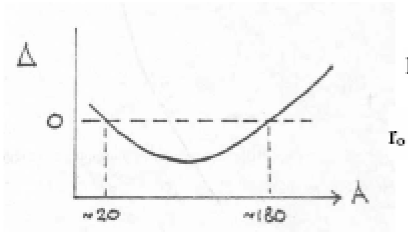


$R = r_o A^{1/3}$,

$r_o \sim 1.2 - 1.4 \times 10^{-13}$ cm.

Delta = M-A

Non-relativistic regime:

$\underline{p} = \hbar \underline{k}$ $E_o >> E_{kin}$, $p = (2m_o E_{kin})^{1/2}$, $\lambda = h/\sqrt{2m_o E_{kin}} = h/m_o v$

Extreme relativsitic regime:

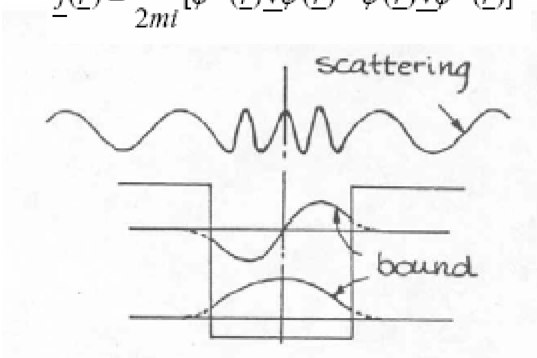
$E_{kin} >> E_o$, $p = E_{kin}/c$, $\lambda = hc/E$

$$\lambda = h/p$$

$$\nu = E/h$$

$$i\hbar \frac{\partial \Psi(\underline{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) \right] \Psi(\underline{r},t)$$

$$\underline{j}(\underline{r}) = \frac{\hbar}{2mi}[\psi^+(\underline{r})\nabla\psi(\underline{r}) - \psi(\underline{r})\nabla\psi^+(\underline{r})]$$



Constant	Value	Unit	
		mks	cgs
Speed of light in vacuum c	2.997925(1)	x 10^{8} m s^{-1}	x 10^{10} cm s^{-1}
Elementary charge e	1.60210(2) 4.80298(7)	10^{-19} C	10^{-20} emu 10^{-10} esu
Avogadro's number N	6.02252(9)	10^{26} $kmole^{-1}$	10^{23} $mole^{-1}$
Mass unit	1.66043(2)	10^{-27} kg	10^{-24} g
Electron rest mass m_0	9.10908(13) 5.48597(3)	10^{-31} kg 10^{-4} u	10^{-28} g 10^{-4} u
Proton rest mass M_P	1.67252(3) 1.00727663(8)	10^{-27} kg u	10^{-24} g u
Neutron rest mass M_n	1.67482(3) 1.0086654(4)	10^{-27} kg u	10^{-24} g u
Faraday constant Ne	9.64870(5) 2.89261(2)	10^{4} C $mole^{-1}$	10^{3} emu 10^{14} esu
Planck constant h $\hbar = h/2\pi$	6.62559(16) 1.054494(25)	10^{-34} J s 10^{-34} J s	10^{-27} erg s 10^{-27} erg s
Charge-to-mass ratio for electron e/m_0	1.758796(6) 5.27274(2)	10^{11} C kg^{-1}	10^{7} emu 10^{17} esu
Rydberg constant $2\pi^2 m_0 e^4/h^3 c$	1.0973731(1)	10^{7} m^{-1}	10^{5} cm^{-1}
Bohr radius $\hbar^2/m_0 e^2$	5.29167(2)	10^{-11} m	10^{-9} cm
Compton wavelength of electron h/m_0c $\hbar/m_0c$	2.42621(2) 3.86144(3)	10^{-12} m 10^{-13} m	10^{-10} cm 10^{-11} cm
Compton wavelength of proton h/M_pc $\hbar/M_pc$	1.321398(13) 2.10307(2)	10^{-15} m 10^{-16} m	10^{-13} cm 10^{-14} cm

Figure by MIT OCW. Adapted from Meyerhof, Appendix D.

For a one-dimensional system the time-independent wave equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \qquad (3.1)$$

$k^2 = 2m(E+V_o)/\hbar^2$ $\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$ $|x| \le L/2$

$\kappa^2 = -2mE/\hbar^2$ $\frac{d^2\psi(x)}{dx^2} - \kappa^2\psi(x) = 0$ $|x| \ge L/2$

$\psi(x) = A\sin kx$ $|x| \le L/2$

$= Be^{-\kappa x}$ x > L/2

$= Ce^{\kappa x}$ x < -L/2

, the boundary conditions are

$$\psi_{int}(x_o) = \psi_{ext}(x_o)$$

$$\left.\frac{d\psi_{int}(x)}{dx}\right|_{x_o} = \left.\frac{d\psi_{ext}(x)}{dx}\right|_{x_o}$$

$\xi = kL/2$, $\eta = \kappa L/2$

$\xi^2 + \eta^2 = 2mL^2|V_o|/4\hbar^2 \equiv \Lambda$

$\xi \cot \xi = -\eta$ (odd-parity)

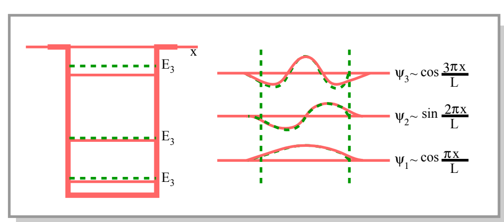


Figure by MIT OCW.

ψ vanishes at $x = \pm L/2$ $E_n = -|V_o| + \frac{n^2\pi^2\hbar^2}{2mL^2}$,   n = 1, 2, …

$\psi_n(x) = A_n \cos(n\pi x/L)$,   n = 1, 3, …

$= A_n' \sin(n\pi x/L)$ n = 2, 4, …

$\nabla^2 = D_r^2 + \frac{1}{r^2}\left[\frac{-L^2}{\hbar^2}\right]$ $\quad D_r^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\right]$ $\quad \left[-\frac{\hbar^2}{2m}D_r^2 + \frac{L^2}{2mr^2} + V(r)\right]\psi(r\theta\varphi) = E\psi(r\theta\varphi)$ $\quad L^2 Y_\ell^m(\theta,\varphi) = \hbar^2\ell(\ell+1)Y_\ell^m(\theta,\varphi)$

$L_z = -i\hbar\partial/\partial\varphi$ its eignefunctions are also $Y_\ell^m(\theta,\varphi)$, with eigenvalues $m\hbar$ $\quad \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\varphi Y_\ell^{m*}(\theta,\varphi) Y_{\ell'}^{m'}(\theta,\varphi) = \delta_{\ell\ell'}\delta_{mm'}$

$-\frac{\hbar^2}{2m}\frac{d^2u_\ell(r)}{dr^2} + \left[\frac{\ell(\ell+1)\hbar^2}{2mr^2} + V(r)\right]u_\ell(r) = Eu_\ell(r)$

notation: **s, p, d, f, g, h, …**
ℓ = **0, 1, 2, 3, 4, 5, …**

$\underline{L} = \underline{r} \times \underline{p}$,

$\psi(xyz) = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z)$

$= (2/L)^{3/2}\sin(n_x\pi x/L)\sin(n_y\pi y/L)\sin(n_z\pi z/L)$

$E_{n_x n_y n_z} = E_{n_x} + E_{n_y} + E_{n_z}$

$= \frac{(\hbar\pi)^2}{2mL^2}\left[n_x^2 + n_y^2 + n_z^2\right]$

In regions I and III

$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$, $\quad k^2 = 2mE/\hbar^2$

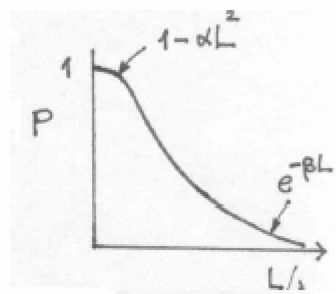
$\psi_1 = a_1 e^{ikx} + b_1 e^{-ikx} \equiv \psi_{1\rightarrow} + \psi_{1\leftarrow}$

$\psi_3 = a_3 e^{ikx} + b_3 e^{-ikx} \equiv \psi_{3\rightarrow}$

In region II,

$\frac{d^2\psi(x)}{dx^2} - \kappa^2\psi(x) = 0$, $\quad \kappa^2 = 2m(|V_o| - E)/\hbar^2$

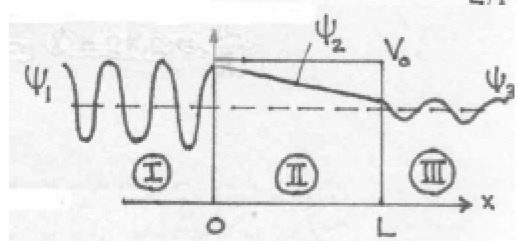
$\psi_2 = a_2($



$T = \left|\frac{a_3}{a_1}\right|^2$, $\quad R = \left|\frac{b_1}{a_1}\right|^2$ $\quad \frac{|a_3|^2}{|a_1|^2} = \left|\frac{a_3}{a_1}\right|^2 = \frac{1}{1 + \frac{V_o^2}{4E(V_o - E)}\sinh^2\kappa L} \equiv P$

$P \sim 1 - \frac{V_o^2}{4E(V_o - E)}(\kappa L)^2 = 1 - \frac{(V_o L)^2}{4E}\frac{2m}{\hbar^2}$ $\quad \kappa L << 1$

$P \sim \frac{16E}{V_o}\left(1 - \frac{E}{V_o}\right)e^{-2\kappa L}$ $\quad \kappa L >> 1$



$n + H^1 \rightarrow H^2 + \gamma$ (2.23 Mev)

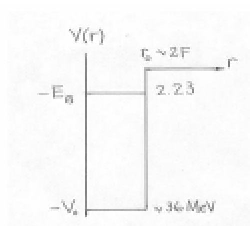
$u(r) = A\sin Kr$ $\quad K = [m(V_o - E_B)]^{1/2}/\hbar$ $\quad r < r_o$

$u(r) = Be^{-\kappa r}$ $\quad \kappa = \sqrt{mE_B}/\hbar$ $\quad r > r_o$

$K\cot Kr_o = -\kappa$, or $\tan Kr_o = -\left(\frac{V_o - E_B}{E_B}\right)^{1/2}$

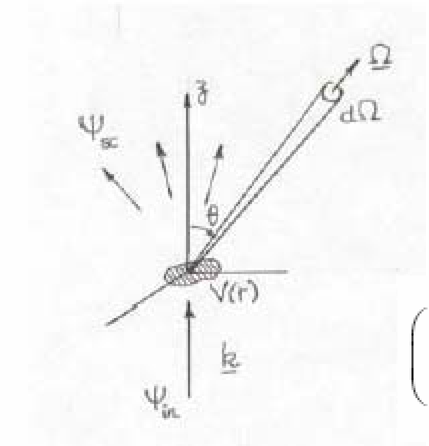
$(\text{radius})^2 \sim (1.4\text{x}A^{1/3})^2 \sim 3.1$ F, or $(1.2\text{x}A^{1/3})^2 \sim 2.3$ F

, Vo >> E_B, $\quad Kr_o \sim \pi/2$



$K \sim \sqrt{mV_o}/\hbar \sim \pi/2r_o$

$V_o r_o^2 \sim \left(\frac{\pi}{2}\right)^2\frac{\hbar^2}{m} \sim$ 1 Mev-barn



$$\Psi_{sc} = f(\theta) b \frac{e^{i(kr-\omega t)}}{r} \qquad \sigma(\theta) = \frac{J_{sc}\cdot\Omega}{J_{in}} = |f(\theta)|^2 \qquad \mu = m_1 m_2/(m_1+m_2)$$

$$\psi_k(\underline{r}) \to_{r>>r_o} e^{ikz} + f(\theta)\frac{e^{ikr}}{r} \qquad \psi(r,\theta) = \sum_{\ell=0}^{\infty} R_\ell(r) P_\ell(\cos\theta)$$

$$\left(\frac{d^2}{dr^2} + k^2 - \frac{2\mu}{\hbar^2}V(r) - \frac{\ell(\ell+1)}{r^2}\right)u_\ell(r) = 0\,,$$

$$u_\ell(r) \to_{kr>>1} (B_\ell/k)\sin(kr - \ell\pi/2) - (C_\ell/k)\cos(kr - \ell\pi/2) \qquad f_\ell = \frac{1}{2ik}(-i)^\ell[a_\ell e^{i\delta_\ell} - i^\ell(2\ell+1)]$$

$$= (a_\ell/k)\sin[kr - (\ell\pi/2) + \delta_\ell] \qquad a_\ell = i^\ell(2\ell+1)e^{i\delta_\ell}$$

$$\sigma = \int d\Omega\,\sigma(\theta) = 4\pi\lambda\!\!\!^{-2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell$$

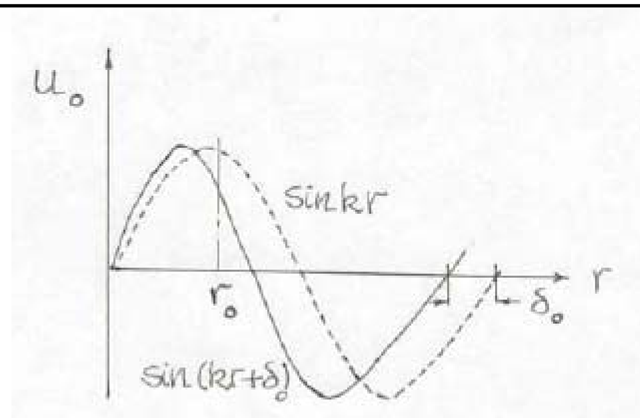
S-wave scattering

$$\sigma = 4\pi\lambda\!\!\!^{-2}\sin^2\delta_o(k)$$

at low energies, as $k \to 0$

$$\lim_{k\to 0}[e^{i\delta_o(k)}\sin\delta_o(k)] = \delta_o(k) = -ak$$

$$\sigma = 4\pi a^2$$



$u(r) = B\sin(K'r)$, $r < r_o$ $\qquad K'\cot(K'r_o) = k\cot(kr_o + \delta_o)$

$$K' = \sqrt{m(V_o + E)}/\hbar$$

$u(r) = C\sin(kr + \delta_o)$, $r > r_o$ $\qquad k = \sqrt{mE}/\hbar$

this series of approximations $k\cot(\delta_o) = -\kappa$ $\qquad \sigma(\theta) \approx \dfrac{1}{k^2+\kappa^2} = \dfrac{\hbar^2}{m}\dfrac{1}{E+E_B} \approx \dfrac{\hbar^2}{mE_B}$

$\sigma(\theta) = (1/k^2)\sin^2\delta_o$ $\qquad \sigma = 4\pi\hbar^2/mE_B \sim 2.3$ barns

$\hbar = 1.055 \times 10^{-27}$ erg sec, m = 1.67×10^{-24} g, and and $E_B = 2.23 \times 10^6 \times 1.6 \times 10^{-12}$ ergs,

$$\sigma(\theta) = \frac{1}{k^2}\left(\frac{1}{4}\sin^2\delta_{os} + \frac{3}{4}\sin^2\delta_{ot}\right) \qquad \sigma \approx \frac{\pi\hbar^2}{m}\left(\frac{3}{E_B} + \frac{1}{E^*}\right)$$