22.101 Applied Nuclear Physics (Fall 2006)

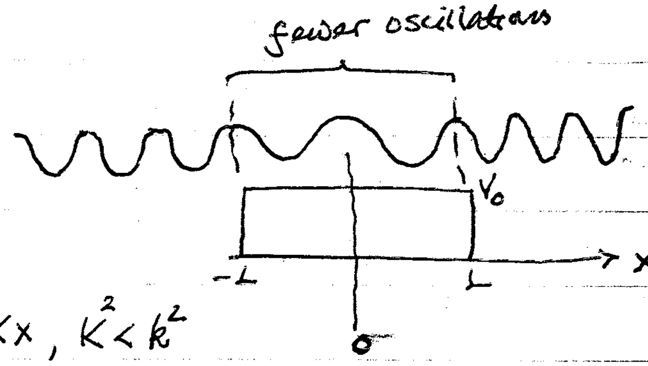
Solutions to Quiz 1 — Oct. 23, 2006

**Prob 1**

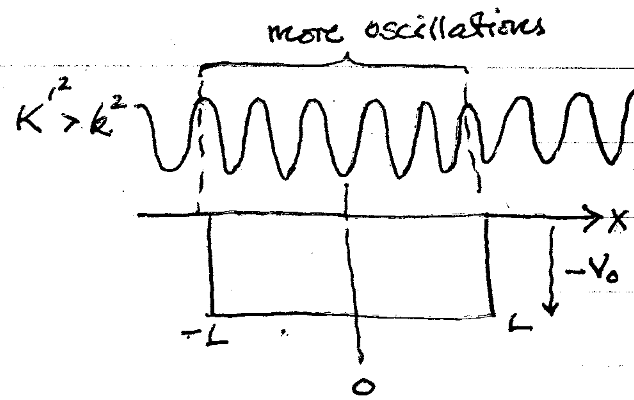
(a.) $\psi \sim \sin kx$, $k^2 = \frac{2m}{\hbar^2} E > 0$

barrier potential $\left[\frac{\hbar^2 d^2}{2m\, dx^2} + (E - V_0)\right]\psi(x) = E\psi$

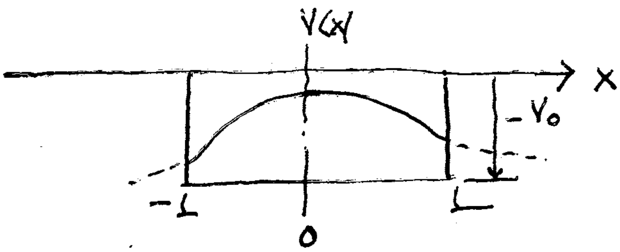
$$\Big[\frac{d^2}{dx^2} + \underbrace{\frac{2m}{\hbar^2}(E - V_0)}_{K^2}\Big]\psi(x) = 0 \qquad \psi \sim \sin Kx, \; K^2 < k^2$$



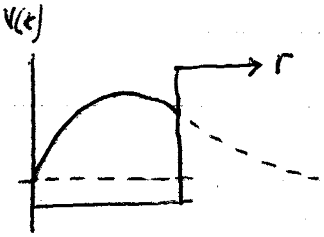

(b) potential well $\Big[\frac{d^2}{dx^2} + \underbrace{\frac{2m}{\hbar^2}(E_0 + V_0)}_{K'^2}\Big]\psi(x) = 0$



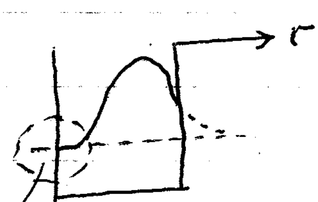

(c) (barely) bound state in well




**Prob 2** Labeling bound states in 3D with spherically symmetric potentials involves orbital angular momentum quantum number $l$, $l = 0, 1, 2, \ldots$ ($s, p, d, f\ldots$ waves). For each $l$, degeneracy is $2l+1$ (number of values magnetic quantum number $m$ can take)




ground state $l = 0$




1st excited state $l = 1$

wave function suppressed by centrifugal part of the potential $\frac{l(l+1)}{mr^2}$

Prob 3

S-wave scattering, scattering is spherically symmetric $f(\theta) \to f_0$

$$\psi(r) \sim e^{ikr\cos\theta} + f(\theta)\frac{e^{ikr}}{r}$$

$$A \sin(kr+\delta_0) = \sin kr + f_0 e^{ikr}$$

$$A\frac{1}{2i}\left\{e^{i(kr+\delta_0)} - e^{-i(kr+\delta_0)}\right\} = \frac{1}{2i}\left\{e^{ikr} - e^{-ikr}\right\} + f_0 e^{ikr}$$

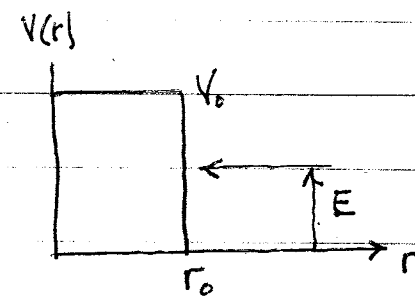
match coefficients of $e^{ikr}$, $e^{-ikr}$

$$\frac{A}{2i}e^{i\delta_0} = \frac{1}{2i} + f_0 \Rightarrow f_0 = \frac{1}{2i}\left(Ae^{i\delta_0} - 1\right)$$

$$-\frac{Ae^{-i\delta_0}}{2i} = -\frac{1}{2i} \Rightarrow A = e^{i\delta_0}$$

Thus $f_0 = \frac{1}{2i}\left[e^{2i\delta_0} - 1\right] = e^{i\delta_0}\sin\delta_0$ this completes the derivation

Prob 4 s-wave scattering, barrier, $E < V_0$




$r < r_0$ $\quad u = A \sinh k'r \qquad k'^2 \sim V_0 - E$

$r > r_0$ $\quad\;\; = B\sin(kr + \delta_0) \qquad k^2 \sim E$

BC at $r = r_0$

$$\boxed{k' \coth k'r_0 = k\cot(kr_0 + \delta_0)}$$

answer to (a)

(*)

Prob 4 - cont'd

low energy scattering $E \ll V_0$,

$$k'^2 \sim k_0^2 = \frac{2m}{\hbar^2} V_0$$

Eq (*) $\rightarrow$ $$k_0 \coth k r_0 = k \cot (k r_0 + \delta_0) \qquad (+)$$

Note: $k r_0 \ll 1$, $\delta_0 \ll 1$

(+) $\rightarrow$ $$k_0 \coth k_0 r_0 \sim k \frac{1}{(k r_0 + \delta_0)}$$

rearrange to give $$\delta_0 = k r_0 \left( \frac{\tanh k_0 r_0}{k_0 r_0} - 1 \right) \equiv - a k$$

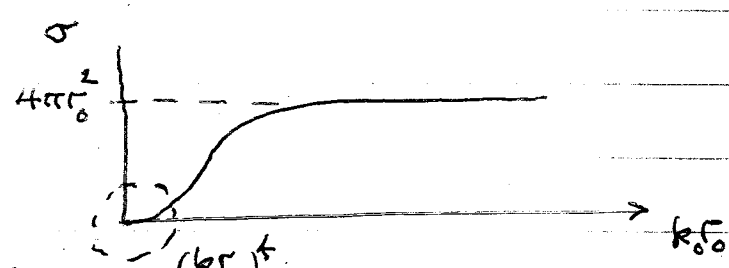
scattering length

$$\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0 \sim 4\pi \left(\frac{\delta_0}{k}\right)^2 = 4\pi a^2$$

$$= 4\pi r_0^2 \left( \frac{\tanh k_0 r_0}{k_0 r_0} - 1 \right)^2$$ answer to (b)

$\sigma \rightarrow 0$ $k_0 r_0 \ll 1$
no scattering when potential is weak

$\sigma \rightarrow 4\pi r_0^2$ billiard ball scatt. when potential
$k_0 r_0 \gg 1$ is strong (hard wall)




(c) rough estimate: $\sigma \sim 4\pi r_0^2 \sim 0.5$ barns
more detailed calc. will give results within a factor of $\sim 2$.
Thus result is much smaller than expt (20.4 barns)

implication: main prob. is we have neglected spin-dep interactions which we know can account for the discrepancy bet. 2.3 b (from potential well calc in class) and expt. Also, using a potential is NOT appropriate