22.38 PROBABILITY AND ITS APPLICATIONS TO RELIABILITY, QUALITY CONTROL AND RISK ASSESSMENT

EXAM 1 – SOLUTION

1. **(50 points)** Consider the flow network shown in Fig. 1. In this network flow occurs between nodes along the links shown, only in the direction from left to right. System success consists of at least two flow paths being open from node 0 to node C.
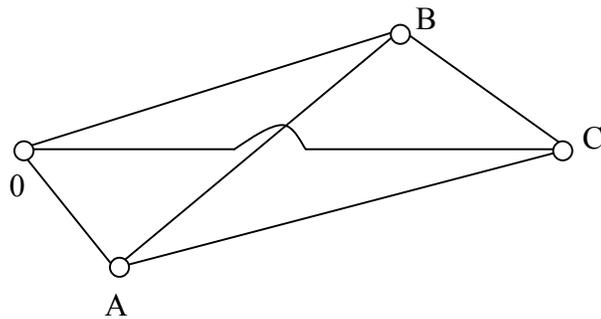

Figure 1

QUESTIONS AND SOLUTIONS FOR PROBLEM #1

a) **(10 points)** In a Venn diagram show the space of event S; and draw the success tree for event S.

Venn Diagram

Success, $S = (0 \cdot C) \cdot (A + B)$
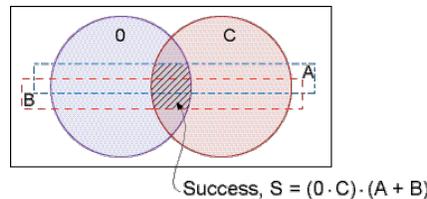
b) **(10 points)** What would be the probability of system success, S, should the probability of blockage of a single node be p and all links are always open?
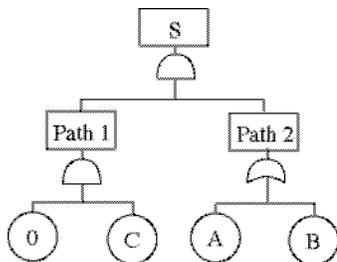
$$\text{Prob.}(S) = (1-p)^2 \cdot \left[2(1-p) - \left(1-p^2\right)\right] = (1-p)^3(1+p)$$

c) **(20 points)** What is the structure function for the system?

$$X_S = (X_0 X_C) \cdot \left[1 - (1 - X_A)(1 - X_B)\right]$$

d) **(30 points)** Should event S occur, what is the probability that the flow path $(0, C)$ participates?

$$\text{Prob.}(0 \cdot C | S) = 1$$

<p align="center">* * * * * * *</p>

2. **(50 points)** Consider a student supplementing his income by delivering pizzas. The distances of his deliveries are measured in units of traffic lights that must be transited.

The probability that a particular delivery will involve a total of n traffic lights is Poisson distributed, having mean and standard deviation of 3 (ignore values of $n > 6$). Treat the case $n = 0$ as a cancelled delivery.

A delivery consists of a succession of transits through m_i successive green lights for the ith transit, where

<p align="center">Prob.(m_i) is governed by the geometric distribution, where</p>

p is the probability of stoppage at a single traffic light, $\displaystyle\sum_{i=1}^{k} m_i = n$ and $p = 1/3$.

The time (in minutes) of a delivery trip (one way), t_t, varies as

$$t_t = \begin{cases} 15, & m_{max} \leq 5 \\ 15\left[1 - 0.2\left(\dfrac{m_{max}}{6}\right)\right], & m_{max} = 6 \end{cases}$$

NOTE: In doing this problem some possible outcomes addressed by the indicated probability distributions are actually impossible (i.e., $n > 6$; $m > 6$). Treating these facts correctly requires re-normalization of the probability distributions used. For this problem do not calculate these renormalizations, but rather treat the errors introduced by failure to do so as being negligible.

QUESTIONS AND SOLUTIONS FOR PROBLEM #2

<p align="center">One-Way Trip Information</p>

No. of Traffic Lights in Trip, n	Max. No. of Successive Traffic Lights Transited Without Stopping, m_{max}	Trip Time, t_t(min.)	Prob.(t_t)*
1	1	15	
2	2		
3	3		
4	4	↓	
5	5		
6	< 6		
6	6	12	

$$* \; \text{Prob.}(n = 6, m_{max} = 6) = \text{Prob.}(t_t = 12) = \underbrace{\left(\frac{\mu^6 e^{-\mu}}{6!}\right)}_{\substack{\text{Prob.}(m=6) \\ \mu=3}} \cdot \underbrace{\left[\left(\frac{2}{3}\right)^6\right]}_{\text{Prob.}(m_{max}=6)} = (0.05)(0.088) = 0.0044$$

$$\text{Prob.}(n < 6, m_{max} < 6) \cong 1 - \text{Prob.}(n = 6, m_{max} = 6) = 0.9956$$

a) **(30 points)** What is the probability mass function for the time spent during a two-trip delivery cycle (i.e., out and return)?

Two-way trip (out and back)

Total Trip Time (min.)	Probability
24	$(0.0044)^2 = 1.9 \times 10^{-5}$
27	$(0.0044)(0.9956) = 0.00438$
30	$(0.9956)^2 = 0.9912$
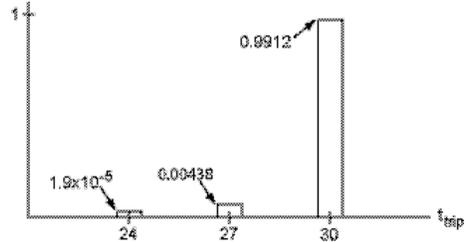
b) **(10 points)** What is the mean time spent on a two-trip delivery cycle?

$$E\left(t_{trip}\right) = \sum_{i=1}^{3} t_{trip_i} \, p\left(t_{trip_i}\right) = 29.85 \text{ min}$$

c) **(10 points)** What is the probability that a particular two-trip delivery cycle will require < 28 min?

$$\text{Prob.}\left(t_{trip} < 28\right) = \sum_{i=1}^{2} P\left(t_{trip_i}\right) = 0.004399$$