Axioms of Quantum Mechanics

22.51 Quantum Theory of Radiation Interaction – Fall 2012

1. The properties of a quantum system are completely defined by specification of its state vector $|\psi\rangle$.
The state vector is an element of a complex Hilbert space $\mathcal{H}$ called the space of states.

2. With every physical property $\mathcal{A}$ there exists an associated linear, Hermitian operator A (called observable), which acts in the space of states $\mathcal{H}$.
The eigenvalues of the operator are the possible values of the physical properties.

3.a *(Born Rule)* If $|\psi\rangle$ is the vector representing the state of a system and $|\varphi\rangle$ represents another physical state, there exists a probability P($|\psi\rangle$,$|\varphi\rangle$) of finding $|\psi\rangle$ in state $|\varphi\rangle$, which is given by the squared modulus of the inner product on $\mathcal{H}$:

$$P(|\psi\rangle, |\varphi\rangle) = |\langle\psi|\varphi\rangle|^2$$

3.b *(Wave function collapse)* If A is an observable with eigenvalues $\{a_n\}$ and eigenvectors $\{|n\rangle\}$, given a system in the state $|\psi\rangle$, the probability of obtaining a_n as the outcome of the measurement of A is

$$P(a_n) = |\langle n|\psi\rangle|^2$$

After the measurement the system is left in the state $|n\rangle$

4. The evolution of a closed system is unitary (reversible). The evolution is given by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$$

where H is the Hamiltonian of the system and $\hbar$ the reduced Planck constant.