Introduction to Plasma Physics I

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1. Consider a charged particle moving in the x-y plane under the influence of a magnetic field varying slowly with x, pointing in the z-direction, $\mathbf{B} = B(x)\hat{\mathbf{e}}_z$ and uniform electric field in the y-direction, $\mathbf{E} = E\hat{\mathbf{e}}_y$, $(E/B < c)$. Show that as a result of the grad-B drift, the particle has work done on it by the electric field. Thus, show explicitly from conservation of energy, that the quantity $\frac{1}{2}mv^2/B$ is constant in this situation.

2. The $\mathbf{E} \wedge \mathbf{B}$ drift cannot exceed the speed of light. Therefore particle motion in crossed electric and magnetic fields needs to be treated relativistically when E/B > c. [Note that this occurs as B → 0.] By integrating the relativistic equation of motion:

$$\frac{d}{dt}\left[\frac{m_o\mathbf{v}}{\sqrt{(1 - v^2/c^2)}}\right] \;=\; q\left(\mathbf{E} + \mathbf{v}\wedge\mathbf{B}\right)$$

   or otherwise, show what happens to the particle orbit when E/B > c, and sketch the orbit when **B** is in the z-direction, **E** is in the x-direction, and z-motion is neglected.

3. Electrons in a magnetic mirror have a distribution function $f_o(\mathbf{v})$ at its center, where the field magnitude is B_o. The highest field, in the mirror throat, is B_1. Calculate the fraction of electrons that is trapped if f_o is

   (a) A Maxwellian $f_o = \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(\frac{-mv^2}{2T}\right)$

   (b) A squared Lorentrian $f_o \propto \frac{1}{\left(1+\frac{v^2}{v_t^2}\right)^2}$

   (c) Proportional to $\frac{v_\perp}{v} \exp\left(\frac{-mv^2}{2T}\right)$

4. A particle of mass m and charge q moves in uniform magnetic field B pointing in the z-direction under the influence of a line-charge of magnitude Q per unit length aligned along the z-axis. (The configuration is invariant in the z-direction.)

   (a) Calculate the trajectory of its guiding center and the time elapsed before the x and y coordinates return to their initial values.

   (b) If the magnetic field is allowed to vary in time with a small constant time derivative dB/dt, calculate the evolution of the radial position of the guiding center ($r_g = \sqrt{(x_g^2 + y_g^2)}$) and of the particle kinetic energy.

(c) Is the sum of particle kinetic energy and electrostatic potential energy ($q\phi$) constant? Do the charge and mass of the particle matter?

5. A uniform hydrogen plasma of infinite length, and temperature T equal for electrons and ions, surrounds a straight infinite cylindrical conductor carrying current I. If the perturbation of the magnetic field by currents in the plasma is negligible,

   (a) What is the mean electron drift?

   (b) What is the mean ion drift?

   (c) What is the mean current density?

   (d) [This last part is quite difficult and will be for bonus marks]. If the perturbation of the plasma currents to the magnetic field is *not* negligible, and the inner edge of the plasma is at $r = a$, calculate the magnetic field as a function of radius.