10/3/05
MWG

22.38 PROBABILITY AND ITS APPLICATIONS TO RELIABILITY, QUALITY CONTROL AND RISK ASSESSMENT

Fall 2005

EXTRA CREDIT

Due Oct. 6, 2005

This is an extra credit problem set (total credit = 0.5 of one problem set) concerning the final three games of the current scheduled baseball season between the Boston Red Sox and the New York Yankees.

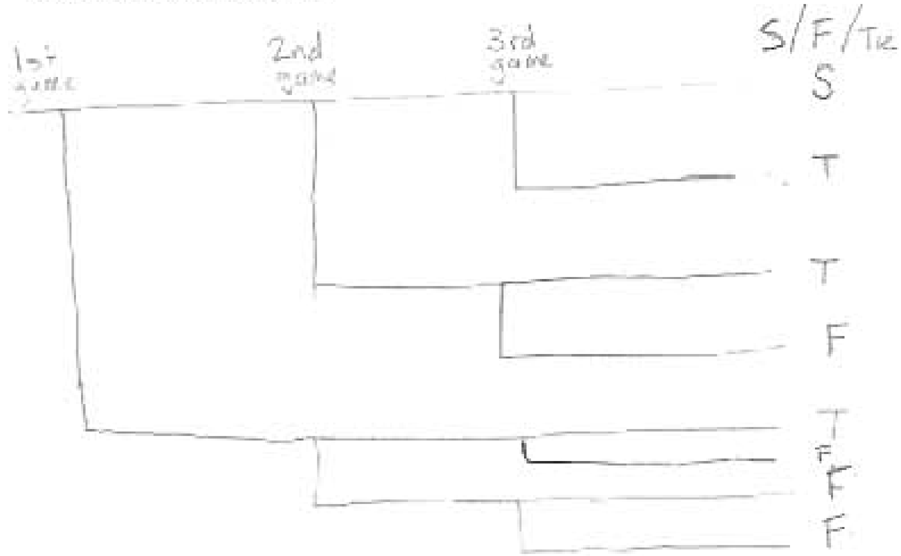
At the start of the last three games of the seasons the Red Sox are behind the Yankees by one game. In order to go into the World Series playoffs the Red Sox must be at least one game ahead of the Yankees at the end of the weekend. Consider the probability of a Red Sox win against the Yankees in a single game to be given by a probability mass function (PMF) having, before the start of the weekend series, equal likelihoods for the Red Sox win-probability values of 0.4, 0.5 and 0.6. Use news reports of the outcomes of the games in answering the questions below.

1. What is the event tree for the possible outcomes of the three game series?

2. As the weekend's games are played-out show how the Red Sox' win-probability PMF evolves as new evidence concerning their ability to win against the Yankees is revealed?

3. How does the probability of an overall Red Sox win of a place in the World Series playoffs change as the series evolves?

1)



extra-credit problem continued

2. $P(H_i/E) = P(E/H_i)\,P(H_i)\,/\,P(E)$; $P(E) = \sum_i P(E/H_i)\,P(H_i)$

Starting:

H_i	win-prob	$P(H_i)$
H_1	.4	.33
H_2	.5	.33
H_3	.6	.33

After E_1 : won 1st game $\Rightarrow P(E_1) = 0.495$

H_i/E_1	$P(E/H_i)$	$P(H_i)$	$P(H_i/E)$ = updated $P(H_i)$ for next step
H_1/E_1	.4	.33	.267
H_2/E_1	.5	.33	.333
H_3/E_1	.6	.33	.400

After E_2 : loss 2nd game $\Rightarrow P(E_2) = .4867$

H_i/E_2	$P(E_2/H_i)$	$P(H_i)$	$P(H_i/E)$
H_1/E_2	.6	.267	.329
H_2/E_2	.5	.333	.342
H_3/E_2	.4	.400	.329

After E_3 : win 3rd game $\Rightarrow P(E) = 0.5$

H_i/E_3	$P(E_3/H_i)$	$P(H_i)$	$P(H_i/E)$
H_1/E_3	.4	.329	0.263
H_2/E_3	.5	.342	0.342
H_3/E_2	.6	.329	0.3948

3. E_0 : $P(\text{in WS}) = [.33(.4) + .33(.5) + .33(.6)]^3 = .1213$

E_1 : $P(\text{in WS}) = [.267(.4) + .33(.5) + .4(.6)]^2 = .2635$

E_2 : $P(\text{in WS}) = 0$

E_3 : $P(\text{in WS}) = 0$

> assuming no wild card, and a tie = loss