22.101 Applied Nuclear Physics
(Fall 2006)

QUIZ No. 2 (closed book) **November 15, 2006**

Problem 1 (15%)
Consider a two-particle system comprising a neutron scattering from a proton.

a) Write the Hamiltonian for this system with the addition of a spin-spin coupling term representing the interaction energy between the spin of the proton S_p and the spin of the neutron S_n. *(Hint: This should be very similar to the spin-orbit coupling we described for a nucleus).*
b) Define a new operator S^2_{total} representing the total spin angular momentum of this system and use S^2_{total} to express the spin-spin coupling component of the Hamiltonian in terms of S^2_{total}, $S^2_{p,}$ and S^2_{n}.
c) For the Hamiltonian in part a) we could have expressed our eigenfunctions as $|s_p,s_n,m_p,m_n>$. Based on your result in part b), choose a new set of eigenfunctions that depends on the total spin angular momentum of the system.
d) Describe the implied constraints on the eigenvalues of the diagonalized operators for this new set of eigenfunctions.

Problem 2 (15%)

a) Sketch a nuclear potential well emphasizing the differences in the potential seen by neutrons and protons. Include a rough schematic of the energy levels for both types of particle.
b) Use this sketch to derive the asymmetry term in the empirical mass formula.

Problem 3 (15%)
Imagine a fictional universe where all nuclides follow a B/A curve of the form

$$B/A = \begin{cases} E_B & A_1 \leq A \leq A_2 \\ 0.1E_B & \text{otherwise} \end{cases}$$

with $A_2 > A_1$.

a) Draw a graph describing B/A vs. A and comment on the stability of the nuclides in the various regions of the graph.
b) Is/Are there region(s) of the graph where fission is favorable for isotopes in that/those region(s)? Explain.
c) Is/Are there region(s) of the graph where fusion is favorable for isotopes in that/those region(s)? Explain.

Problem 4 (15%)
Consider a radioisotope that decays through β^+ decay and electron capture with decay constants λ_β and λ_{EC}, respectively. An amount of this isotope is present at t=0.

a) What fraction of the nuclei present at t=0 will decay between arbitrary t_1 and t_2?
b) What fraction of the nuclei that decayed in part a will have done so via β^+ decay?

c) What fraction of the total nuclei present at t=0 will decay through β^+ decay between arbitrary t_1 and t_2? *Note: This is not the same as part b.*

d) Is your result in part c) reasonable? Interpret your result physically.

Problem 5 (20%)

Discuss the stopping power of electrons in a high Z absorber.

i) Sketch the stopping power of electrons in a high Z absorber such as lead over an energy range from zero to 10 times the rest mass energy.

ii) Explain the physical origin of all characteristic features.

iii) For portions of the curve where possible, give simple formulas describing the shape of the curve.

Problem 6 (20%)

The solution to the Q-equation is given as $\sqrt{E_3} = s \pm \sqrt{s^2 + t}$, where

$$s = \frac{\sqrt{M_1 M_3 E_1}\cos\theta}{M_3 + M_4} \quad \text{and} \quad t = \frac{M_4 Q + (M_4 - M_1)E_1}{M_3 + M_4}$$

a) For an endothermic reaction with $M_4 > M_1$, what is the constraint on E_3 for the described reaction to occur?

b) From this constraint, what can you conclude about s and t?