22.312 **ENGINEERING OF NUCLEAR REACTORS**

Tuesday, October 20th, 2015, 1:00 – 2:30 p.m.

OPEN BOOK QUIZ 1 (solutions)

Problem 1 (50%) – Thermodynamic analysis of a cooling tower

i)

Taking the whole cooling tower as the system, the conservation of mass yields:

$$0 = \dot{m}_{w1} + \dot{m}_a - \dot{m}_{w3} - (\dot{m}_a + \dot{m}_{w4}) \quad \Rightarrow \quad \dot{m}_{w3} = \dot{m}_{w1} - \dot{m}_{w4} \tag{1}$$

where $\dot{m}_{w4}$ is the mass flow rate of water in the air at Point 4. The partial pressure of water in the air is easily found $P_{w4} = \phi_4 P_{sat}(T_4)$ = 4.24 kPa, where ϕ_4 = 1 and $P_{sat}(T_4)$ = 4.24 kPa is the saturation pressure of water at 30°C. The corresponding density (from the table) is ρ_{w4}=0.03 kg/m^3. Since air and water exiting the cooling tower at Point 4 occupy the same volume, the volumetric flow rate of water is equal to the volumetric flow rate of air:

$$\frac{\dot{m}_{w4}}{\rho_{w4}} = \frac{\dot{m}_a}{\rho_{a4}} \quad \Rightarrow \quad \dot{m}_{w4} = \rho_{w4}\frac{\dot{m}_a}{\rho_{a4}} \approx 431.4 \text{ kg/s}$$

where the density of air at Point 4 is calculated from the equation of state $\rho_{a4} = \frac{P_{a4}}{R_a T_4} \approx 1.11$ kg/m^3, and $P_{a4} = P_{atm} - P_{w4} \approx 96.76$ kPa is the partial pressure of air at Point 4. Finally, Eq. (1) yields $\dot{m}_{w3} \approx 16{,}569$ kg/s. Note that it is therefore necessary to provide make-up water for 431.4 kg/s to compensate for evaporative losses in this cooling tower.

ii)

Again taking the cooling tower as the system, the conservation of energy yields:

$$0 = \dot{m}_{w1}h_{w1} + \dot{m}_a h_{a2} - \dot{m}_{w3}h_{w3} - (\dot{m}_a h_{a4} + \dot{m}_{w4}h_{w4})$$

or

$$\dot{m}_{w3}(h_{w1} - h_{w3}) = \dot{m}_a(h_{a4} - h_{a2}) + \dot{m}_{w4}(h_{w4} - h_{w1}) \tag{2}$$

Now, let us expand each term in Eq. (2):

- Treating subcooled water as an incompressible fluid and recognizing that since P_1=P_3=P_{atm}, we have $h_{w1} - h_{w3} = c_w(T_1 - T_3) + \frac{P_1 - P_3}{\rho_w} = c_w(T_1 - T_3)$.
- Treating air as an ideal gas, we have $h_{a4} - h_{a2} = c_{pa}(T_4 - T_2)$

- Note that $h_{w4} = h_g$ =2556 kJ/kg from the table; and h_{w1} =146.7 kJ/kg from the problem statement.

The only unknown in Eq. (2) is T_3, from which $T_3 \approx 17.7$°C.

iii)
The answer depends on the humidity of the air at Point 3. If the humidity is low (i.e. <100%), then there is evaporation from the water into the air, and as a result the water discharged at Point 3 will be cooler than the water entering at Point 4, $T_3 < T_1$. On the other hand, if the humidity is 100%, there is no driving force for evaporation or heat transfer, and as result $T_3 = T_1 = T_2 = T_4$, and thus the cooling tower fails to cool the water coming from the condenser.

iv)
Upon coming in contact with the cooler atmosphere, the warm humid air exiting the tower will experience some condensation, which results in the generation of tiny droplets. Those droplets deflect the sun light thus making the plume visible.

Courtesy of Michael Kappel on Flickr. Used with permission.

Problem 2 (50%) – Transient analysis of a firebrick-based energy storage system

i)
The mass of air in the vessel is found from the equation of state:

$$M_a = \frac{P_1 V_a}{R_a T_1} \approx 1324 \text{ kg}$$

where V_a=180 m^3 and P_1 = 2 MPa. Then the air thermal capacity is $M_a c_{va} \approx 9.5\times10^5$ J/K, which is much lower than the firebrick thermal capacity $M_b c_b \approx 2.5\times10^9$ J/K, where M_b =3.6×10^6 kg.

ii)
Taking the firebricks and the air as the system, the energy equation is:

$$\frac{\partial E}{\partial t} = -\dot{W}_{electric} \quad \Rightarrow \quad (M_b c_b + M_a c_{va})\frac{dT}{dt} = -\dot{W}_{electric} \quad \Rightarrow$$

$$T(t) = T_1 + \frac{-\dot{W}_{electric}}{M_b c_b + M_a c_{va}} t \tag{3}$$

where we assumed $E(t) = (M_b c_b + M_a c_{va})T(t)$ (note that it would be acceptable to neglect the thermal capacity of the air, per the result in 'i'), $\dot{W}_{electric}$ = -100 MW, the vessel is well insulated ($\dot{Q}$=0), and $T(0)$ = T_1 = 950 K. Equation (3) is a linearly increasing function of time and is plotted (for $t < t_2$ = 3 hours) in the figure below. The maximum temperature reached by the system is $T(t_2)$ ≈ 1378 K. Note that the pressure of that air at time has risen to $P_2 = P_1 \frac{T(t_2)}{T_1}$ ≈ 2.9 MPa.

iii)
Again taking the firebricks and the air within the vessel as the system, the conservation of mass equation is:

$$\frac{\partial M}{\partial t} = \dot{m}_{in} - \dot{m}_{out}$$

Since $\dot{m}_{in} = \dot{m}_{out}$ = 300 kg/s, it follows that $\frac{\partial M}{\partial t}$ = 0, and thus the mass of air in the vessel is constant.

The conservation of energy equation is:

$$\frac{\partial E}{\partial t} = \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out} \;\Rightarrow\; (M_b c_b + M_a c_{va})\frac{dT(t)}{dt} = \dot{m}_{in}(h_{in} - h_{out}) \;\Rightarrow$$

$$(M_b c_b + M_a c_{va})\frac{dT(t)}{dt} = \dot{m}_{in} c_{pa}[T_{in} - T(t)] \tag{4}$$

where T_{in} = 950 K constant in time, and it was assumed that the air outlet temperature is equal to the system temperature at any given time (per the problem statement), and $c_{pa} = c_{va} + R_a$ = 1005 J/kg-K. Integrating Eq. (4) with the initial condition $T = T(t_2)$ at t_2, we get:

$$T(t) = [T(t_2) - T_{in}]e^{-(t-t_2)/\tau} + T_{in} \tag{5}$$

where $\tau = \dfrac{M_b c_b + M_a c_{va}}{\dot{m}_{in} c_{pa}} \approx 8361$ sec is the time constant of the system. Equation (5) is plotted in the figure below for $t > t_2$.

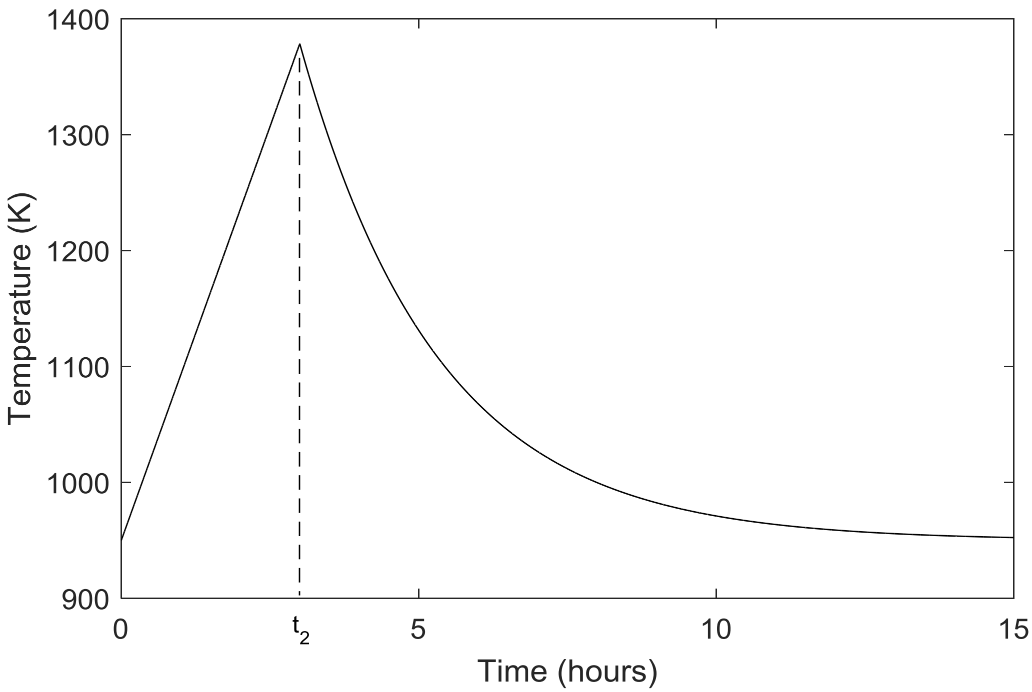

Figure. Temperature history of the FIRES system