Introduction to Plasma Physics I

Course 22.611j, 8.613j, 6.651j — I.H. Hutchinson

2 Oct 03 — Problem Set 4 — due 9 Oct 03

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1. Calculate $\ln\Lambda$ for a plasma with $n_e = n_i = 10^{20} m^{-3}$ and $T_e = 5$ keV, in the following situations:
   (a) A high energy electron beam of energy E=100keV, colliding with stationary hydrogen ions.
   (b) This beam colliding with the electrons in this plasma.
   (c) The thermal collisions in this plasma.

2. This exercise is to explore the thermal collision frequency derivation by applying it to a different distribution function, namely the shifted squared Lorentzian:

$$f(\mathbf{v}) = n\frac{1}{\pi^2 v_t^3}\left(\frac{1}{(\mathbf{u}-\mathbf{u}_d)^2+1}\right)^2$$

where v_t is a constant representing the thermal velocity, $\mathbf{u} \equiv \mathbf{v}/v_t$, and $\mathbf{u}_d = \hat{\mathbf{e}}_x u_d$, $u_d \ll 1$ is the normalized drift velocity. Expand this distribution to first order in u_d. Substitute into the formula for the rate of momentum loss dp/dt to stationary targets. Use the same trick based on the isotropy of the unshifted distribution to express the result in terms of an integral over speed u. Perform that integral and hence derive $\overline{\nu_{ei}}$ for this electron distribution function.

3. Calculate the mean free path for momentum loss (equal to the characteristic velocity divided by the collision frequency) for
   (a) An electron at thermal energy in a tokamak plasma of equal electron and ion temperatures 10keV, and density 0.5×10^{20} m^{-3}.
   (b) A thermal ion in the same plasma.
   (c) A thermal electron in a processing plasma of temperature 5 eV, and density 5×10^{18} m^{-3}.

4. A toroidal hydrogen plasma with circular cross-section has uniform temperature $T_e = 2$ keV across its minor radius, a = 0.3m. The major radius is R = 1.2m. Calculate the toroidal electric field E_ϕ required to drive a current of $4 \times 10^5 A$ the long way round the torus, and hence the required one-turn toroidal E.M.F. (called the "loop voltage"). [You may do this calculation to lowest order in a/R, and adopt a generic value of $\ln\Lambda$]

Calculate, ignoring relativity, the minimum parallel energy at which an electron becomes a "runaway" if the density of this plasma is 10^{19} m^{-3}. Does this energy justify your ignoring relativistic effects?