

22.103 Microscopic Theory of Transport

(Fall 2003)

QUIZ (closed book)

Nov. 14, 2003

Question 1 (60% total - 12% each)

Answer briefly each of the following (answer all parts in the order asked, stick to the essentials and do not write more than is necessary)

- What is the Green-Kubo relation in the case of diffusion? What assumptions are involved, if any? Is this relation valid for all physical systems? In what way is it useful?
- What do we know about the dynamic structure factor in the hydrodynamic limit? Give a sketch of the spectrum and discuss the characteristic features?
- What is the equation governing the phase-space density correlation function for a low-density gas? What is meant by collisional invariants, and what are the invariants associated with this equation?
- What is the approximation made to the linearized Boltzmann equation that gives the Nelkin-Ghatak kinetic model? Discuss the physical meaning of this approximation.
- What is the difference between the Boltzmann equation as derived in class and the neutron transport equation? What is the closest analogue to the neutron transport equation that one can obtain from the conventional Boltzmann equation in the study of rarefied gas dynamics?

Question 2 (20%)

The Nelkin-Ghatak kinetic model can be used to describe the velocity autocorrelation function of a gas. Define this function (which is not normalized) as

$$\psi(t) = \int d^3v d^3v' \underline{v} \cdot \underline{v}' f(\underline{v}, \underline{v}', t)$$

where $f(\underline{v}, \underline{v}', t)$ is the distribution function satisfying the Nelkin-Ghatak model,

$$\left(\frac{\partial}{\partial t} + \alpha \right) f(\underline{v}, \underline{v}', t) = \alpha f_M(v) \int d^3v'' f(\underline{v}'', \underline{v}', t) \quad (1)$$

where α is the collision frequency, and $f_M(v)$ is the Maxwellian velocity distribution. Notice that equation (1) is an equation in the variable \underline{v} , but not the variable \underline{v}' . Use equation (1) to calculate $\psi(t)$ subject to the initial condition, $f(\underline{v}, \underline{v}', 0) = \delta(\underline{v} - \underline{v}') f_M(v)$. Discuss whether your result is surprising considering what we have discussed in class about the nature of the approximation in the Nelkin-Ghatak model.

Question 3 (20%)

You are given the velocity autocorrelation function as $3v_o^2 \exp(-t/\tau)$, where v_o is a thermal speed, and τ is a relaxation time (a constant). Use this result to determine the corresponding mean square displacement $\langle \Delta^2 r(t) \rangle$. Inspect your result for short and long times, and discuss whether these are the proper limiting behavior for a fluid.