22.38 PROBABILITY AND ITS APPLICATIONS TO RELIABILITY, QUALITY CONTROL AND RISK ASSESSMENT

Fall 2004

EXAMPLES

Consider two sets of valves tested to failure.

Set 1, $m_1 = 15$	Set 2, $m_2 = 10$
T_{fail} (months)	T_{fail} (months)
49.7	59.7
51.4	68.6
55.0	69.0
62.0	69.8
62.1	77.2
65.4	78.1
65.5	80.2
65.8	86.0
66.2	87.7
72.0	98.6
73.3	$\bar{x}_2 = 77.49$ mo
73.7	$s_2 = 11.34\ \text{mo}^2$
75.8	
76.7	
79.7	
$\bar{x}_1 = 66.29$ mo	
$s_1 = 9.17\ \text{mo}^2$	

$$\frac{(\bar{x}_1 - \bar{x}_2)}{\underbrace{s_P\left[\frac{1}{m_1} + \frac{1}{m_2}\right]^{1/2}}_{4.11}} = \frac{66.29 - 77.49}{10.07(1/5 + 1/10)^{1/2}} = -2.724$$

$$s_P = \frac{(m_1 - 1)s_1^2 + (m_2 - 1)s_2^2}{(m_1 + m_2 - 2)} = \left[\frac{(15 - 1)9.17^2 + (10 - 1)11.34^2}{15 + 10 - 2}\right]^{1/2} = 10.07\ \text{mo}$$

For value test data:

$$n = 15 \Rightarrow \upsilon = df = 14, \qquad \sqrt{14} = 3.74$$

$$x_n = 66.3 \text{ mo}, \qquad s_n^2 = 83.6 \text{ mo}^2, \qquad s_n = 9.14$$

$$P\left\{x_n - \left[t_{(n-1),(1-\alpha/2)}\right]\left(\frac{s_n}{\sqrt{n}}\right) < \mu < \bar{x} + \left[t_{(n-1),\alpha}\frac{s_n}{\sqrt{n}}\right]\right\} = 1 - \alpha$$

α	$\alpha/2$	$t_{1-\alpha/2} = -t_{\alpha/2}$	$t_{1-\alpha/2}\frac{s_n}{\sqrt{n}}$	$\bar{x} - (\ )$	$\bar{x} + (\ )$	$\frac{2(\ )}{\bar{x}}$
0.05	0.025	2.145	5.05	61.25	71.35	0.15
0.01	0.005	2.977	7.02	59.3	73.3	0.21

Now, test whether $\mu_1 = \mu_2$, at 95% confidence level ($\alpha = 0.05$). $t_{0.025} = -2.069$ for df - 23.

$$\text{Prob}\left\langle\left\{\left[t_{\alpha/2}\right]\left[s_P\right]\left[\frac{1}{m_1} + \frac{1}{m_1}\right]^{1/2}\right\} - (\bar{x}_1 - \bar{x}_2)\right\} < (\mu_1 - \mu_2) < \left\{\left[t_{(1-\alpha/2)}\right]\left[s_P\right]\left[\frac{1}{m_1} + \frac{1}{m_1}\right]^{1/2}\right\} - (\bar{x}_1 - \bar{x}_2)\right\}\right\rangle$$

$$= 1 - \alpha$$

or,

$$\text{Prob}\{[-2.069(4.11) - 11.2] < (\mu_1 - \mu_2) < [2.069(4.11) - 11.2]\} = 0.95$$

or,

$$\text{Prob}[-22.39 < (\mu_1 - \mu_2) < -2.69] = 0.95$$

$\therefore$ It is unlikely that $\mu_1 = \mu_2$.

For 15 elements in valve testing data set 1, obtain result:

$$\chi^2 = \frac{(n-1)s_n^2}{\sigma^2}$$

$$\bar{x} = 66.29 \text{ mo} \qquad s_n^2 = 83.6 \text{ mo}^2$$

$$P\left[\chi_\alpha^2 \le \frac{(n-1)s_n^2}{\sigma^2}\right] = 1 - \alpha$$

or,

$$P\left[\sigma^2 \le \frac{(n-1)s_n^2}{\chi_\alpha^2}\right] = 1 - \alpha.$$

Here n = 15 □ υ = df = (n - 1) = 14.

α	χ_α^2	$\frac{(n-1)s_n^2}{\chi_\alpha^2}$	$\sqrt{\frac{(n-1)s_n^2}{\chi_\alpha^2}} > \sigma$	$\frac{\sigma_{max}}{\bar{x}}$
0.05	6.57	177.8	13.3	0.20
0.01	4.66	250.8	15.8	0.24