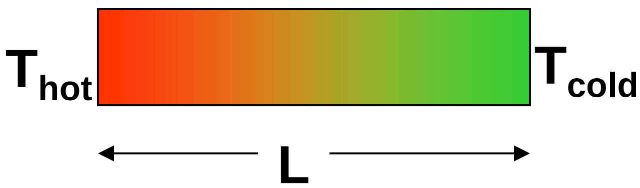
Heat Transfer Modes

Heat Conduction



- **Fourier Law**

$$\dot{Q} = -kA\frac{dT}{dx}\ [\text{W}]$$

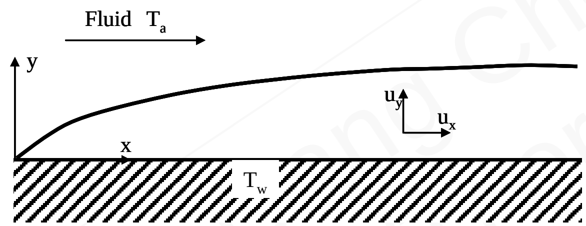
Thermal Conductivity [W/m-K] Materials Property

Cross-Sectional Area

- **Heat Flux**

$$\dot{q} = -k\frac{dT}{dx}(= -k\nabla T)\ \left[\text{W/m}^2\right]$$

Convection



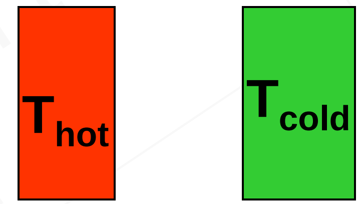
- **Newton's law of cooling**

$$\dot{Q} = hA\left(T_w - T_a\right)$$

Convective Heat Transfer Coefficient [W/m²K] Flow dependent

- **Natural Convection**
- **Forced Convection**

Thermal Radiation



- **Stefan-Boltzmann Law for Blackbody**

$$\dot{Q} = A\sigma T^4$$

Stefan-Boltzmann Constant
σ=5.67x10^{-8} W/m^{2}K^4

- **Heat transfer**

$$\dot{Q} = AF\ \varepsilon\sigma\left(T_{hot}{}^4 - T_{cold}{}^4\right)$$

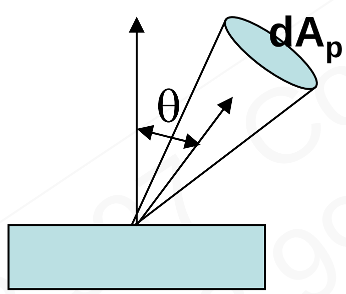
View factor F=1 for two parallel plates

Emissivity of two surfaces

Thermal Radiation: Planck's Law

- **Intensity: power per unit solid angle in the direction of propagation**

$$I_\lambda = \frac{Power}{dA_\perp d\Omega d\lambda}$$



Solid Angle

$$d\Omega = \frac{dA_p}{R^2} = \sin\theta d\theta d\varphi$$

- **Emissive power: power per unit surface area**

$$e_\lambda = \int_{2\pi} I_\lambda \cos\theta d\Omega$$

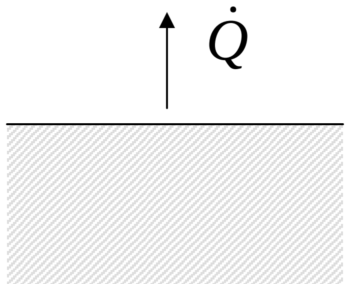
$$= \int_0^{2\pi} d\varphi \int_0^{\pi/2} I_\lambda \cos\theta \sin\theta d\theta = \pi I_\lambda$$

- **Planck's law**

$$I(\lambda) = \frac{4\pi c^2 \hbar}{\lambda^5} \frac{1}{\exp\left(\frac{2\pi\hbar c}{k_B T\lambda}\right) - 1}$$

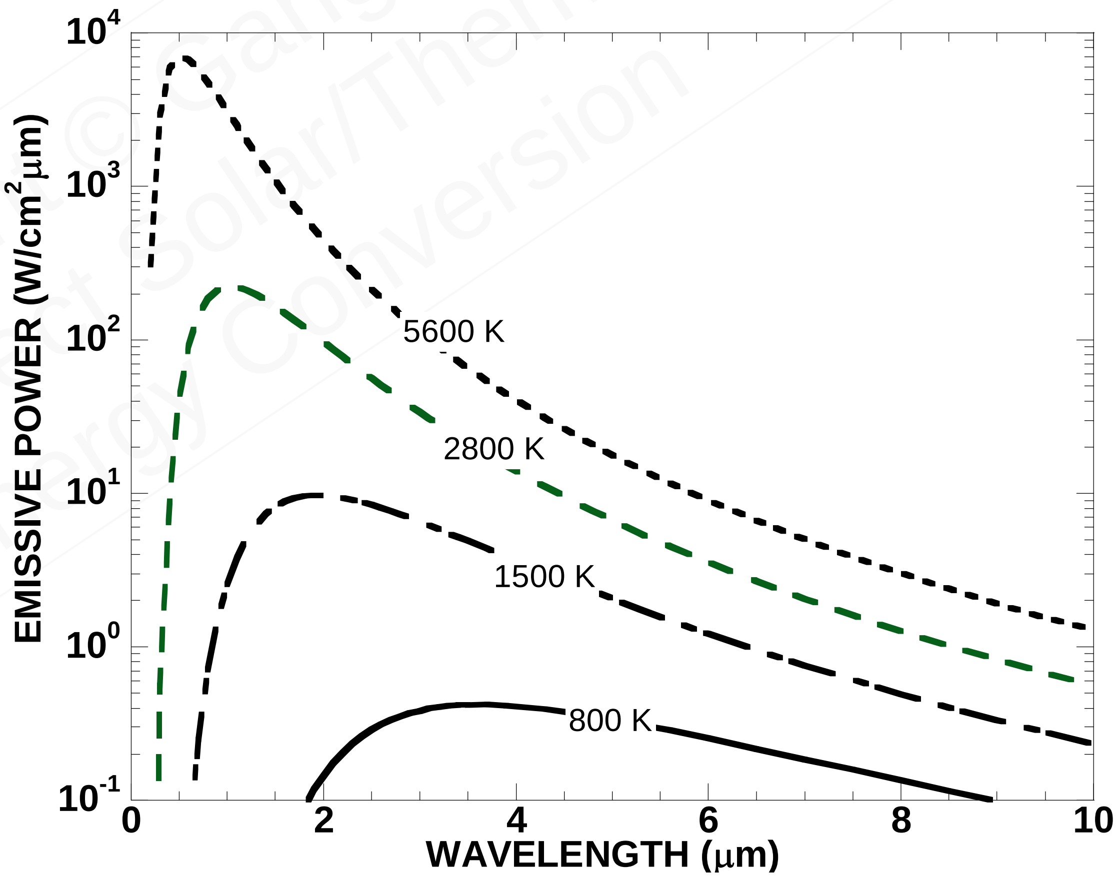
$$e(\lambda) = \frac{4\pi^2 c^2 \hbar}{\lambda^5} \frac{1}{\exp\left(\frac{2\pi\hbar c}{k_B T\lambda}\right) - 1}$$

Thermal Radiation: Planck's Law



Wien's displacement law

$$\lambda_{max} T = 2898 \text{ K}\mu\text{m}$$



Total

$$\dot{Q} = \int_0^\infty \dot{Q}(\lambda) d\lambda = A\sigma T^4$$

$$e_b = \sigma T^4$$

Stefan-Boltzmann constant

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 K^4$$

Universal Blackbody Curve

Function of λT only

$$\frac{e_{b\lambda}}{T^5} = \frac{4\pi^2 c^2 \hbar}{(\lambda T)^5} \frac{1}{\exp\left(\frac{2\pi\hbar c}{k_B \lambda T}\right) - 1}$$

$$= \frac{C_1}{(\lambda T)^5} \frac{1}{\exp\left(\frac{2\pi\hbar c}{k_B \lambda T}\right) - 1}$$

Fraction of Energy between [0,λ] Relative to Total Blackbody

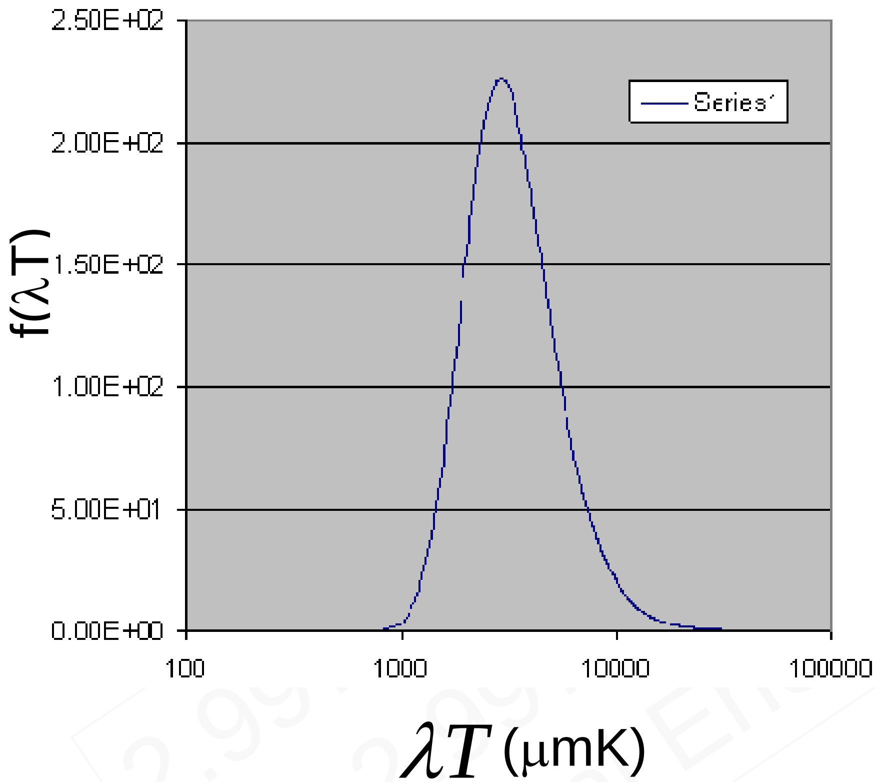
$$F = \frac{\int_0^\lambda e_{b\lambda} d\lambda}{\sigma T^4} = \int_0^\lambda \frac{C_1}{(\lambda T)^5} \frac{d\lambda}{\exp\left(\frac{2\pi\hbar c}{k_B \lambda T}\right) - 1}$$

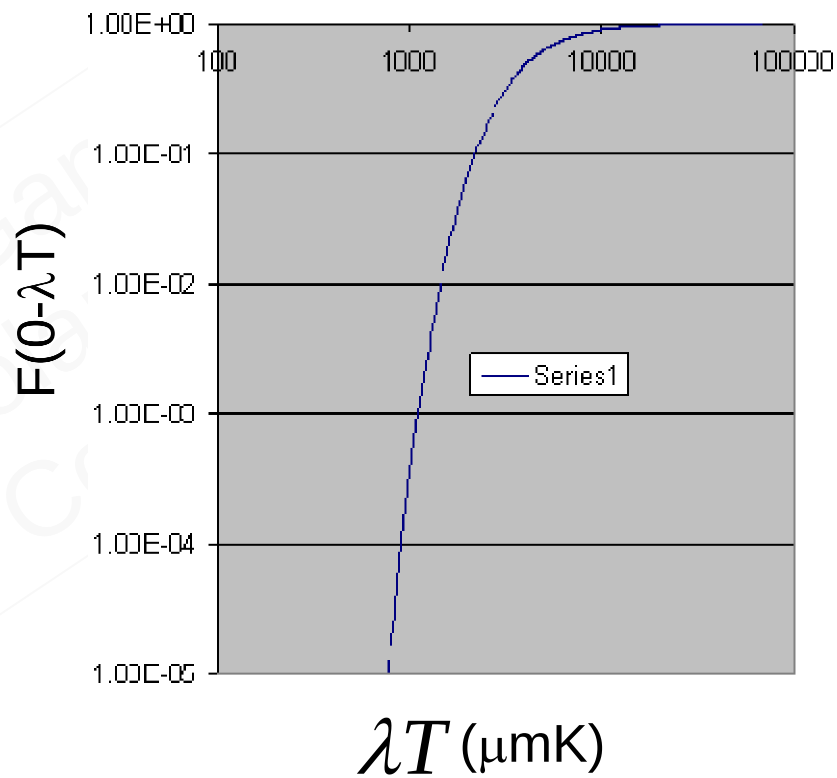
$$= \frac{1}{\sigma} \int_0^{\lambda T} \frac{C_1}{(\lambda T)^5} \frac{d(\lambda T)}{\exp\left(\frac{2\pi\hbar c}{k_B \lambda T}\right) - 1}$$

$$= \frac{C_1}{\sigma} \int_0^{\lambda T} \frac{1}{x^5} \frac{dx}{\exp\left(\frac{C_2}{x}\right) - 1} = \int_0^x f(x) dx$$

$$= \mathrm{F}(0 - \lambda \mathrm{T})$$

Universal Function





Earth Orbital

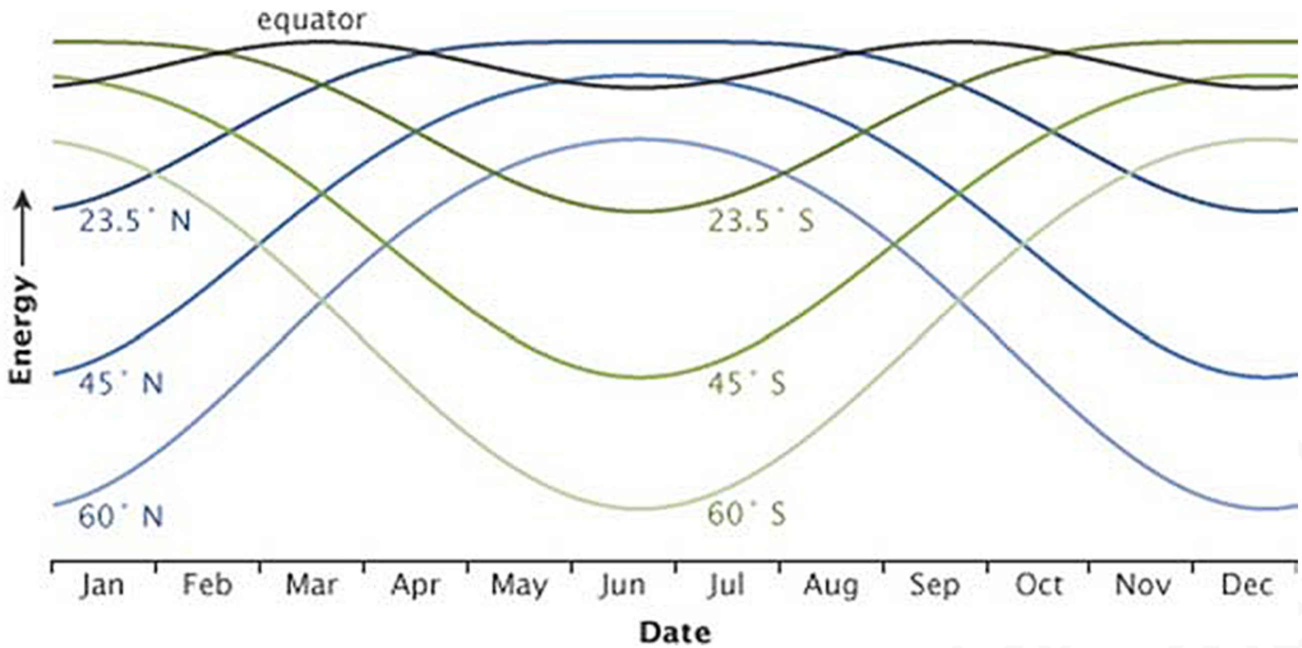


Image by Robert Simmon (NASA).

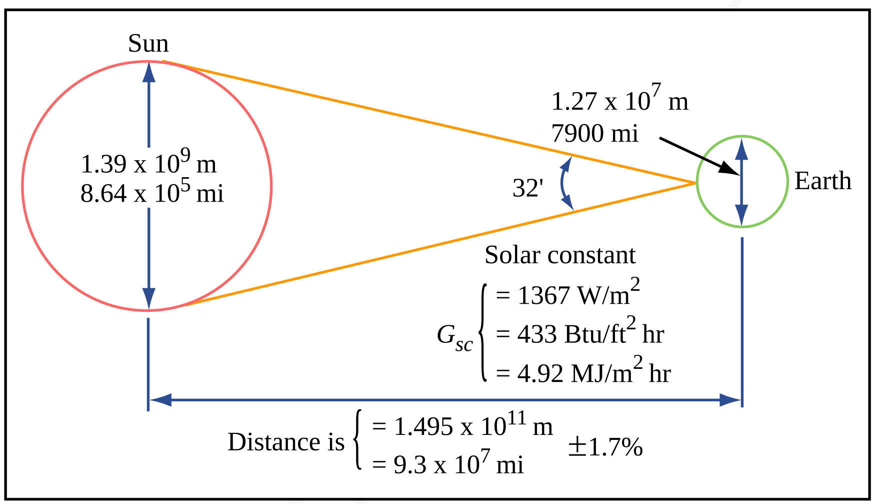
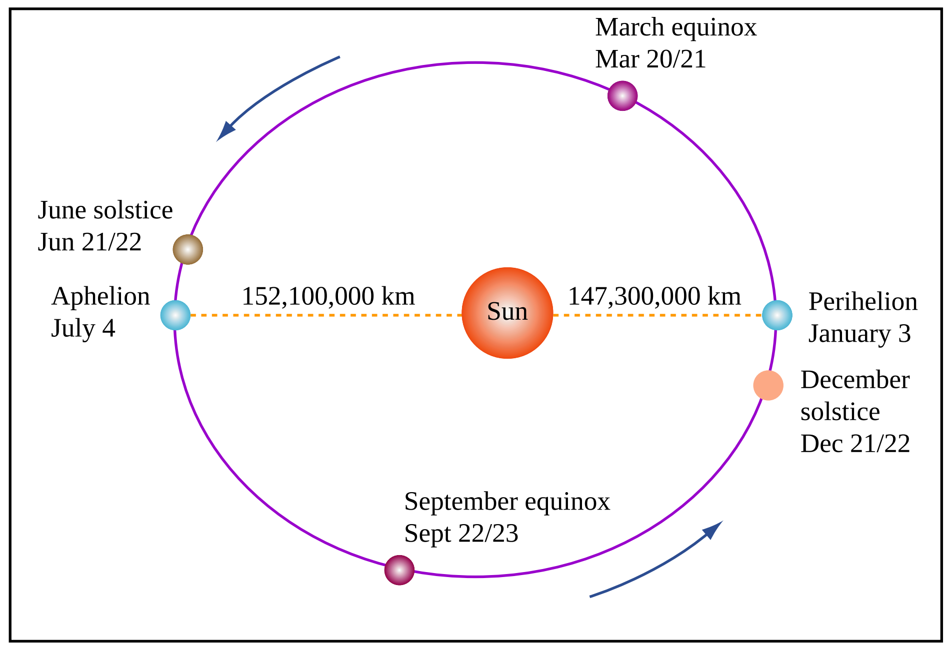


Figure by MIT OpenCourseWare.



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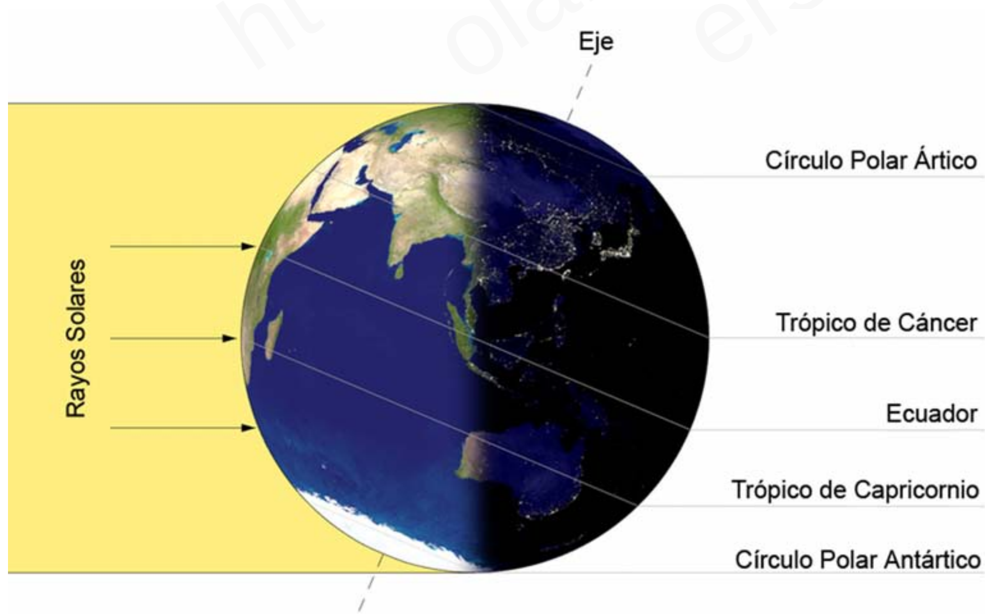
Earth Tilt



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Solar Spectral Outside Atmosphere

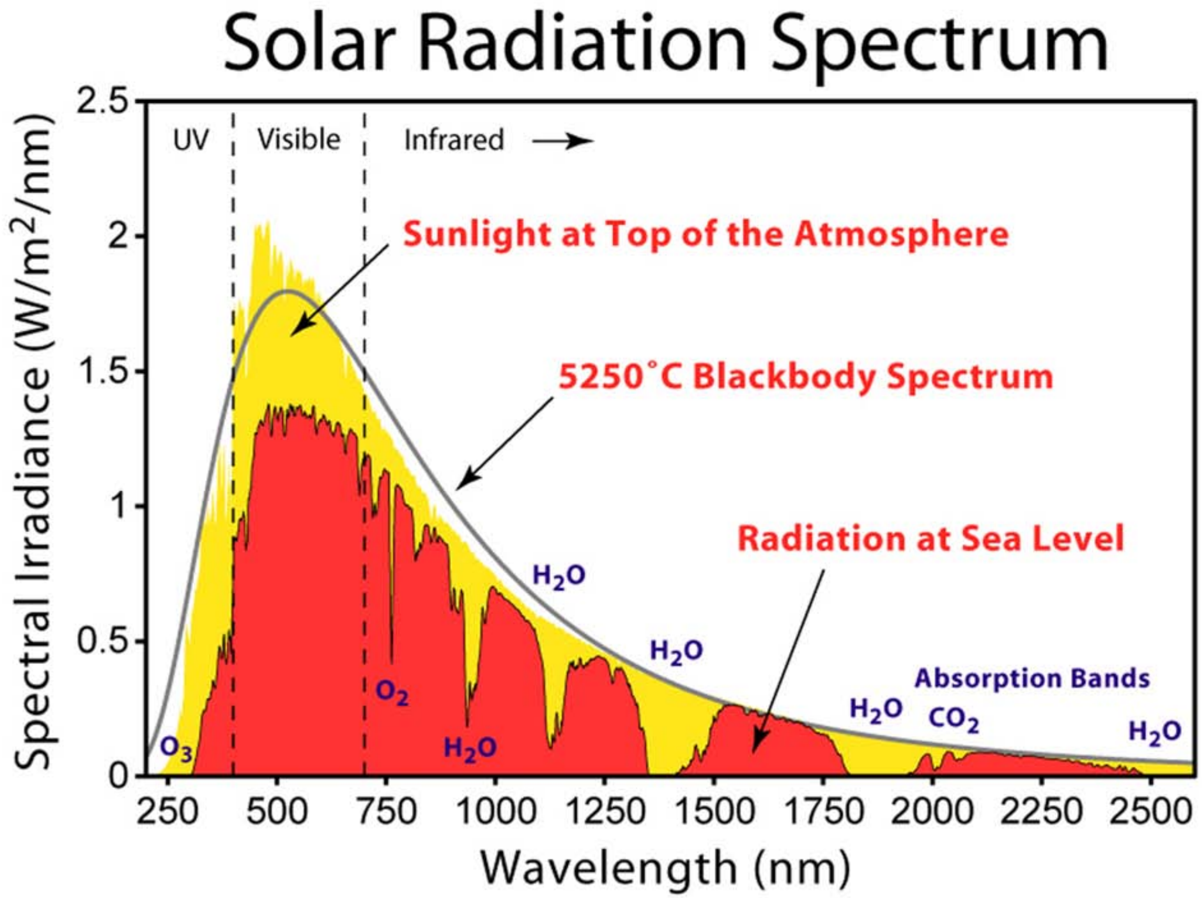
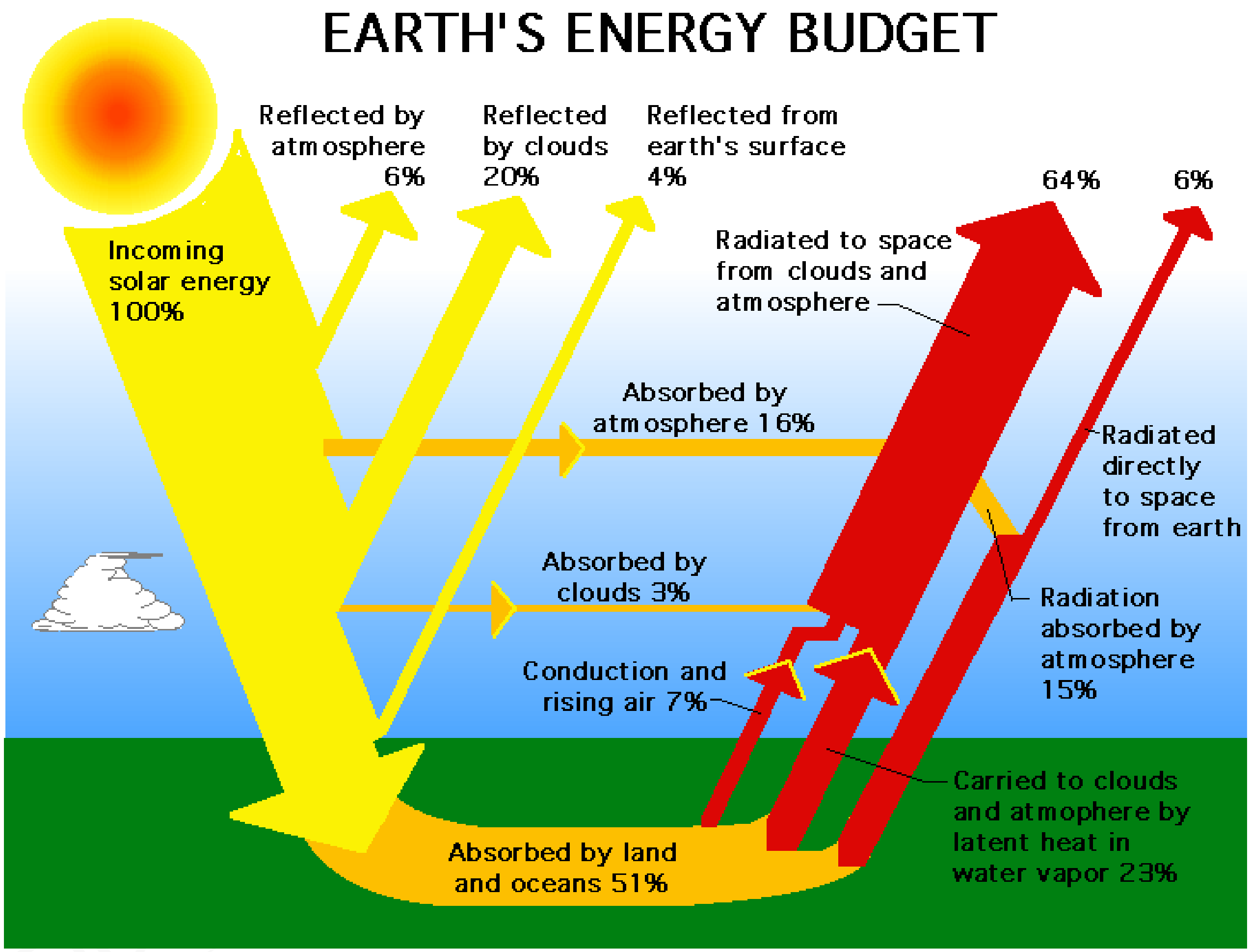


Image by Robert A. Rohde/Global Warming Art.

http://www.newport.com/images/webclickthru-EN/images/798.gif

Solar Going Through Atmosphere



Source: http://marine.rutgers.edu/mrs/education/class/yuri/erb.html

Image by NASA.

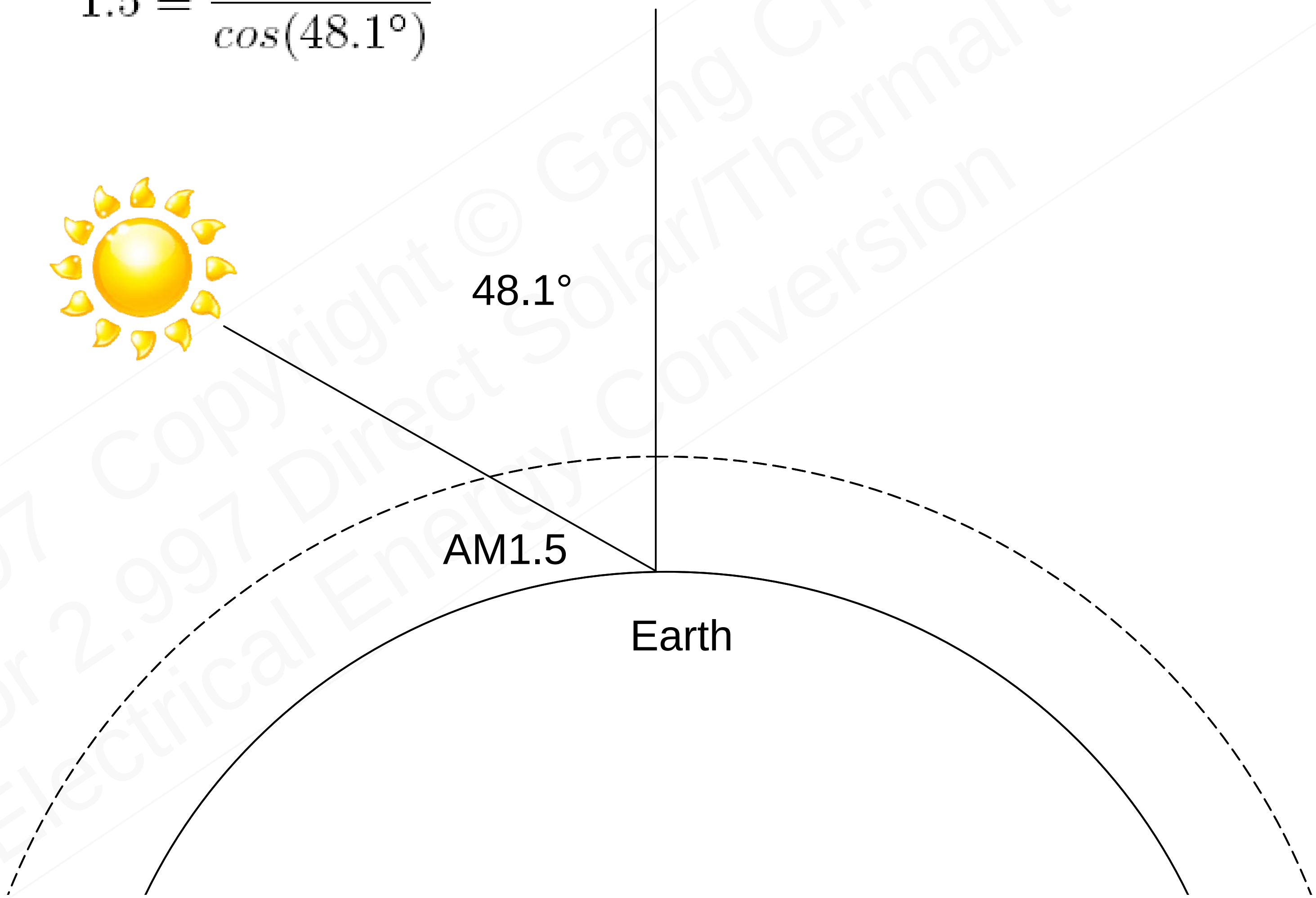
Solar spectra

Solar spectra are named by their air mass (AM), which is the ratio of the path length of air the rays travel through to the shortest possible pathlength (i.e. directly overhead). Thus the AM0 solar spectrum is measured above the earth's atmosphere, AM1 occurs when the sun is directly overhead, and AM1.5 occurs when the sun's rays travel through 50% more of the atmosphere than when the sun is directly overhead. Note 1/cos(48.2°) = 1.5, so AM1.5 occurs when the solar zenith angle is 48.2°.

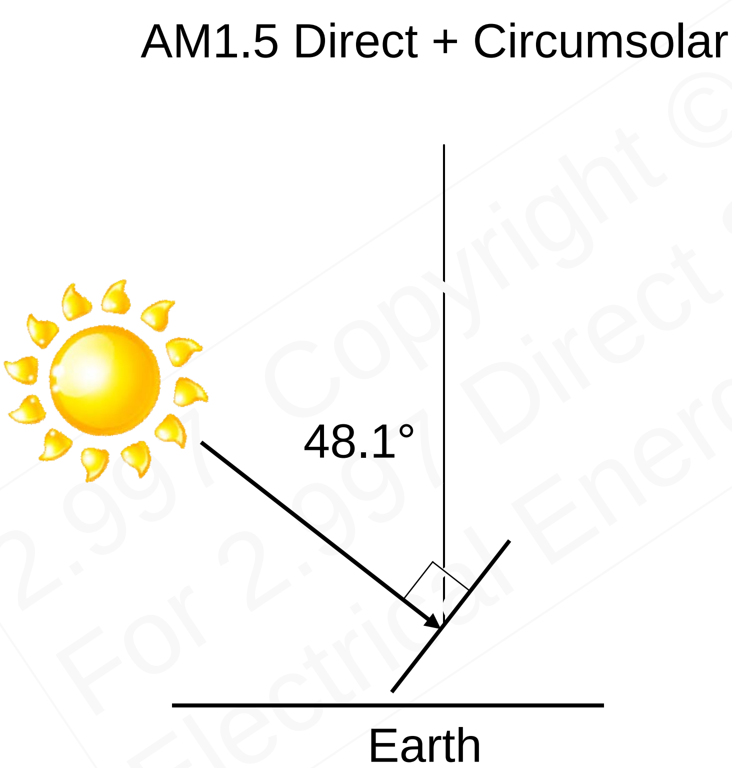
- AM0 is an average of measured data from many satellites, the space shuttle, high-altitude aircraft, sounding rockets, etc.
- AM1.5G and AM1.5 Direct + Circumsolar are calculated values based on atmospheric constituent and particle concentrations, humidity, ground surface albedo, the U.S. Standard Atmosphere, and various other parameters. They are defined as follows:
  - AM1.5 Direct + Circumsolar is only the solar radiation that comes from the sun and the cone of sky of half-angle 2.5 degrees surrounding the sun. The reference surface is normal to the sun, with an air mass of 1.5 (solar zenith 48.2°). (Defined in ASTM G173-03)
  - AM1.5G accounts for radiation from the sun, the entire sky, and reflections off the ground. The solar zenith is 48.2°, but the panel is tilted at an angle of 37°. This results in an angle of incidence of the sun of 11.2°. (ASTM G173-03)
  - AM1.5G is the spectrum used to calibrate solar cells. This spectrum gives higher solar cell efficiency than AM0, and higher power per square meter than AM1.5 Direct + Circumsolar. (ASTM E490-00a)

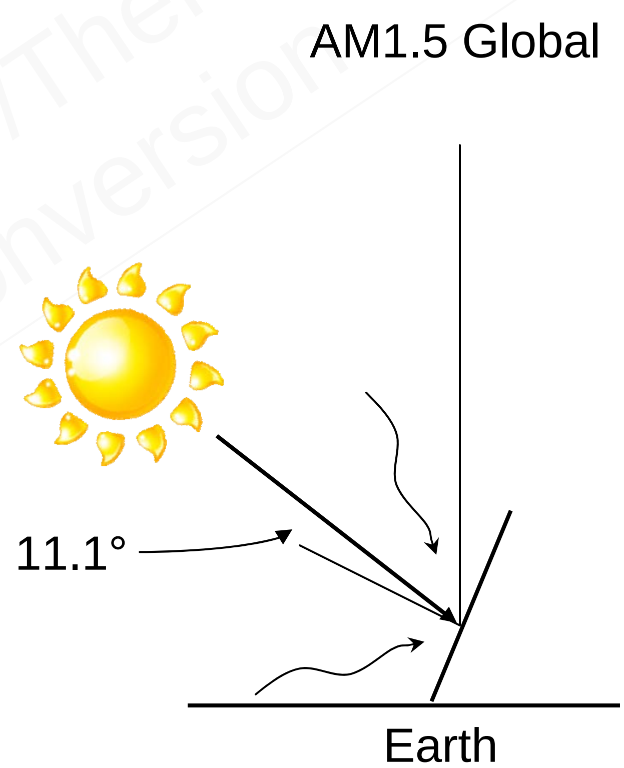
Air mass standards

$$1.5 = \frac{1}{cos(48.1°)}$$



Air mass standards





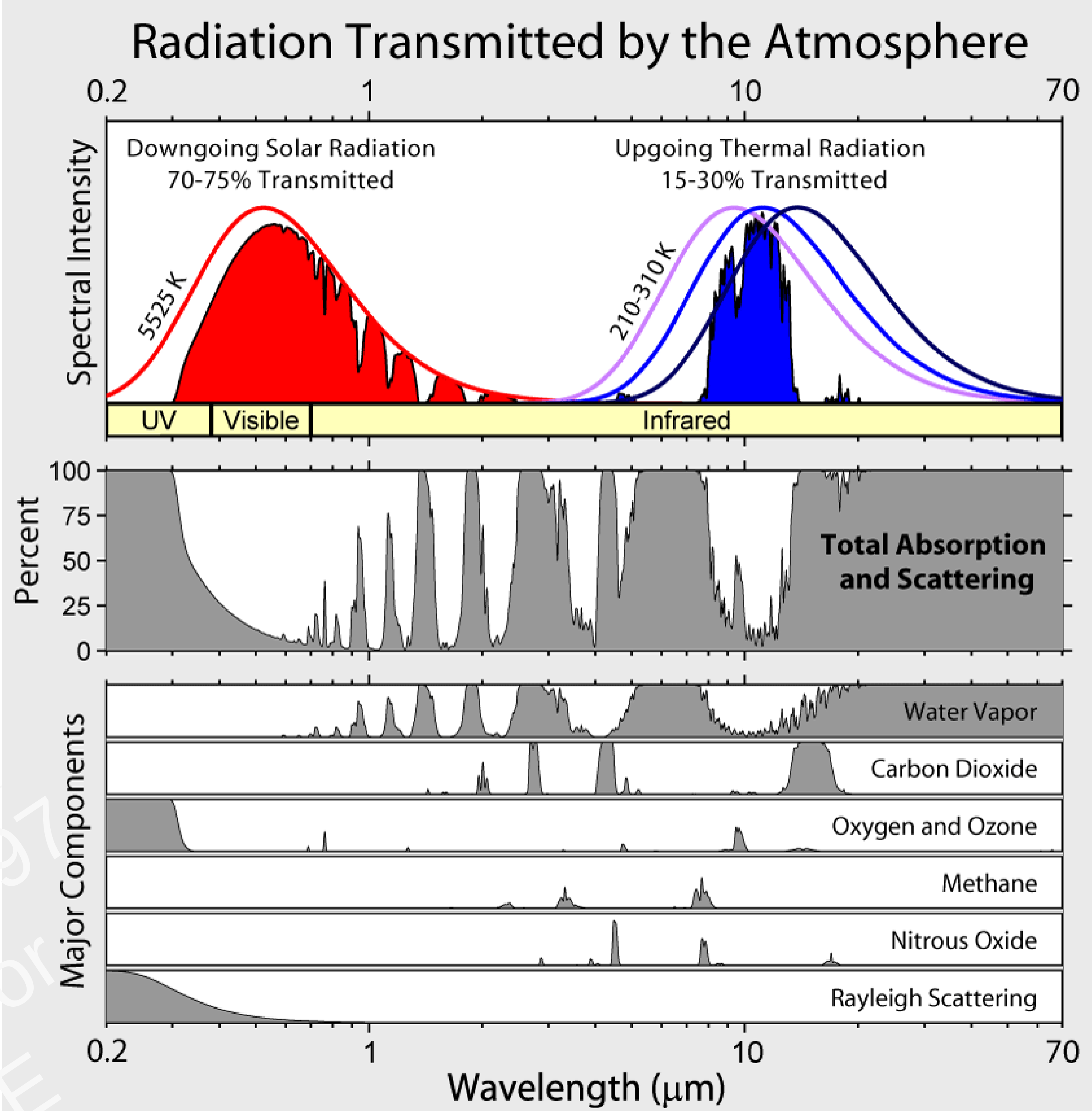
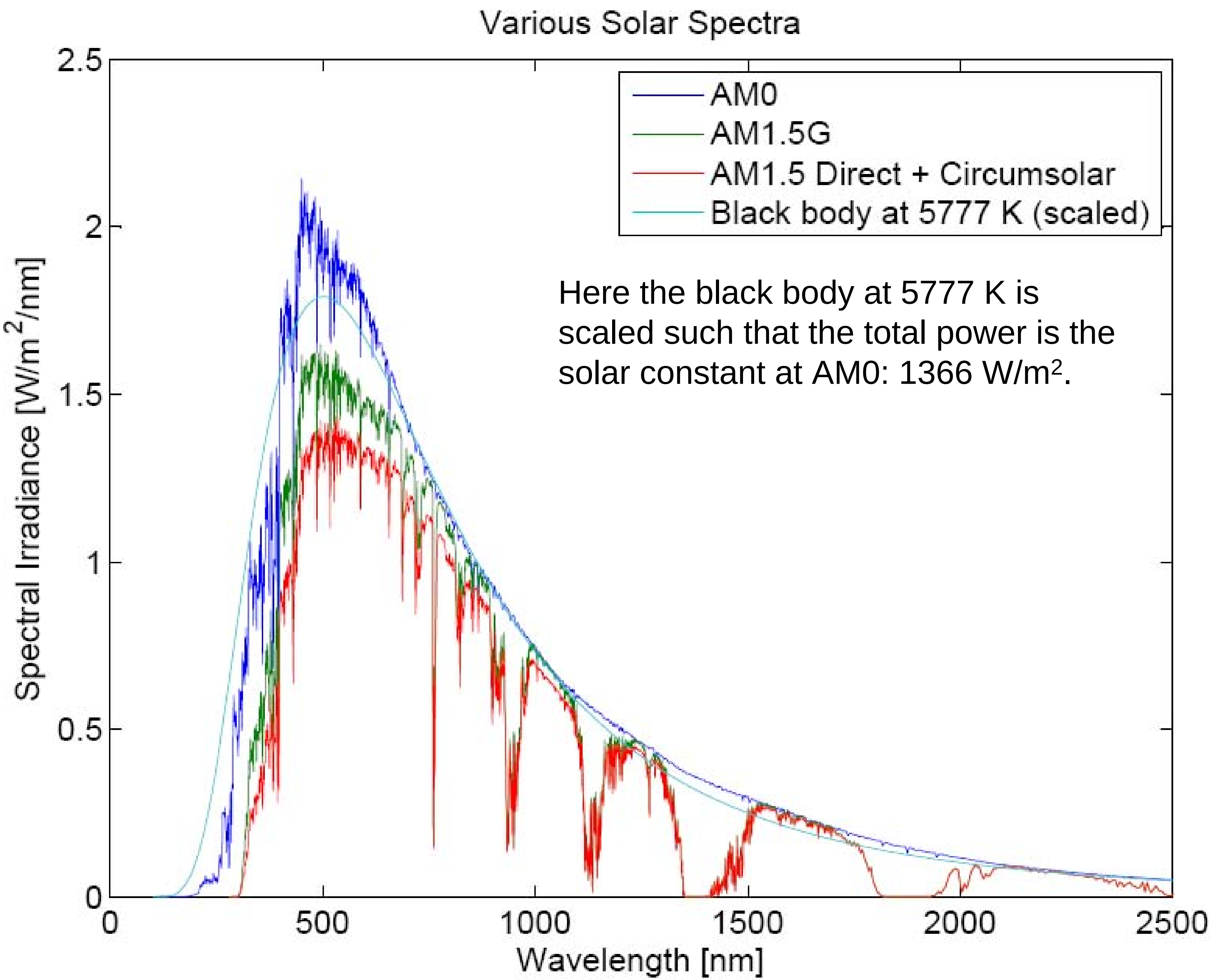


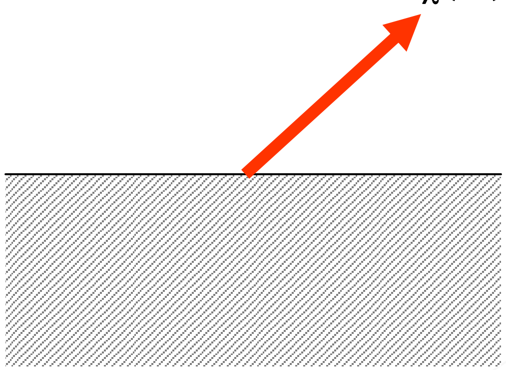
Image by Robert A. Rohde/Global Warming Art.

Source: http://en.wikipedia.org/wiki/File:Atmospheric_Transmission.png



Here the black body at 5777 K is scaled such that the total power is the solar constant at AM0: 1366 W/m^2.

Surface Properties: Emissivity



Directional-spectral $$\varepsilon'_\lambda(T) = \frac{I_\lambda(\Omega)}{I_{b\lambda}}$$

Hemispherical-spectral $$\varepsilon_\lambda(T) = \frac{e_\lambda}{e_{b\lambda}}$$

Directional-total $$\varepsilon'(T) = \frac{I(\Omega)}{I_b}$$

Hemispherical-total $$\varepsilon(T) = \frac{e}{e_b}$$

Diffuse Emitter

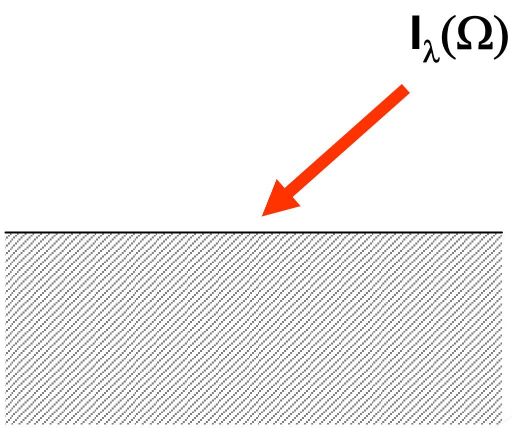
$$\varepsilon'_\lambda = \varepsilon_\lambda$$

Gray Emitter

$$\varepsilon'_\lambda = \varepsilon'$$

Diffuse-Gray Emitter

Surface Properties: Absorptivity



Directional-spectral

$$\alpha'_\lambda(T) = \frac{power\text{ absorbed}}{I_\lambda(\Omega)d\Omega}$$

Hemispherical-spectral $\alpha_\lambda(T)$

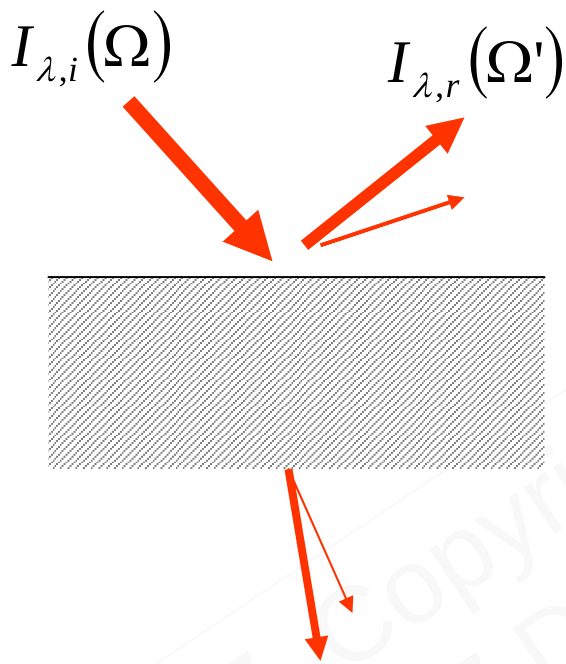
Directional-total $\alpha'(T)$

Hemispherical-total $\alpha(T)$

Kirchoff's Law

$$\varepsilon'_\lambda = \alpha'_\lambda$$

Surface Properties: Reflectivity&Transmissivity



Diffuse Reflector

Gray Reflector

Diffuse Gray Reflector

Reflectivity:

Bi-directional Reflectivity $\rho_\lambda^{''} = \frac{I_{\lambda,r}(\Omega')}{I_{\lambda,i}(\Omega)}$

Directional-spectral: $\rho_\lambda^{'}$

Hemispherical-spectral: ρ_λ

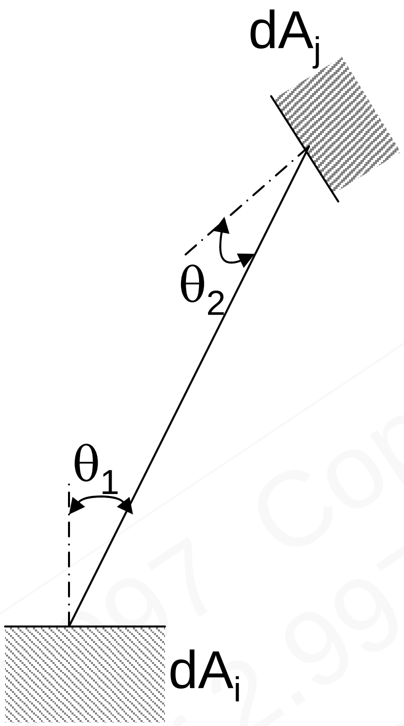
Directional-total: $\rho^{'}$

Hemispherical-total: ρ

Transmissivity: $\tau_\lambda^{''}, \tau_\lambda^{'}, \tau_\lambda, \tau^{'}, \tau$

Energy Conservation $\rho_\lambda^{'} + \alpha_\lambda^{'} + \tau_\lambda^{'} = 1$

View Factor



$$F_{dA_i-dA_j} = \frac{\text{power reaching } \mathrm{dA_j}}{\text{power leaving } \mathrm{dA_i}}$$

$$= \frac{I_i dA_i \cos\theta_i \dfrac{\cos\theta_j dA_j}{R_{ij}^{\ 2}}}{\pi \mathrm{I_i} dA_i}$$

$$= \frac{\cos\theta_i \cos\theta_j dA_j}{\pi R_{ij}^{\ 2}}$$

$$F_{A_i-A_j} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R_{ij}^{\ 2}} dA_i dA_j$$

Assumptions: Diffuse surface
Radiation leaving A_i is uniform

View Factor Relations

Reciprocity

$$F_{dA_i-dA_j}dA_i = F_{dA_j-dA_i}dA_j \qquad F_{A_i-A_j}A_i = F_{A_j-A_i}A_j$$

Summation

$$F_{A_i-(A_j+A_k)} = F_{A_i-A_j} + F_{A_i-A_k}$$

Earth-Sun Radiation

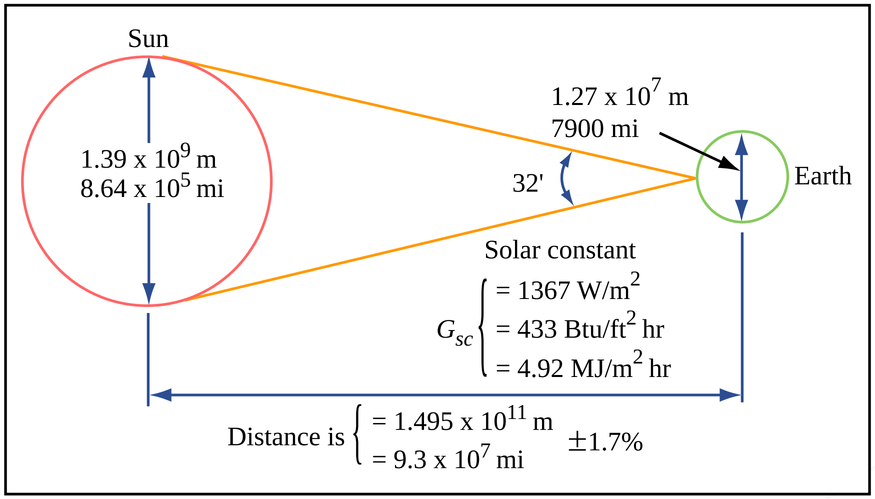


Figure by MIT OpenCourseWare.

Radiation Reaching Earth

$$e_s A_s F_{s-e}$$

$$A_s = \frac{\pi D_s^{\ 2}}{4} \qquad A_e = \frac{\pi D_e^{\ 2}}{4}$$

$$F_{s-e} = \frac{\pi D_e^{\ 2}/4}{R_{se}^{\ 2}} = 5.67\times 10^{-9}$$

$$F_{e-s} = \frac{\pi D_s^{\ 2}/4}{R_{se}^{\ 2}} = 6.79\times 10^{-5}$$

$$A_s F_{s-e} = A_e F_{e-s}$$

Solar Radiation Per Unit Area Normal to Sun on Earth Outside Atmosphere

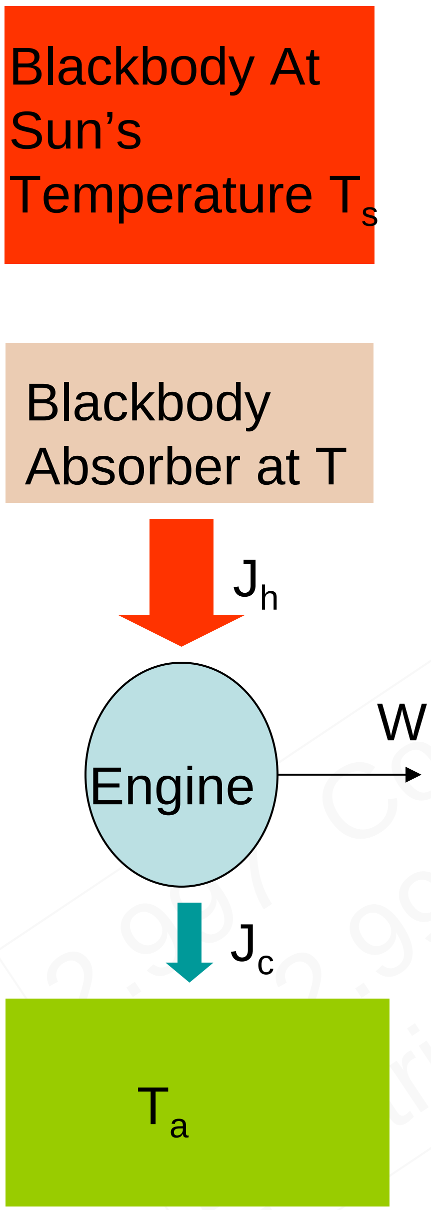
$$J_s = \frac{e_s A_s F_{s-e}}{A_e} = e_s F_{e-s}$$

$$= 5.67\times 10^{8} \times (5777)^4 \times 6.79\times 10^{-5}$$

$$= 1365 \, \frac{\mathrm{W}}{\mathrm{m}^2}$$

Solar Constant: 1366 W/m²

Maximum Efficiency of a Solar Thermal Engine



Heat Transferred to Absorber

$$Q_h = \sigma\left(T_s^4 - T^4\right)$$

Thermal Efficiency

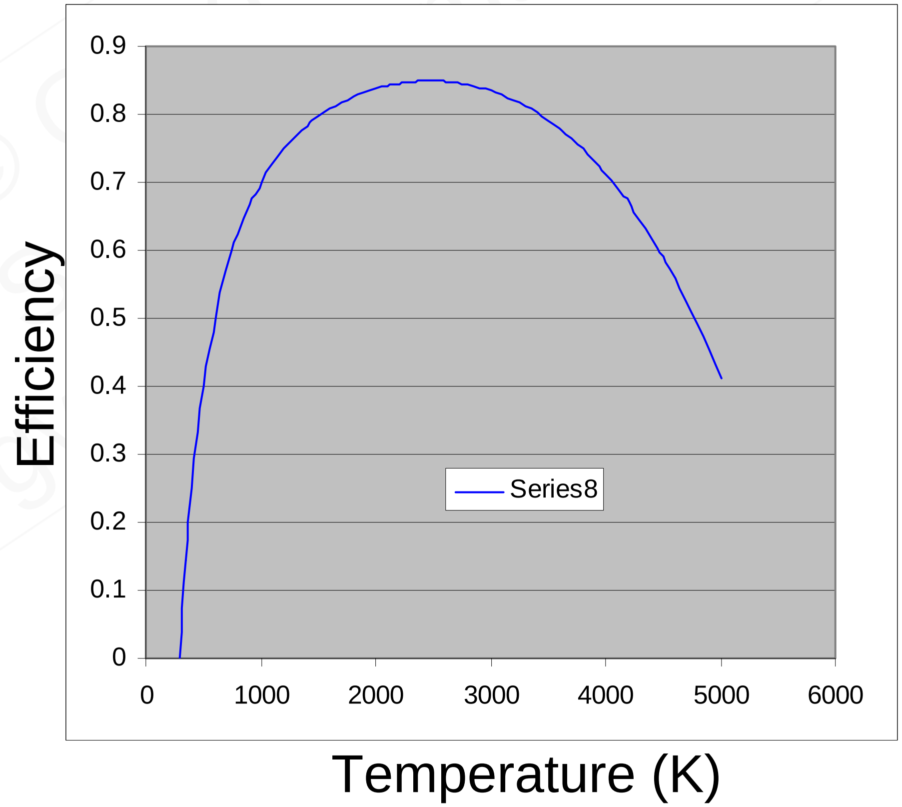
$$\eta_{th} = \frac{\sigma\left(T_s^4 - T^4\right)}{\sigma T_s^4} = 1 - \frac{T^4}{T_s^4}$$

Carnot Efficiency

$$\eta = 1 - \frac{T_a}{T}$$

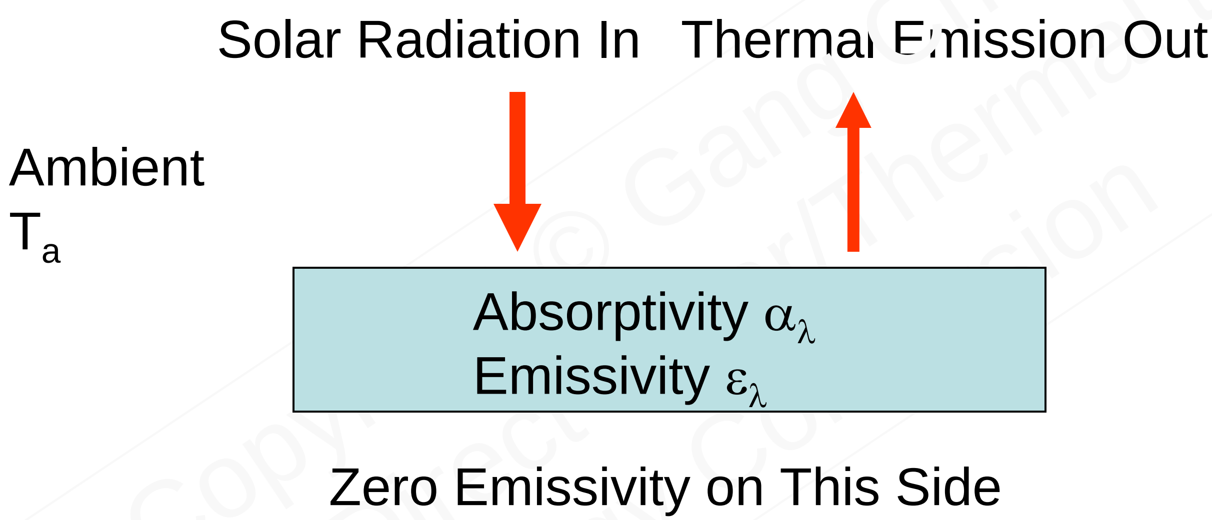
Maximum Efficiency of a Solar Thermal Engine

$$\eta = \eta_{th}\eta_C = \left(1 - \frac{T^4}{T_s^4}\right)\left(1 - \frac{T_a}{T}\right)$$



Maximum: 85% @ T=2450K
For T_a=300 K

How Hot a Surface Can Get By Solar Radiation?

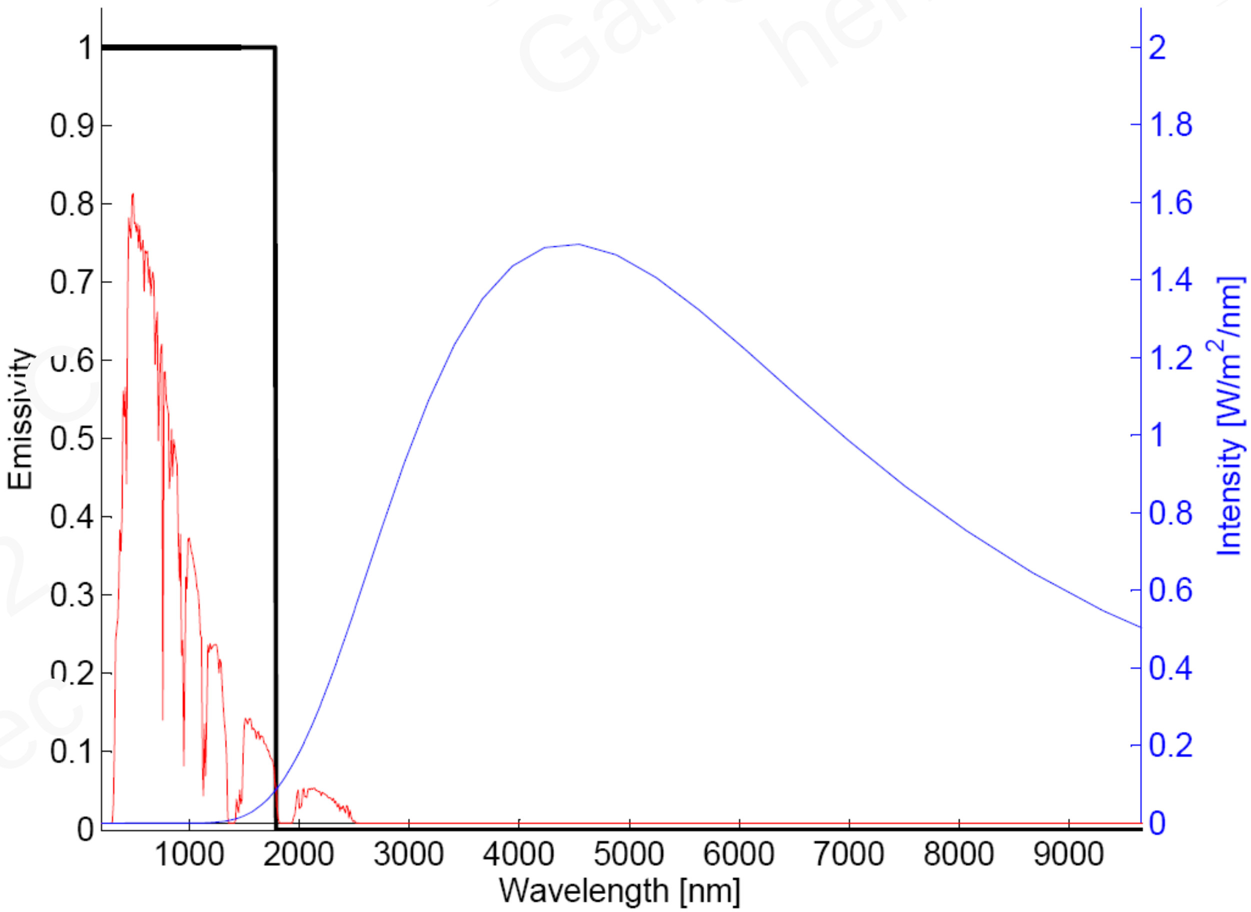


$$C\int_0^\infty \alpha_\lambda J_\lambda d\lambda = \int_0^\infty \varepsilon_\lambda \left[e_{b\lambda}(T) - e_{b\lambda}(T_o)\right] d\lambda$$

↑ Concentration

Selective surface

- Black body: emissivity = 1 for all wavelengths
- Selective surface: emissivity is high in the solar spectrum and low in the infrared. Ideally the transition would occur abruptly at the cutoff wavelength λ_c.



Approximate the Sun as A Blackbody

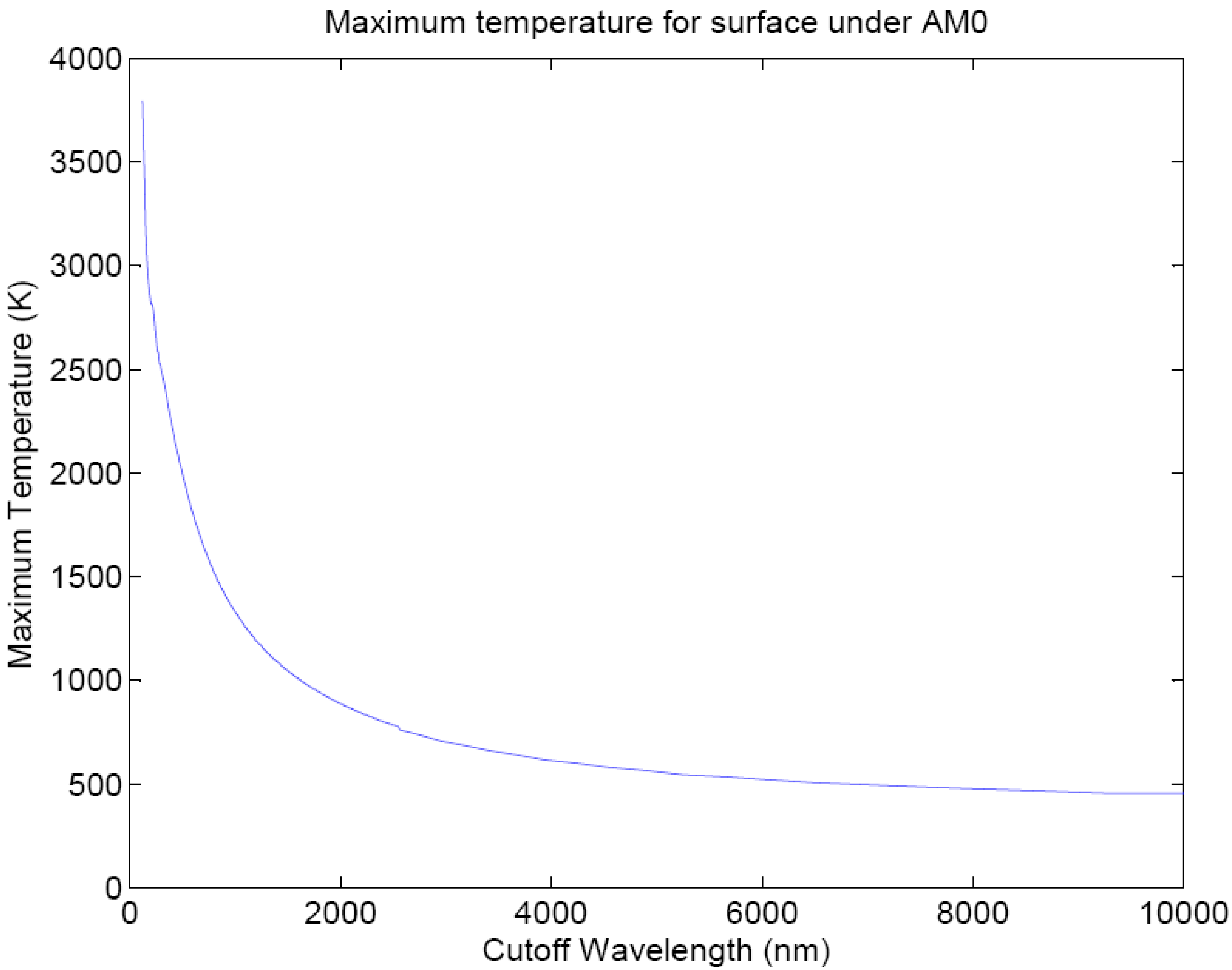
One Sun C=1

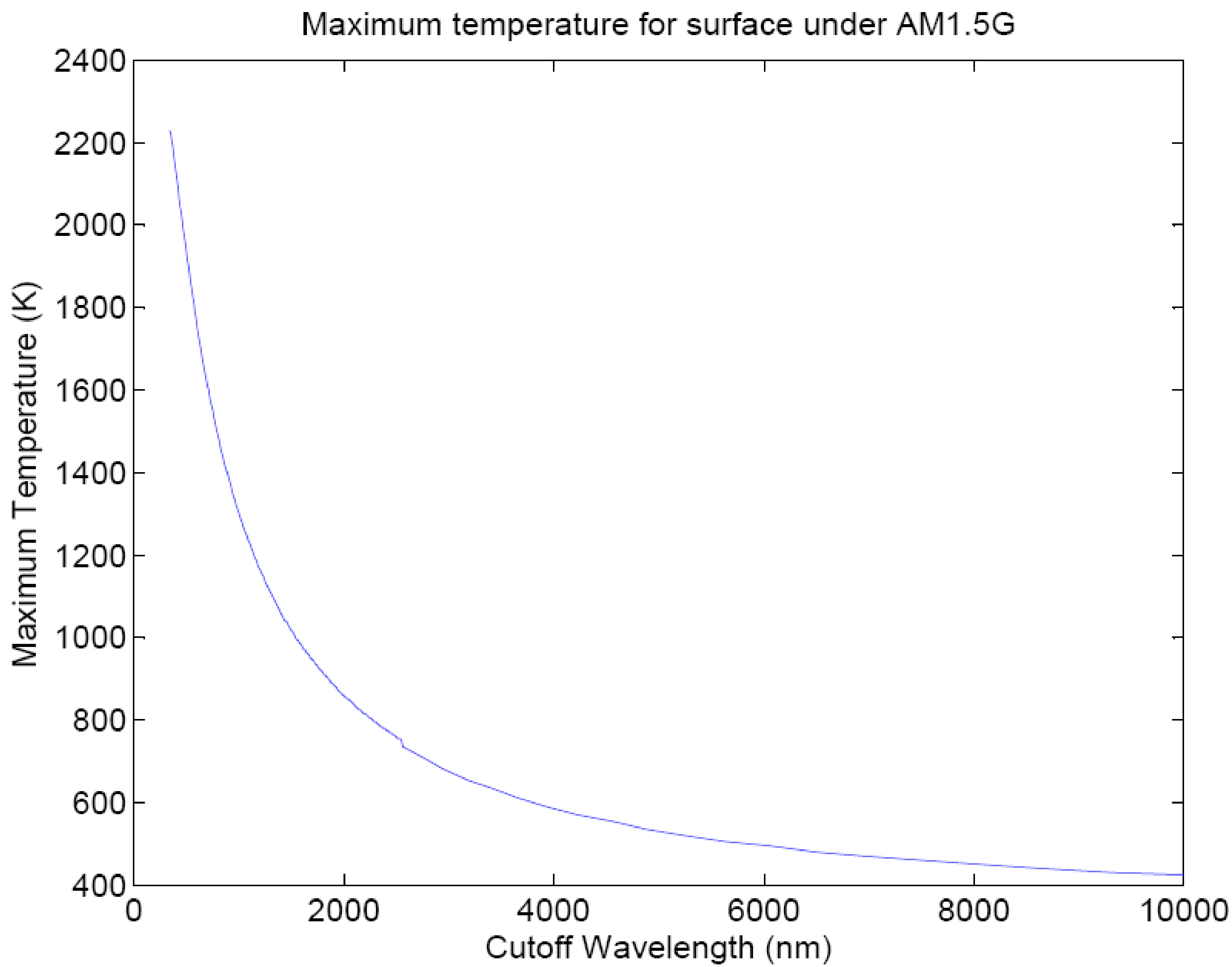
$$\int_0^{\lambda} A_{sun} F_{sun-surface} e_{b\lambda}(T_{sun}) d\lambda = \int_0^{\lambda} A[e_{b\lambda}(T) - e_{b\lambda}(T_o)] d\lambda$$

$$\int_0^{\lambda} A F_{e-s} e_{b\lambda}(T_{sun}) d\lambda = \int_0^{\lambda} A[e_{b\lambda}(T) - e_{b\lambda}(T_o)] d\lambda$$

$$F_{e-s} \frac{F(0-\lambda T_s)}{\sigma T_{sun}^{\ 4}} = \frac{F(0-\lambda T)}{\sigma T^4} - \frac{F(0-\lambda T_a)}{\sigma T_a^{\ 4}}$$

Maximum Temperature AM0

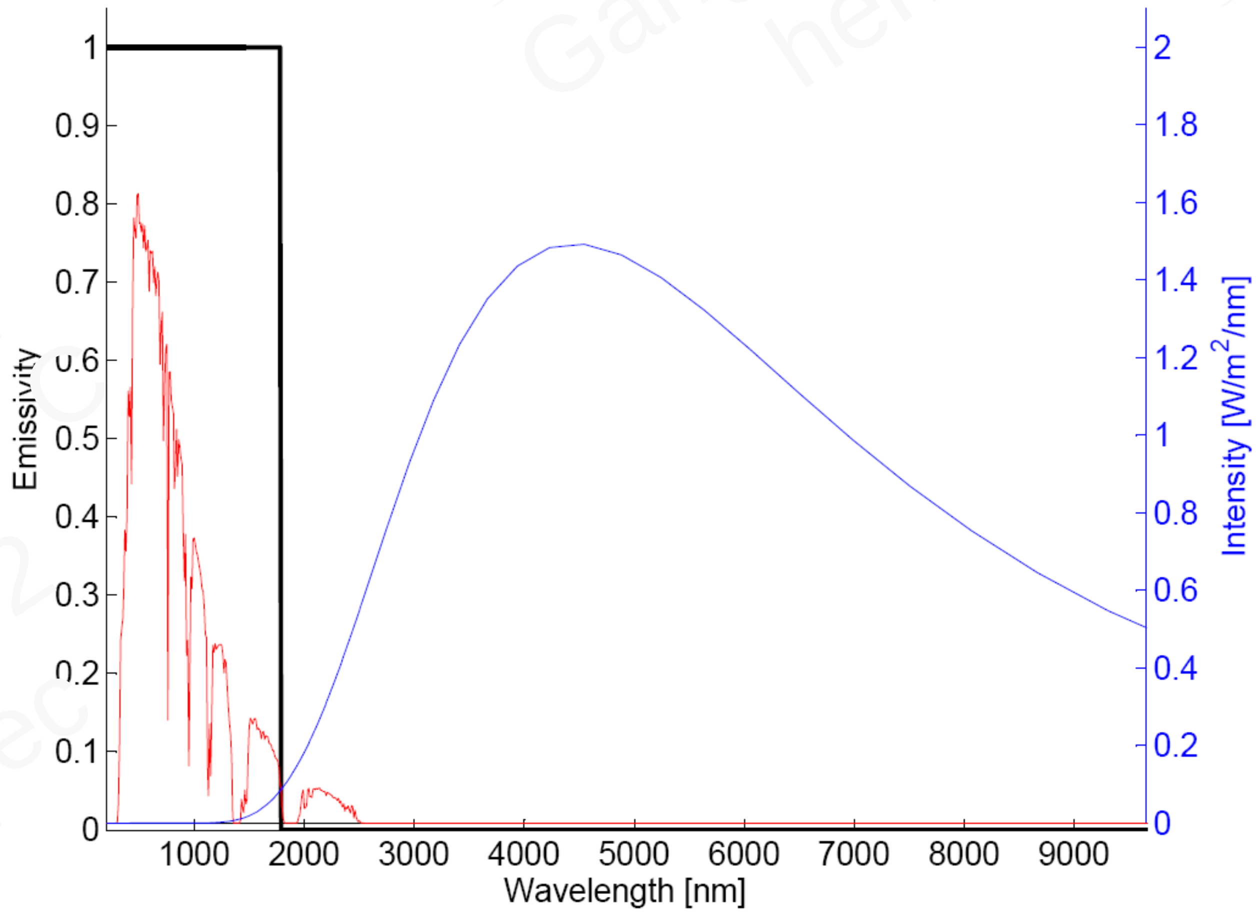


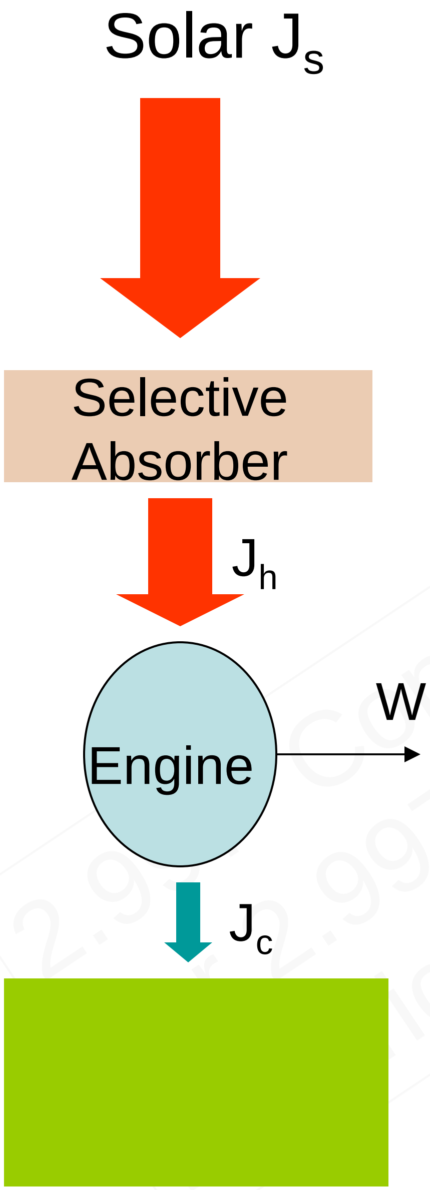


Maximum temperature for surface under AM1.5G
Maximum Temperature (K)
Cutoff Wavelength (nm)
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
0
2000
4000
6000
8000
10000

Selective surface

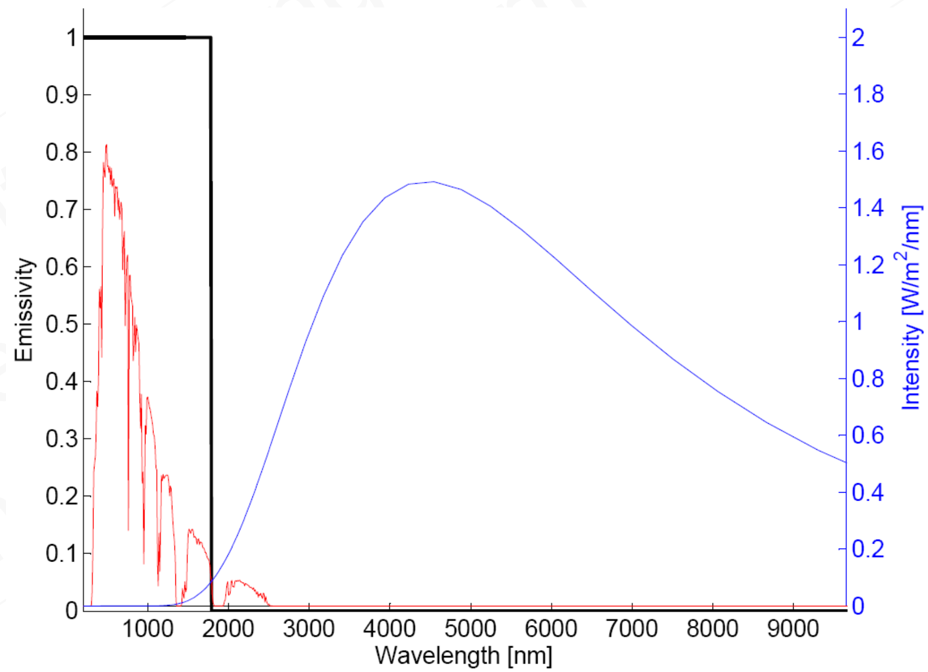
- Fix Temperature, choosing a cut-off wavelength





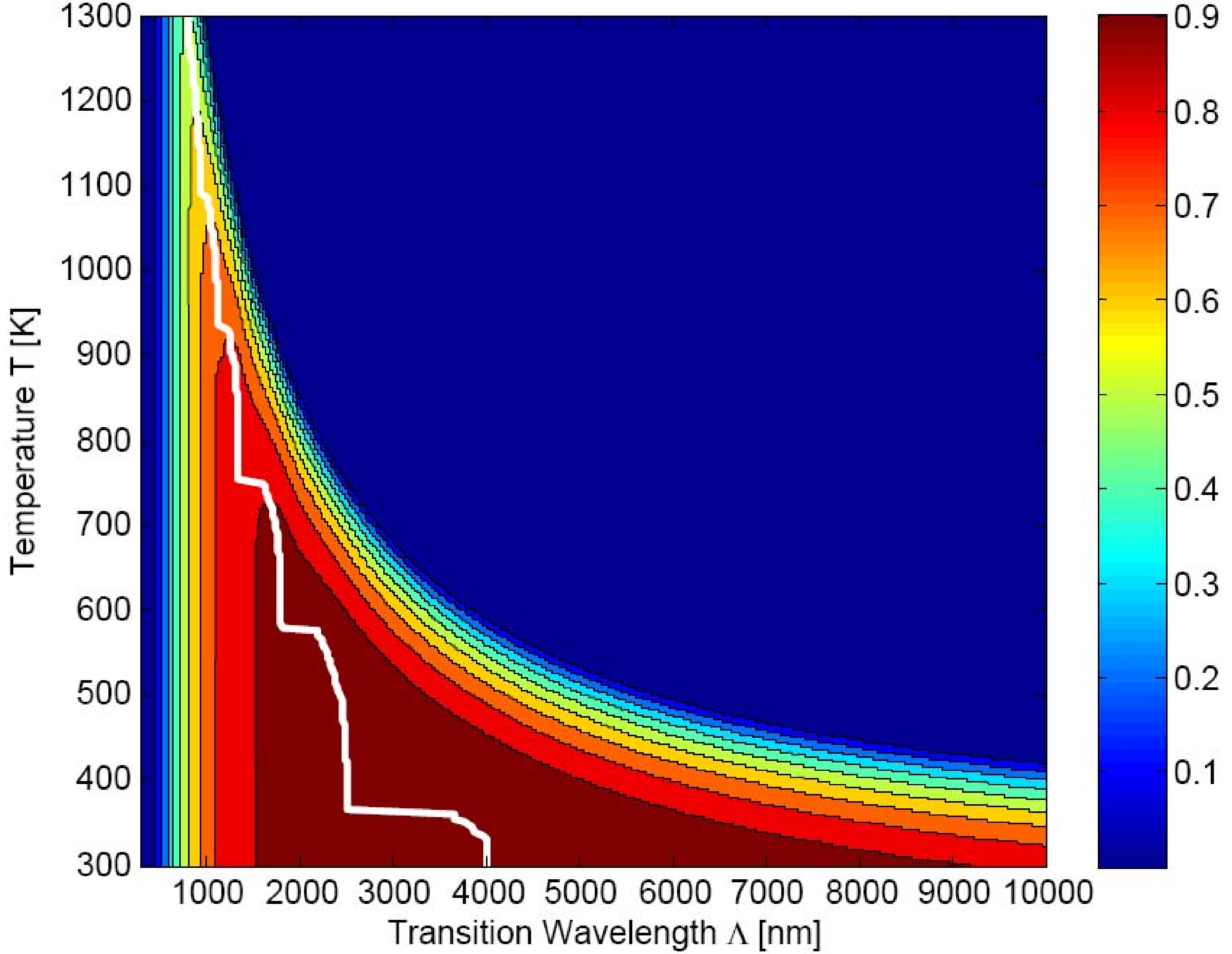
Efficiency of a Solar Thermal Engine: 1 Sun

- Fix Temperature, choosing a cut-off wavelength

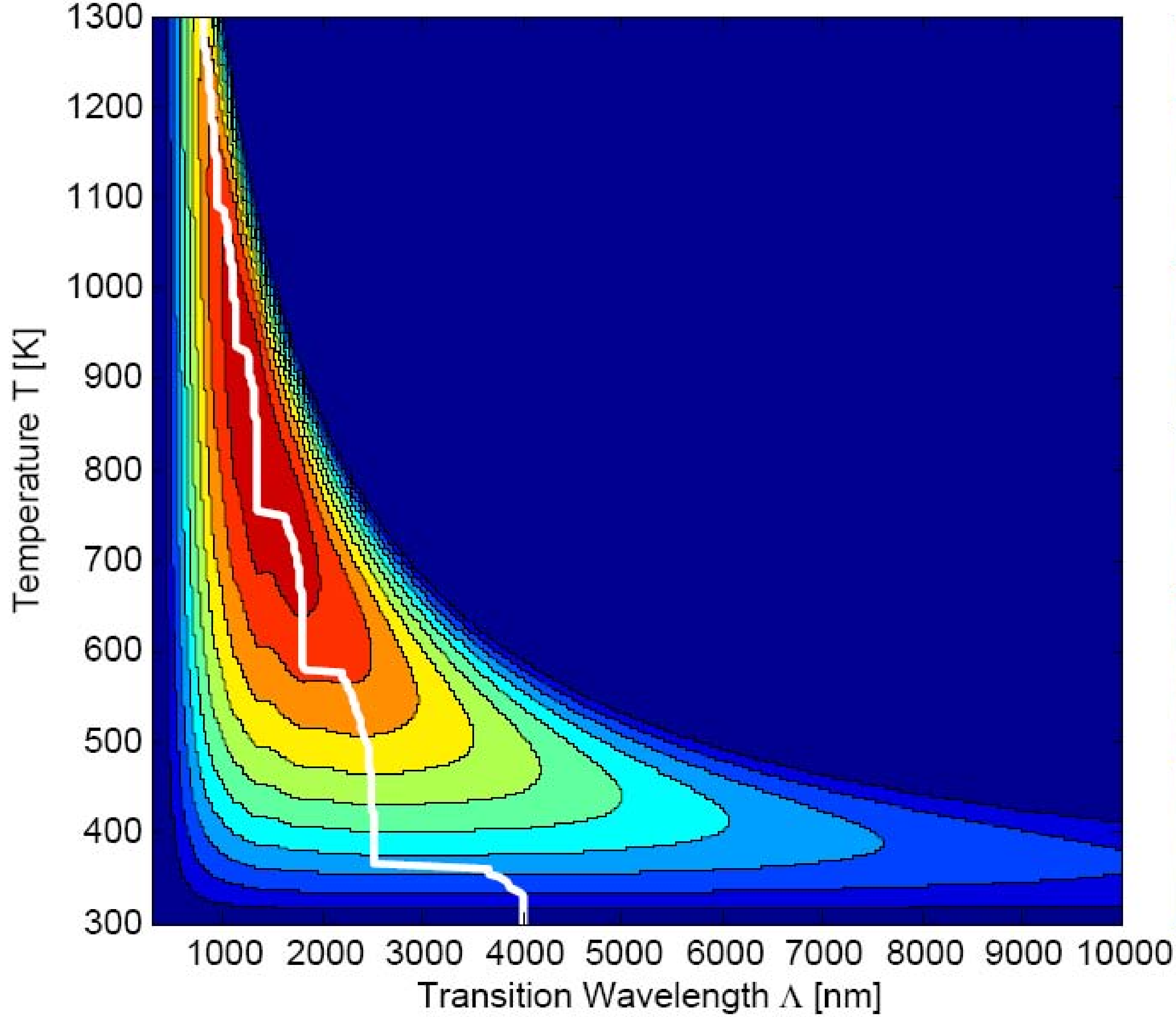


η_{th} Thermal Efficiency

Thermal Efficiency for AM1.5G with No Concentration

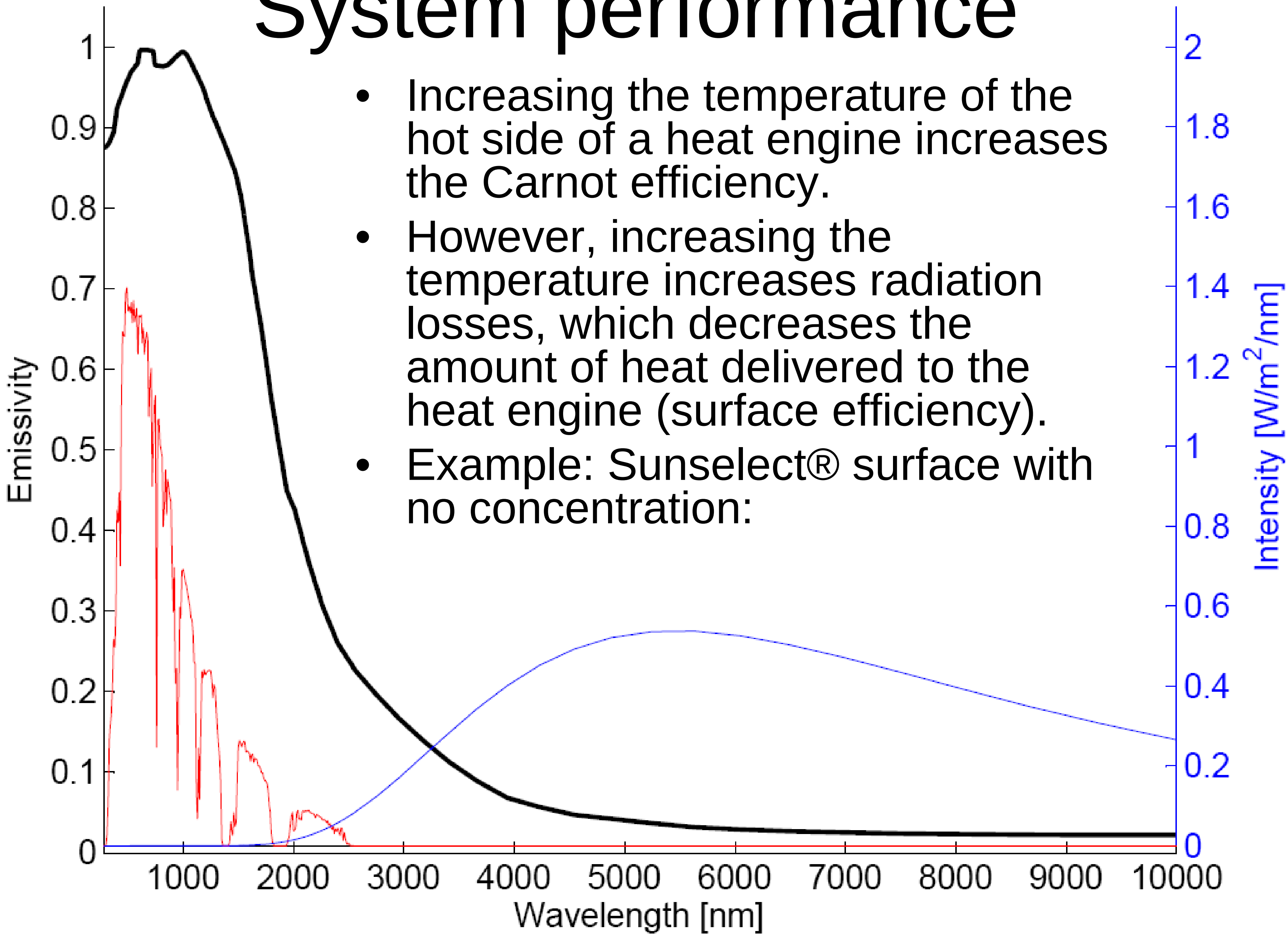


System (Surface x Carnot) Efficiency for AM1.5G with No Concentration

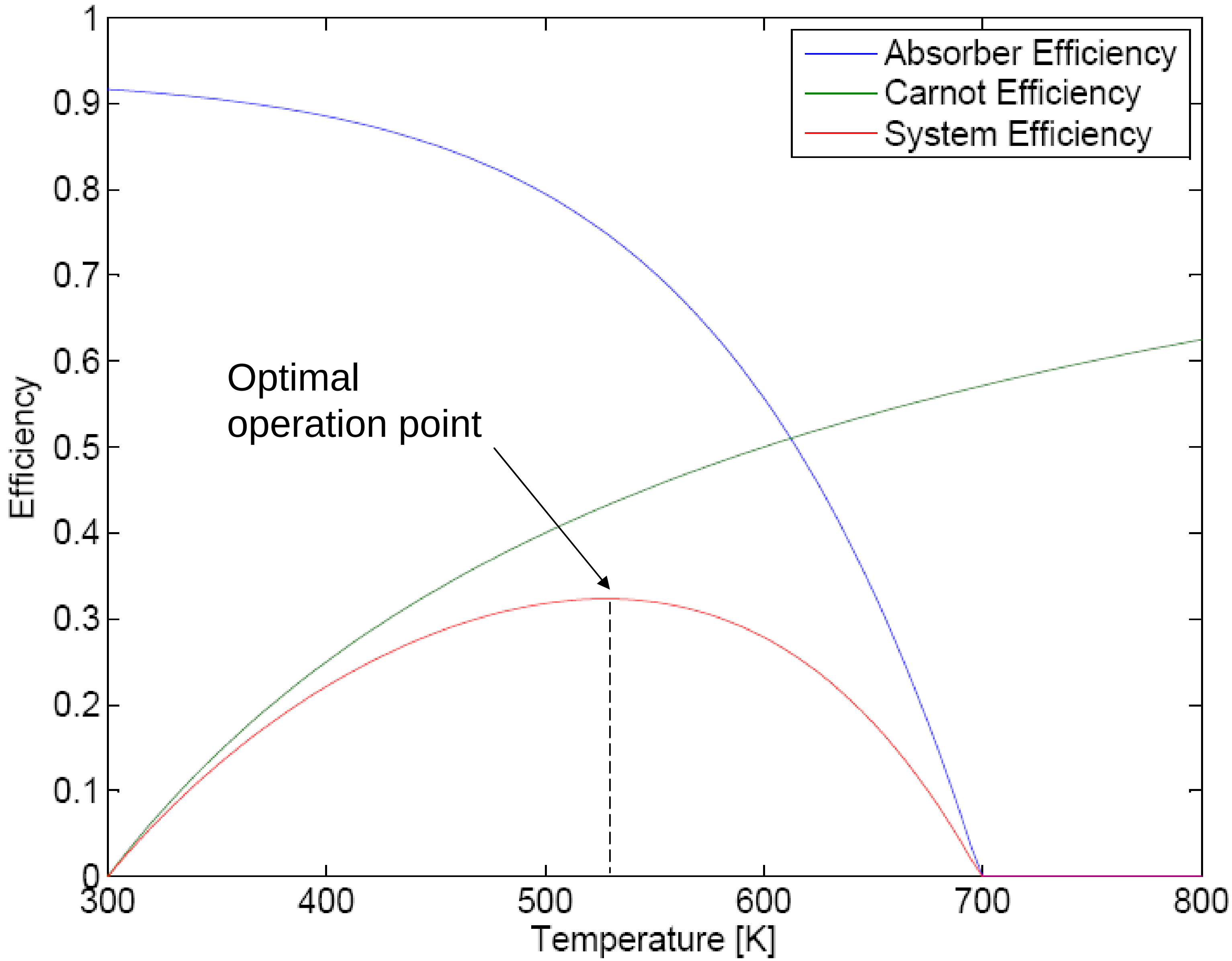


System performance

- Increasing the temperature of the hot side of a heat engine increases the Carnot efficiency.
- However, increasing the temperature increases radiation losses, which decreases the amount of heat delivered to the heat engine (surface efficiency).
- Example: Sunselect® surface with no concentration:

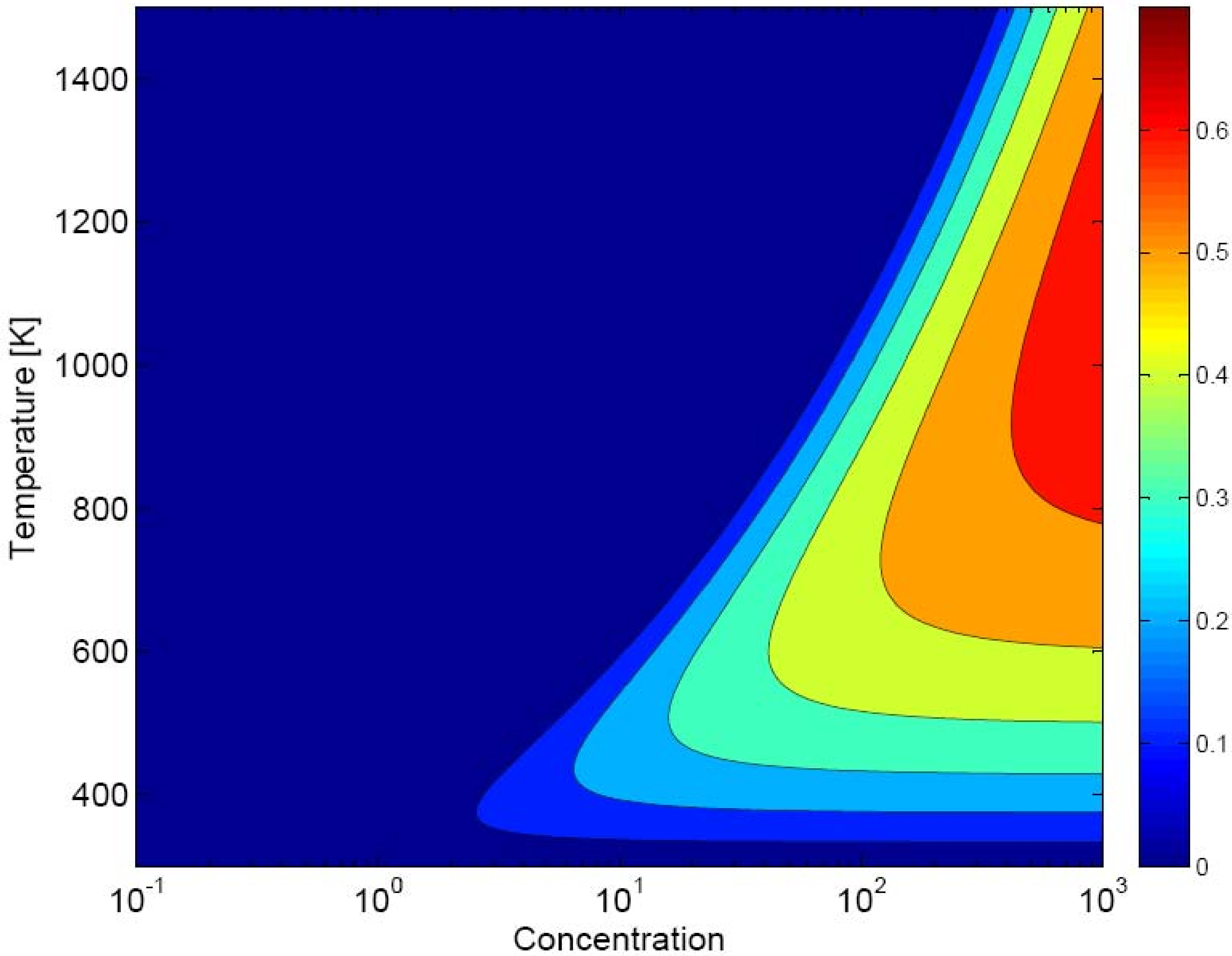


Sunselect absorber

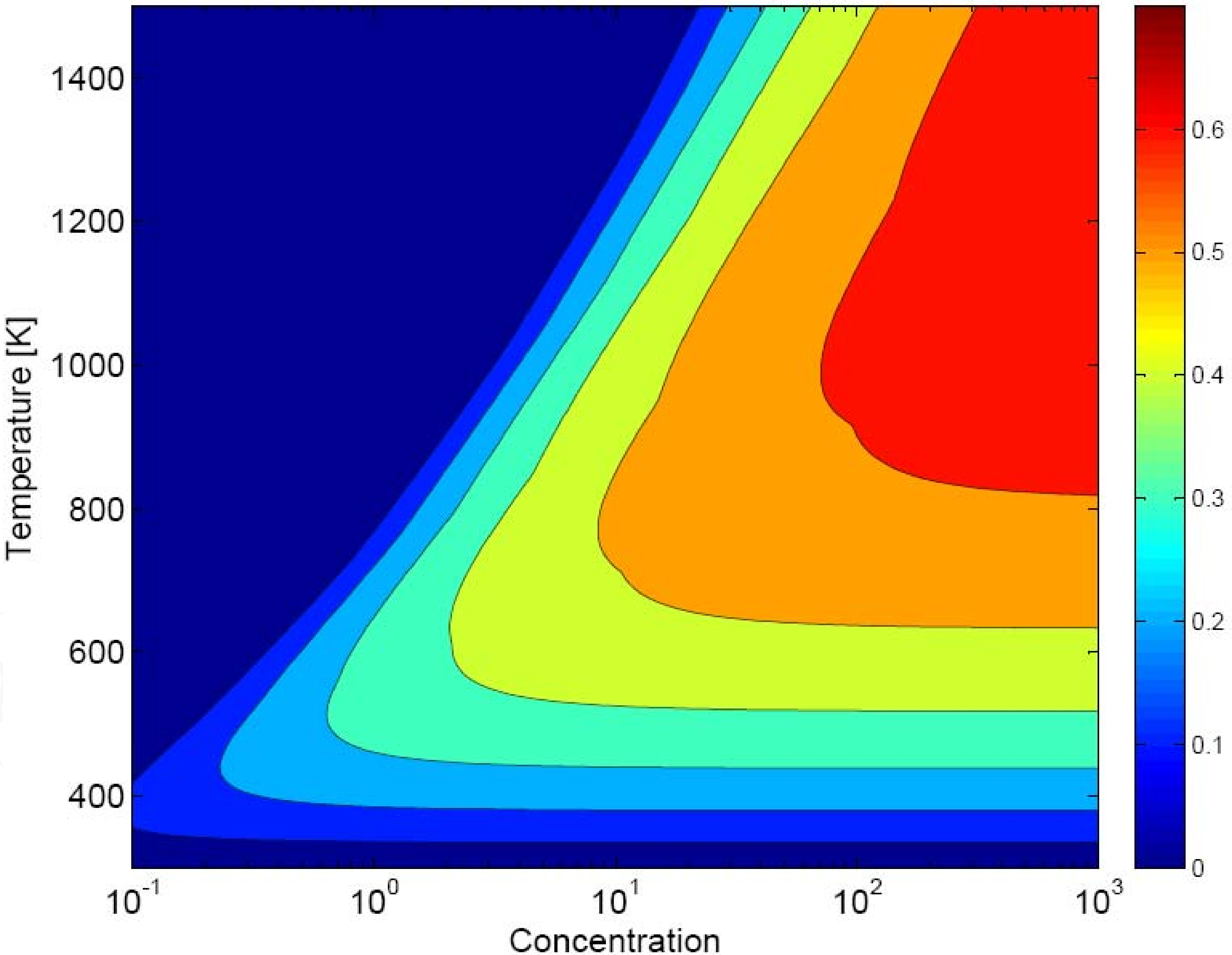


- This same analysis can be performed to find the ideal absorber for various concentrations and operating temperatures. This is the absolute upper limit on system efficiency.
- We can compare a blackbody, the Sunselect absorber, and an ideal absorber

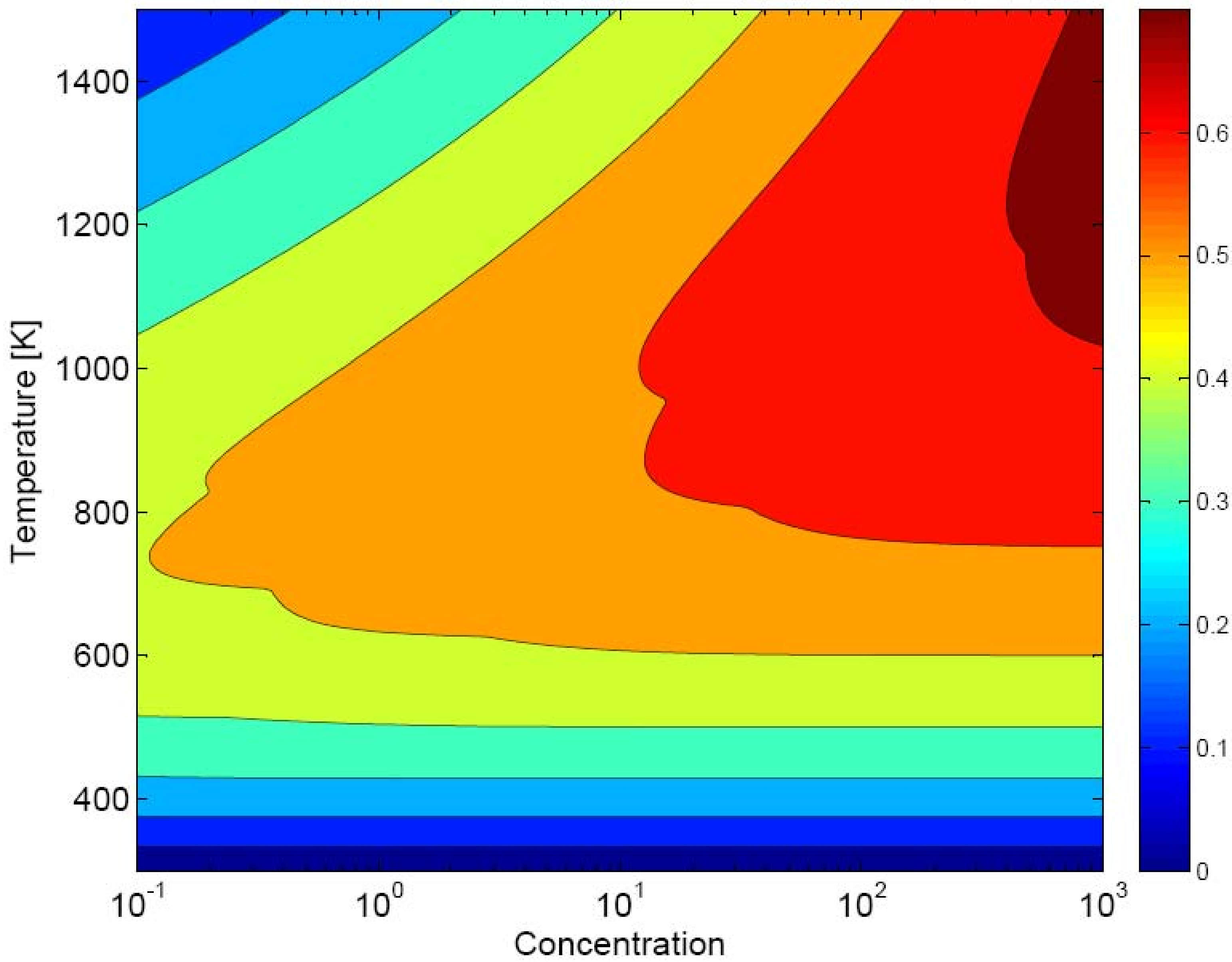
Blackbody absorber



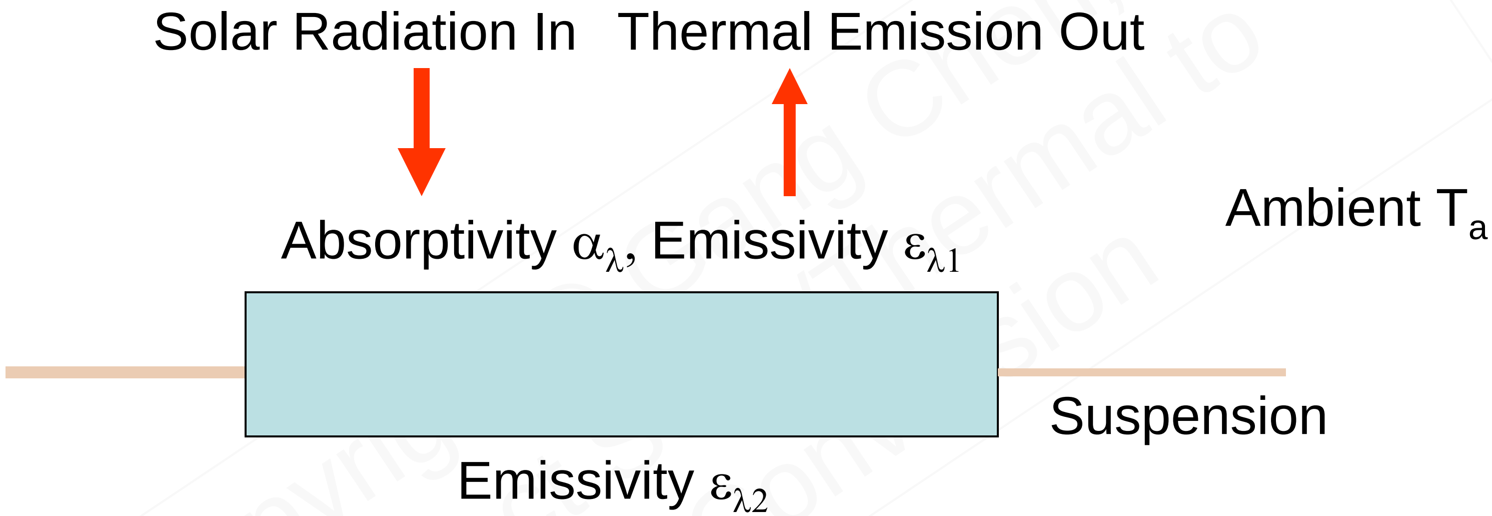
Sunselect absorber



Ideal absorber



Heat Transfer on A Suspended Surface



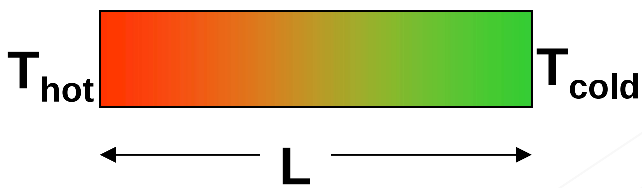
$$\int_0^\infty \alpha_\lambda J_\lambda d\lambda = \int_0^\infty \varepsilon_\lambda \left[e_{b\lambda}(T) - e_{b\lambda}(T_o)\right] d\lambda + \frac{T - T_a}{R_{th}}$$

↑

Thermal Resistance by Conduction

Heat Conduction

Heat Conduction



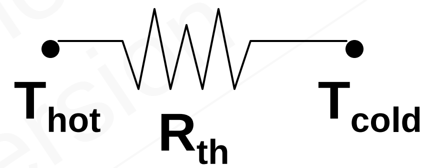
1D, no heat generation

$$\dot{Q} = kA\frac{T_{hot} - T_{cold}}{L} = \frac{T_{hot} - T_{cold}}{R_{th}}$$

Thermal Resistance

$$R_{th} = \frac{L}{kA}$$

Heat Current $\dot{Q}$



Convection

$$R_{th} = \frac{1}{hA}$$

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