} \text{ m}} = \mathbf{3.258 \text{ ly}}$$

c) 1 pc = $\mathbf{3.086 \times 10^{16}}$ **m** 1 ly = $\mathbf{9.46 \times 10^{15}}$ **m**

Problem 1.9 (*Mean Density of Stars – Ohanian Problems 1.37 and 1.38*)

1.37 For density, we use the Greek letter rho (ρ).

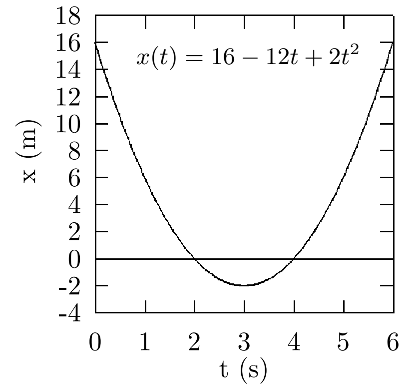
$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{2.0 \times 10^{30} \text{ kg}}{\frac{4}{3}\pi(7.0 \times 10^8 \text{ m})^3} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = \mathbf{1.4 \text{ g/cm}^3}$$

1.38 Same thing.

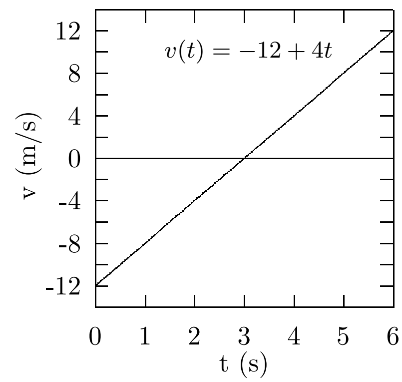
$$\rho = \frac{2.0 \times 10^{30} \text{ kg}}{\frac{4}{3}\pi(20 \times 10^3 \text{ m})^3} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \cdot \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = 6.0 \times 10^{13} \frac{\text{g}}{\text{cm}^3} \cdot \frac{1 \text{ tonne}}{10^6 \text{ g}} = \mathbf{6.0 \times 10^7 \text{ tonne/cm}^3}$$

Problem 1.10 (*Position, Velocity, and Acceleration*)

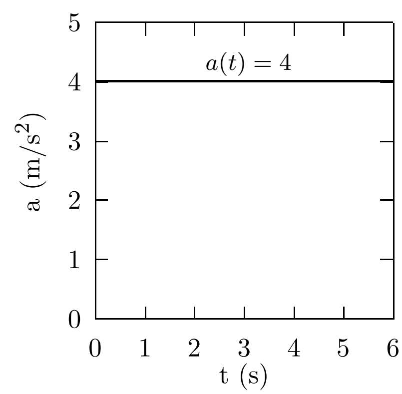
a) $x(t) = 16 - 12t + 2t^2$ (see plot)



b) By differentiating, $v(t) = -12 + 4t$ (see plot)



c) Another differentiation yields $a(t) = 4$ (see plot)



d) Using the equation in part **b)**: $\boldsymbol{v(0) = -12}$ **m/s**, $\boldsymbol{v(2) = -4}$ **m/s**, $\boldsymbol{v(4) = 4}$ **m/s**

e) Using the equation in part **c)**: $\boldsymbol{a(0) = 4}$ $\mathbf{m/s^2}$, $\boldsymbol{a(2) = 4}$ $\mathbf{m/s^2}$, $\boldsymbol{a(4) = 4}$ $\mathbf{m/s^2}$

f) Set $v(t) = -12 + 4t = 0$ to find that $\boldsymbol{t = 3}$ **s**. Now, plug this t into the equation for $x(t)$ to find the position of the object when the velocity is 0. To summarize $\boldsymbol{x(3) = -2}$ **m**, $\boldsymbol{v(3) = 0}$ **m/s**, and of course $\boldsymbol{a(3) = 4}$ $\mathbf{m/s^2}$.

g) The average velocity between two times is defined as

$$\bar{v}_{t_1,t_2} \equiv \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

$$\rightarrow \bar{v}_{-1,3} = \frac{x(3) - x(-1)}{3 - (-1)} = \frac{-2 - 30}{4} = \mathbf{-8\ m/s}$$

h) We use the same formula

$$\bar{v}_{0,6} = \frac{x(6) - x(0)}{6 - 0} = \frac{16 - 16}{6} = \mathbf{0\ m/s}$$

i) The average speed s is the total distance traveled over the time taken. The total distance is found by adding up each one-way segment of the complete journey. Looking at the plot, we can see that the object traveled from $x = 16$ m to $x = -2$ m and back to $x = 16$ m (this is why plots are useful). The total distance is 36 m and the time taken is 6 s. Therefore, the average speed is $s = (36\text{ m})/(6\text{ s}) = $ **6 m/s**.

j) What does it mean to reverse direction? Of course, it means that you're going one way, and then you turn around and go the other way. If you're going in the negative direction, you then start going in the positive direction and vice versa. In other words, your velocity changes sign. The point at which this happens is at zero (this is an important concept). You can also think about it in terms of the calculus. If you reverse direction, then your position will be a local minimum or maximum at that point. To find where this happens, you take the derivative of position and set it equal to zero. This is exactly the same as setting velocity equal to zero since *velocity is the derivative of position*!

¿From part **f)**, we know that $v(3) = 0$ so the object reverses direction at $\boldsymbol{t = 3}$ **s**.

Problem 1.11 (*Car Crash and Seat Belts – Ohanian Problem 2.35*)

This is a tough problem because the answer requires many steps, and at the end it is *essential* to ask the all-important question: "Does my answer make sense?"

First of all, what exactly does it mean to crash into something? It means that two objects (e.g., a passenger and a dashboard) come into contact, i.e., that their positions are equal at the same time. This is the first step – describing the position of each object during the period of deceleration.

We know that for uniform acceleration,

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

and now we must figure out what x_0, v_0, and a are for the passenger and for the dashboard.

We need to set up a coordinate system. Imagine freezing time right as the car begins to crash. We'll put our origin at the position of the passenger. His initial conditions are $x_0 = 0$ and $v_0 = 50$ km/h $= 14$ m/s. The dashboard is 0.6 m in front of the passenger and has initial conditions $x_0 = x_{\text{sep}} = 0.6$ m and $v_0 = 14$ m/s. What about acceleration[1]? Since the dashboard is attached to the car, it will accelerate uniformly at $a = -200$ m/s^2, but, since the passenger isn't wearing a seat belt and therefore isn't connected to the car, he experiences no acceleration as the car *begins* to crash; he "flies out of his seat." He of course accelerates quite quickly when he hits the dashboard. Until then, he continues to travel forward with velocity v_0.

The equations of position for the passenger (p) and dashboard (d) are

$$x_p(t) = 0 + v_0 \cdot t + \frac{1}{2} \cdot 0 \cdot t^2 = v_0 \cdot t$$

$$x_d(t) = x_{\text{sep}} + v_0 \cdot t + \frac{1}{2} \cdot (-a) \cdot t^2$$

Okay, so we've described the positions, but we need to keep in mind that these equations are only valid during the period of uniform acceleration, i.e., from when the car starts accelerating until the time it comes to rest.

We have position as a function of time $x(t)$, and we can take a derivative to get velocity as a function of time $v(t)$. So, let's solve for the specific time, t_{bang} when the position of the passenger is equal to the position of the dashboard. Then we can plug that into our equations for $v(t)$ and find the (almost) final answer.

$$\begin{aligned}
x_p(t_{\text{bang}}) &= x_d(t_{\text{bang}}) \\
v_0 \cdot t_{\text{bang}} &= x_{\text{sep}} + v_0 \cdot t_{\text{bang}} - \frac{1}{2} \cdot a \cdot (t_{\text{bang}})^2 \\
0 &= x_{\text{sep}} - \frac{1}{2} \cdot a \cdot (t_{\text{bang}})^2 \\
t_{\text{bang}} &= \sqrt{2\frac{x_{\text{sep}}}{a}} = \sqrt{2\frac{0.6 \text{ m}}{200 \text{ m/s}^2}}
\end{aligned}$$

[1]It may sound awkward at first to speak of something crashing into a wall as "accelerating," but this is the language of physics. Remember, to "accelerate" merely means to change velocity, whether it be increasing velocity or decreasing velocity. Non-physicists generally use the term "decelerate" to indicate that an object's *speed* is decreasing. For example, suppose the car were traveling backward with velocity $v = -14$ m/s and then crashed until its velocity were $v = 0$ m/s. Strictly speaking, it's velocity increased, but because its speed decreased there might be confusion over whether to call it an "acceleration" or a "deceleration." We overcome the confusion in 8.01 by calling ***ALL*** changes in velocity "accelerations."

$$t_{\text{bang}} = 0.077 \text{ s}$$

Notice that I didn't plug in any numbers until the *last* step. This is very important.

To find the speed of the passenger and the dashboard as functions of time, we simply take the derivatives of $x_p(t)$ and $x_d(t)$ with respect to t.

$$v_p(t) = \frac{dx_p(t)}{dt} = \frac{d}{dt}v_0 \cdot t = v_0$$

$$v_d(t) = \frac{dx_d(t)}{dt} = \frac{d}{dt}(x_{\text{sep}} + v_0 \cdot t - \frac{1}{2} \cdot a \cdot t^2) = v_0 - a \cdot t$$

The relative velocity of the passenger to the dashboard is the difference between the two

$$\begin{aligned} v_{\text{rel}}(t) &= v_p(t) - v_d(t) \\ &= v_0 - (v_0 - a \cdot t) \\ &= a \cdot t \\ \rightarrow v_{\text{rel}}(t_{\text{bang}}) &= 200 \text{ m/s}^2 \cdot 0.077 \text{ s} \\ v_{\text{rel}}(t_{\text{bang}}) &= 15.4 \text{ m/s} \end{aligned}$$

Does this answer make sense?? ***NO!*** Why not? Because the car and passenger were originally traveling at 14 m/s. During the collision, the passenger continues to travel at 14 m/s, but the automobile uniformly accelerates *on its way to a stop.* So the maximum relative velocity could be at most 14 m/s. Anything higher than this would indicate that the car has "bounced back" which we know does not happen. (Remember that our original equations were only valid for the period of uniform acceleration. If we continue to use them after the car has come to a stop, we are implying that it begins to accelerate *away* from the wall!)

We realize that the car must have come to a stop before the passenger hit the dashboard (in fact, it stopped at $t = 0.069$ s). The relative velocity is therefore $\boldsymbol{v_{\text{rel}} = 14}$ **m/s**.

Problem 1.12 (*Brain Teaser – Returning to the same Point on Earth*)

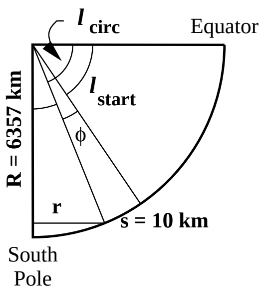
There's not much methodology to brain teasers, but one helpful hint is to consider "special circumstances." In this case, we're looking for special points on the globe. Good starting points would be the North and South Poles and the Equator.

The **North Pole** is one point that meets the stated conditions — if you walk south from it a certain distance d, walk in a circle for however long you want, and then walk north the same distance d, you'll be back at the North Pole. Great.

The other points that meet the condition have to do with special circles around the South Pole that have the following property: their circumferences are $10/n$ km where $n = 1, 2, 3, \ldots$

Let circle$_1$ be the circle with 10 km circumference. After walking east for 10 km, you're back where you started. Let circle$_2$ be the circle with $10/2 = 5$ km circumference. After walking 10 km east, you're still back where you started. And so on for $n =$ any positive integer.

Each of these circles will be the second leg of the journey. The starting point of the journey will be any point 10 km north of where the special circle lies. (These starting points also form a circle.) Then the first leg would be to travel 10 km south to the special circle, the second leg would be to walk east around the special circle for 10 km arriving back where the second leg started, and the third leg would be to walk north for 10 km (retracing exactly the steps of the first leg) to arrive back at the starting point.



Assuming the Earth is a perfect sphere, we can use the figure on the left to figure out the latitude l_{circ_1} of the circle of circumference 10 km. From the figure, we see that this leads to the condition $2\pi\mathbf{r} = 10$ km, where $\mathbf{r} = 6357$ km $\cdot \sin(90° - l_{\text{circ}_1})$. Solving for l_{circ_1}, we find that circle$_1$ is at $l_{\text{circ}_1} = 89°59'08''$ South. (The error introduced by our assumption that the Earth is a perfect sphere is less than $1''$.)

Now we need to find the latitude l_{start_1} of the circle 10 km north of this. From the figure and what we know of arclength, we find that

$$\frac{10 \text{ km}}{2\pi \cdot 6357 \text{ km}} = \frac{\phi}{360°} \rightarrow \phi = 5'24''$$

The latitude l_{start_1} of the starting point is then $l_{\text{circ}_1} - \phi$.

$$\boldsymbol{l_{\text{start}_1} = 89°53'44'' \textbf{ South}}$$

This procedure can be repeated for the second circle of circumference 5 km to find $\boldsymbol{l_{\text{start}_2} = 89°54'10''}$ **South**, and so on. There are an infinite number of these circles, each with an infinite number of points.

Problem 1.13 (*Human Femur*)

a) I measured d to be 1.6 ± 0.2 cm and l to be 24.5 ± 0.5 cm. The errors are large because I was not very confident in my ability to measure the image accurately. The ratio is $d/l = 1.6/24.5 = 0.065$ To get the uncertainty when we're dividing measured quantities, we can use one of two methods which are equivalent. Remember that these are simplified methods that we use in 8.01; later in your MIT career, you'll probably take more formal courses that will teach how to arrive at a final uncertainty which is derived from several quantities.

Method One: Figure out the maximum that the computed value can be, i.e., add the error in the numerator, and subtract the error in the denominator:

$$\frac{1.6 + 0.2 \text{ cm}}{24.5 - 0.5 \text{ cm}} = \frac{1.8 \text{ cm}}{24.0 \text{ cm}} = 0.075$$

Then subtract the computed value without uncertainties from this to get the range of uncertainty:

$$0.075 - \frac{1.6 \text{ cm}}{24.5 \text{ cm}} = 0.075 - 0.065 = 0.010$$

Method Two: When multiplying or dividing measured quantities, add the relative uncertainties of each quantity to get the total relative uncertainty:

$$\frac{0.2 \text{ cm}}{1.6 \text{ cm}} + \frac{0.5 \text{ cm}}{24.5 \text{ cm}} = 12.5\% + 2.0\% = 14.5\%$$

Multiply this by the computed value without uncertainties to get the total absolute uncertainty:

$$14.5\% \cdot \frac{1.6}{24.5} = 14.5\% \cdot 0.065 = 0.010$$

So the final answer is $\boldsymbol{d/l = 0.065 \pm 0.010}$.

b) To average we just add up all the quantities and divide by the total number of quantities. When adding or subtracting numbers with uncertainties attached, we add the absolute uncertainties together (at least, we do this in 8.01).

$$d/l_{\text{avg}} \quad = \quad \frac{(0.063 \pm 0.009) + (0.092 \pm 0.007) + \ldots + (0.085 \pm 0.004)}{7}$$

$$
\begin{aligned}
&= \frac{0.611 \pm 0.039}{7} \\
\boldsymbol{d/l_{\mathrm{avg}}} &= \mathbf{0.087 \pm 0.006}
\end{aligned}
$$

c) The following interesting note is from Professor Bernie Burke.

> Among mammals, d/l appears to be approximately constant. This is not what one would expect from Galileo Galilei's reasoning as discussed in lectures.
>
> This follows from a correct and complete analysis of the scaling for failure of columns. Columns can fail two ways: by having a load so great that the material yields (this is what we evaluated in lectures), or by having a load so great that the column buckles (not considered by Galileo).
>
> The point at which the material yields is described by the yield modulus. The yield modulus for well-aged concrete is about 2000 pounds per square inch, and it may yield (i.e. crumble) sooner if it has only been poured for a few days; mild steel is 25 times stronger, and there are special alloys that have a yield modulus 50 times that of concrete.
>
> Another form of columnar failure, on the other hand, is determined by the elastic modulus, sometimes called Young's modulus. Imagine a thin metal column under a load. Theoretically, if it were perfectly vertical, it could sustain a load up to the yield point. If it is not in stable equilibrium, however, any sideways displacement, no matter how small, will develop into a bend, and the column acts like a spring. Try this by pushing down on a vertical plastic ruler; you will see the bending, and you will feel the spring action. The greater the load, the more the sideways bend. If the column is too slender, the bend will increase uncontrollably, and the column will buckle. The great mathematician Leonard Euler tackled this problem. We suggest you look up on the web "Euler Buckling".

It is unclear why the value of d/l is significantly lower for humans than for the average four-legged mammal. Apparently, we are still far enough from the danger zone of buckling that it does not pose a problem.