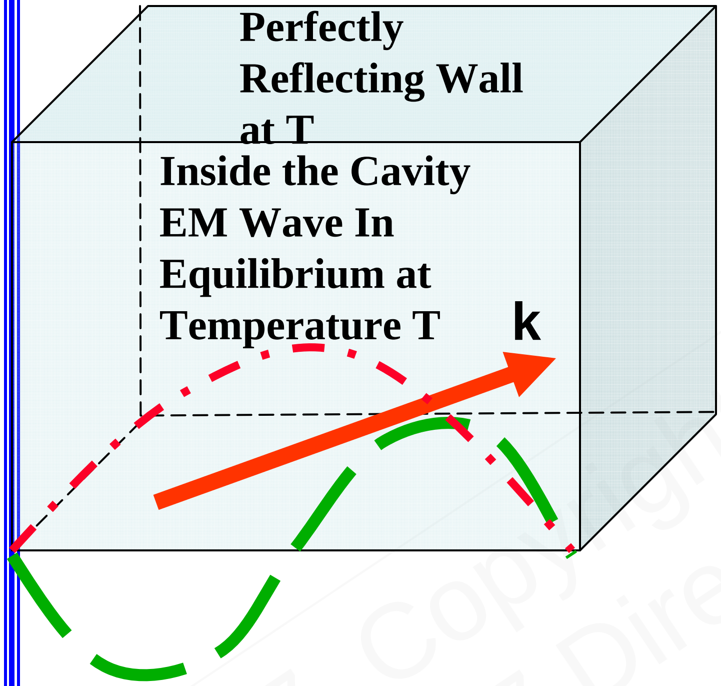
# Thermal Radiaton: Planck's Law



**Perfectly Reflecting Wall at T**

**Inside the Cavity EM Wave In Equilibrium at Temperature T**

**k**

$$L_x = \frac{\lambda_x}{2}, 2\frac{\lambda_x}{2}, \ldots, n_x\frac{\lambda_x}{2}, \ldots$$

$$k_x = n_x \frac{2\pi}{2L_x}$$

**Two polarization**

## Basic Relations

Frequency ν

Angular Frequency ω=2πν

Wavelength λ

Wavevector magnitude k=2π/λ

Wavevector **k**=($k_x$,$k_y$,$k_z$)

$$c = \nu\lambda \quad \Rightarrow \quad \omega = ck = c\sqrt{k_x^2 + k_y^2 + k_z^2}$$

ω(k): Dispersion relation (linear)

## How much energy in the cavity?

$$U = 2\sum_{n_x=1}^{\infty}\sum_{n_y=1}^{\infty}\sum_{n_z=1}^{\infty}\hbar\omega f(\omega,T) =$$

$$2\int_0^{\infty}\frac{dk_x}{(2\pi/2L_x)}\int_0^{\infty}\frac{dk_y}{(2\pi/2L_y)}\int_0^{\infty}\frac{dk_z}{(2\pi/2L_z)}\hbar\omega f(\omega,T)$$

$$= 2\int_{-\infty}^{\infty}\frac{dk_x}{(2\pi/L_x)}\int_{-\infty}^{\infty}\frac{dk_y}{(2\pi/L_y)}\int_{-\infty}^{\infty}\frac{dk_z}{(2\pi/L_z)}\hbar\omega f(\omega,T)$$

# Thermal Radiaton: Planck's Law

$$U = \frac{2V}{8\pi^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hbar\omega f(\omega,T)\,dk_x dk_y dk_z$$

$$= \frac{2V}{8\pi^3}\int_0^{\infty} \hbar\omega f(\omega,T)\,4\pi k^2 dk$$

$$= \frac{2V}{8\pi^3}\int_0^{\infty} \hbar\omega f(\omega,T)\,4\pi\left(\frac{\omega}{c}\right)^2 d\left(\frac{\omega}{c}\right)$$

$$\frac{U}{V} = \int_0^{\infty} \hbar\omega f(\omega,T)\frac{\omega^2}{\pi^2 c^3}d\omega$$

$$= \int_0^{\infty} \hbar\omega f(\omega,T)D(\omega)d\omega$$

$$= \int_0^{\infty} u(\omega)d\omega$$

**D(ω)-density of states per unit volume per unit angular frequency interval**

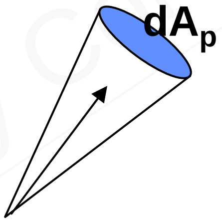
- **Energy density per ω interval**

$$u(\omega) = \hbar\omega f(\omega,T)D(\omega)$$

$$= \frac{\hbar\omega^3}{\pi^2 c^3}\frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right)-1}$$

**Planck's law**

- **Intensity: energy flux per unit solid angle**



**Solid Angle**

$$d\Omega = \frac{dA_p}{R^2}$$

**whole space 4π**

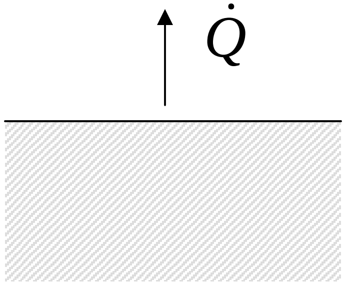
$$I(\omega) = \frac{cu(\omega)}{4\pi} = \frac{\hbar\omega^3}{4\pi^3 c^2}\frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right)-1}$$

**Per unit wavelength interval**

$$I(\lambda) = \left|\frac{I(\omega)d\omega}{d\lambda}\right| = \frac{4\pi c\hbar}{\lambda^5}\frac{1}{\exp\left(\frac{2\pi\hbar c}{k_B T\lambda}\right)-1}$$

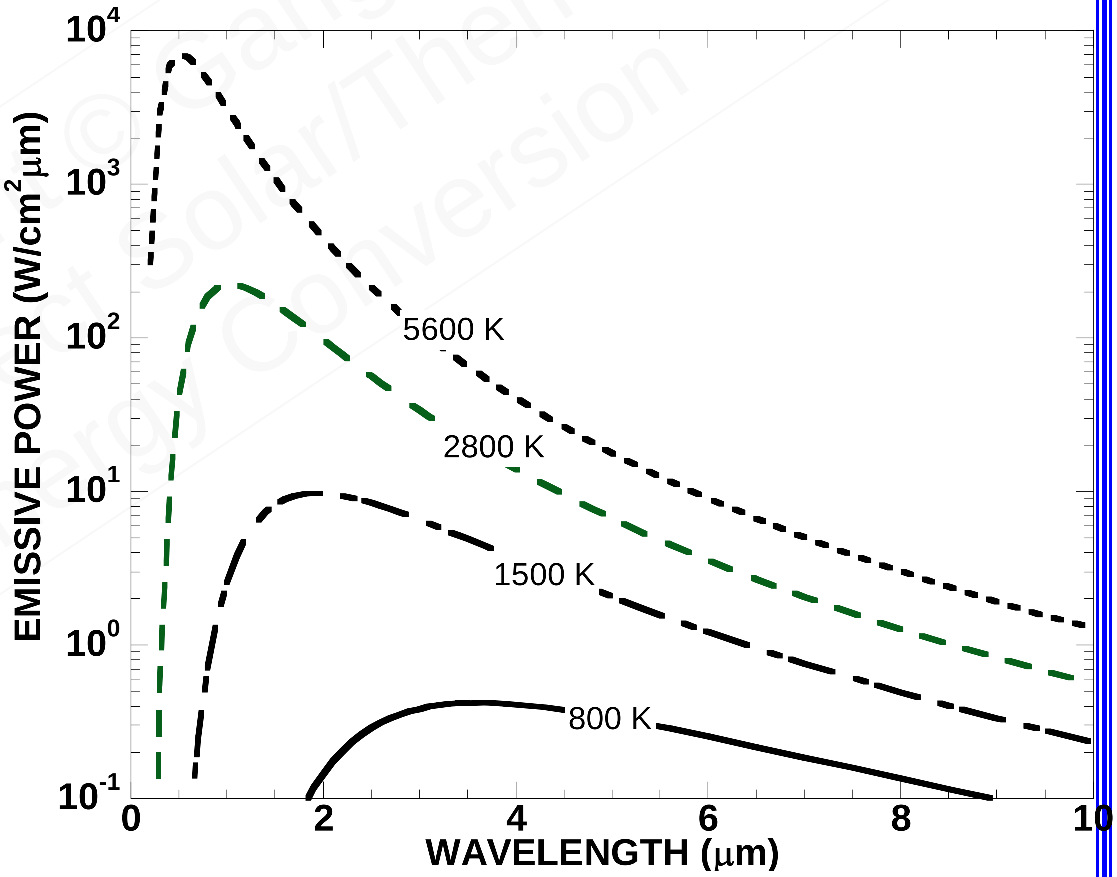
**Planck's law**

# Thermal Radiaton: Planck's Law



## Wien's displacement law

$$\lambda_{\max} T = 2898 \ \mathrm{K}\mu\mathrm{m}$$



## Emissive Power

$$\dot{Q}(\lambda) = A\pi I(\lambda)$$

$$= A\frac{\hbar\omega^3}{4\pi^2 c^2}\frac{1}{\exp\left(\dfrac{\hbar\omega}{k_B T}\right)-1}$$

## Total

$$\dot{Q} = \int_0^\infty \dot{Q}(\lambda)d\lambda = A\sigma T^4$$

# Introduction to Thermoelectricity

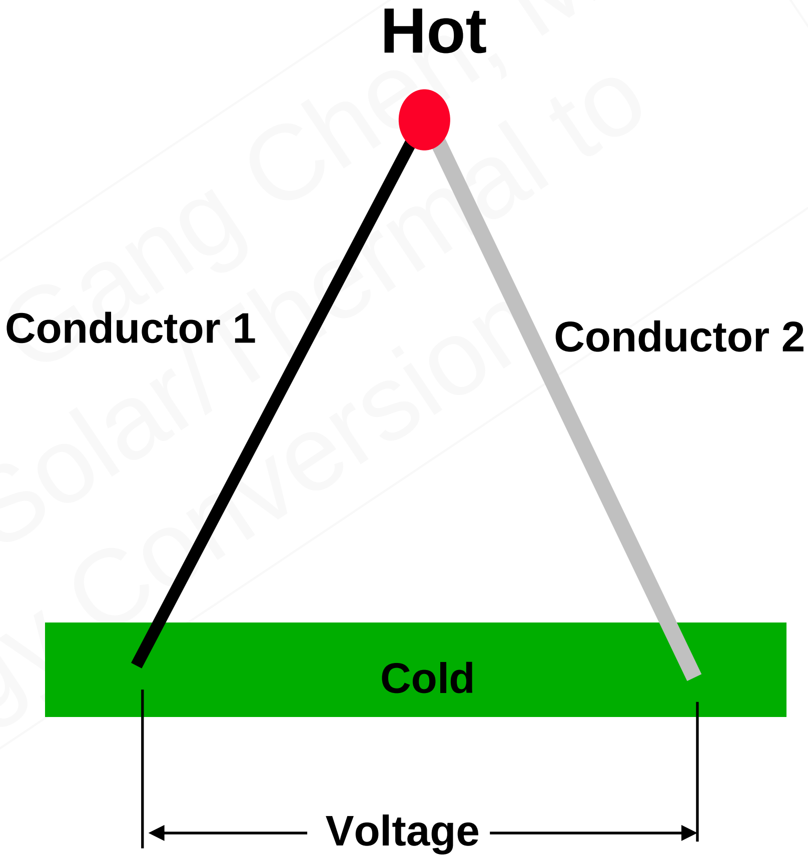
**Gang Chen**

**Mechanical Engineering Department**
**Massachusetts Institute of Technology**

**URL: http://web.mit.edu/nanoengineering**

# Seebeck Effect

**Thomas Johann Seebeck**
**1770-1831**

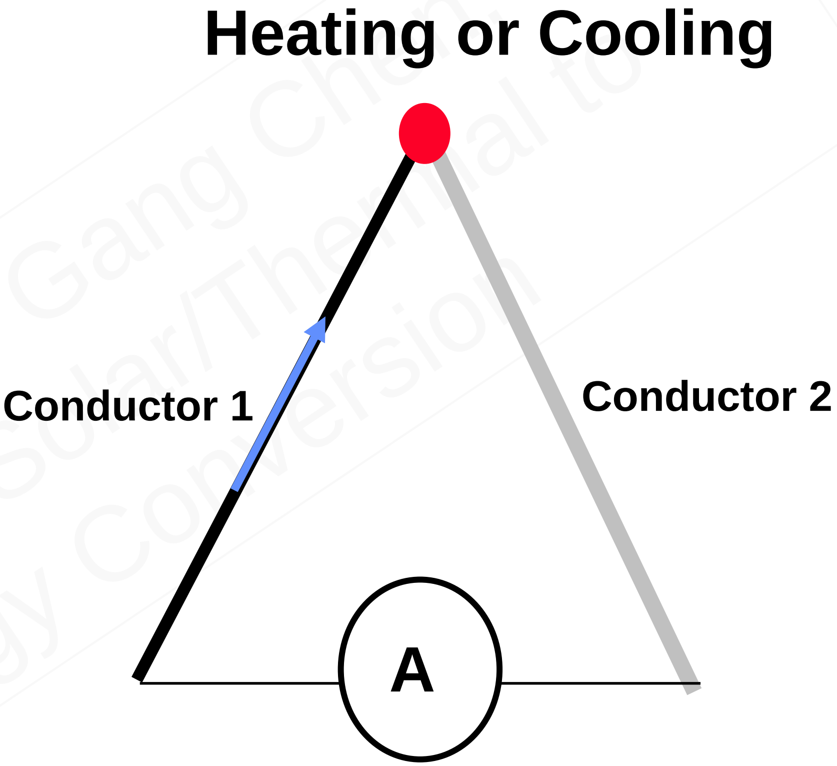


**Seebeck effect: Discovered in 1821**
**Temperature difference generates voltage**

http://www.sil.si.edu/silpublications/dibner-library-lectures/scientific-discoveries/text-lecture.htm

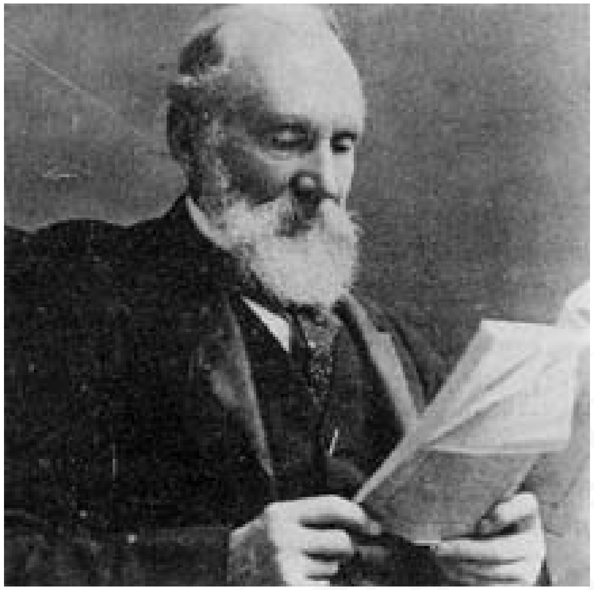
# Peltier Effect

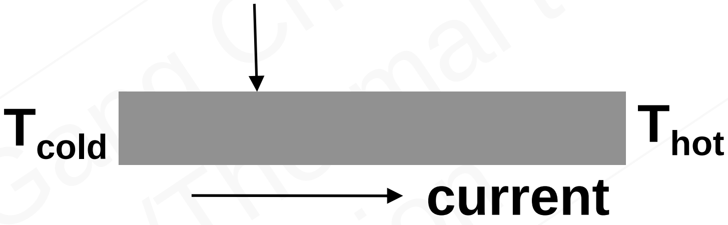
Jean Charles Athanase Peltier
1785-1845



**Peltier Effect: Discovered in 1834**
**An electrical current creates a cooling or heating effect at the junction depending on the direction of current flow.**

# Thomson Effect





**Thomson effect predicted, 1855**

**William Thomson**
**(Lord Kelvin)**
**1824 – 1907**

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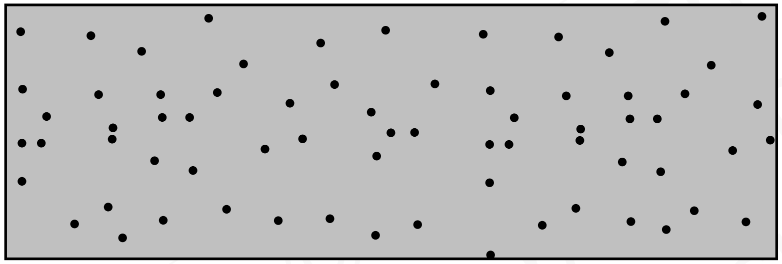
# An Intuitive Picture of Thermoelectric Effects

- ## Current Flow in an Isothermal Conductor



**Current Density** $\mathbf{J}_e = en\mathbf{v} = en\mu\boldsymbol{\varepsilon} = \sigma\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}/\rho$

**Electron Charge [C]** — $e$

**Electron Density [1/m³]** — $n$

**Drift Velocity [m/s]** — $\mathbf{v}$

**Mobility [m²/s.V]** — $\mu$

**Electrical Field [V/m]** — $\boldsymbol{\varepsilon}$

**Electrical Conductivity [1/Ωm]** — $\sigma$

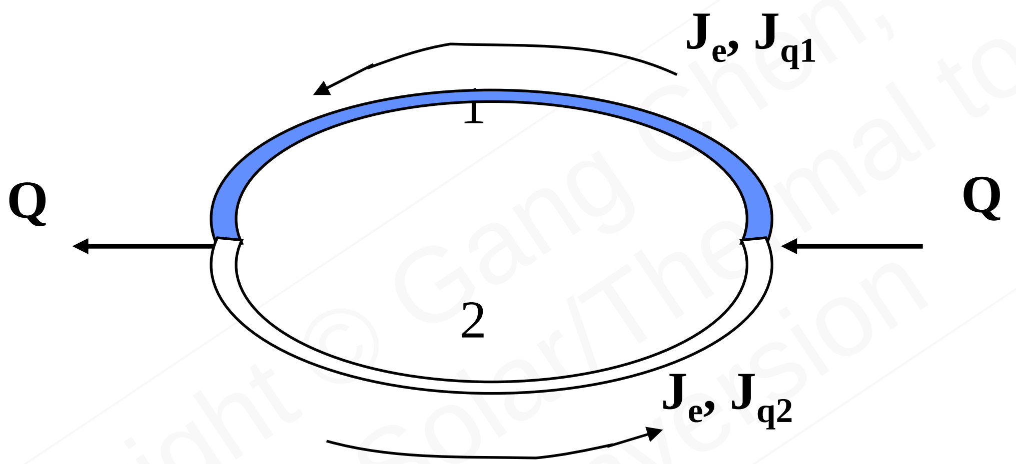
**Electrical Resistivity [Ωm]** — $\rho$

**Heat Flux** $\mathbf{J}_q = qn\mathbf{v} = \frac{q}{e}\mathbf{J}_e = \Pi\mathbf{J}_e$

**Heat Per Charge [J]** — $q$

**Peltier Coefficient [J/A]** — $\Pi$

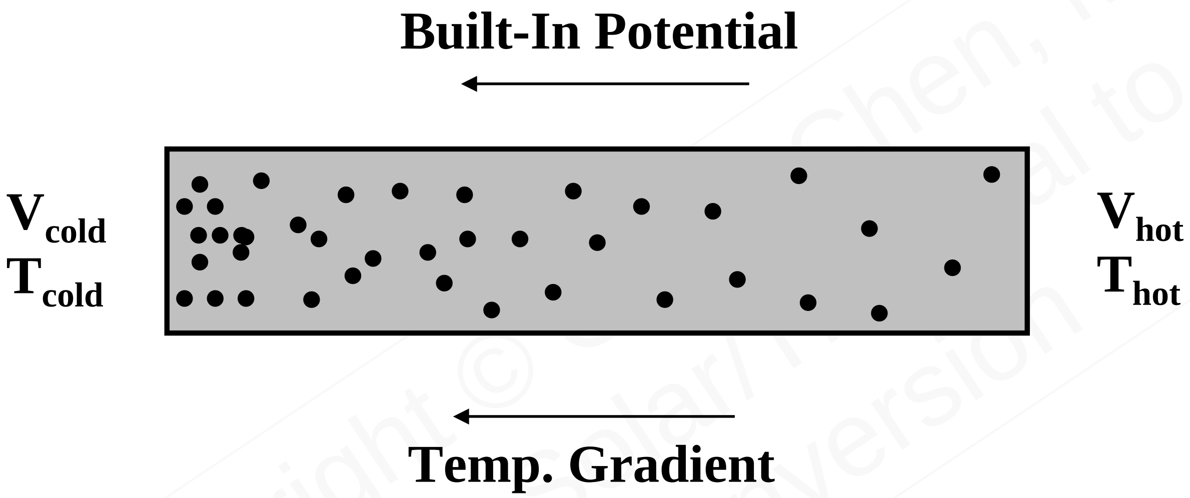
# Peltier Effect



$$Q\ (\text{Peltier}) = (\Pi_1 - \Pi_2)J$$

- **Heating and cooling at junctions**
- **Reversible with current direction**

# Seebeck Effect



**Built-In Potential**

**$V_{cold}$**
**$T_{cold}$**

**$V_{hot}$**
**$T_{hot}$**

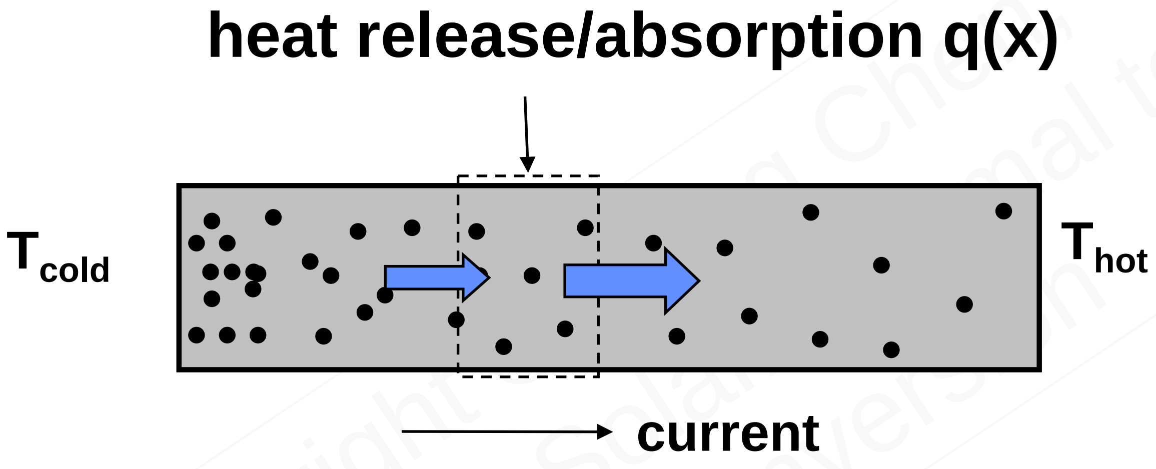
**Temp. Gradient**

- Charge diffusion under a temperature gradient
- Built-in potential resisting diffusion

$$S = -\Delta V/\Delta T = -(V_{hot}-V_{cold})/(T_{hot}-T_{cold})$$

S --- Seebeck Coefficient

# Thomson Effect



**Thomson Coefficient**

$$\tau = \frac{1}{I}\frac{dq}{dx} \bigg/ \frac{dT}{dx}$$

- **Kelvin Relations:** $\Pi = ST; \quad \tau = T\, dS/dT$

# Properties are Temperature Dependent

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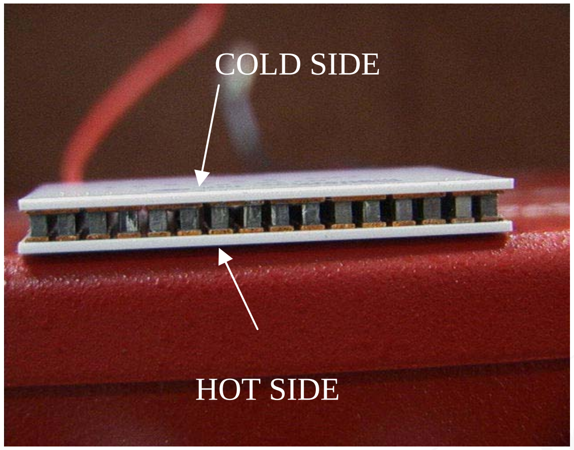
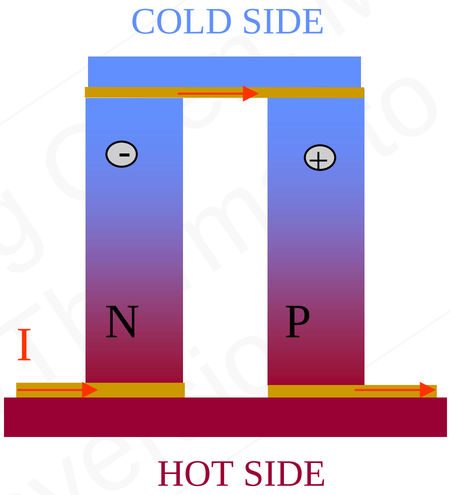
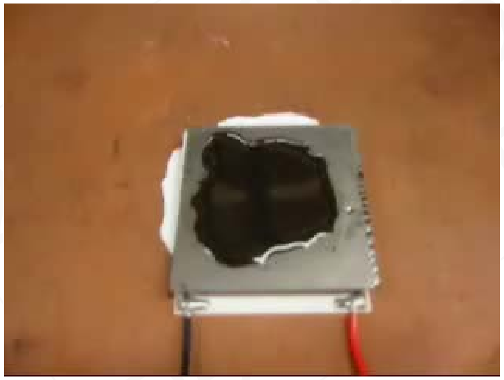
Please see Fig. 2a,b in Poudel, Bed, et al. "High-Thermoelectric Performance of Nanostructured Bismuth Antimony Telluride Bulk Alloys." *Science* 320 (May 2, 2008): 634-638.

# Thermoelectric Devices

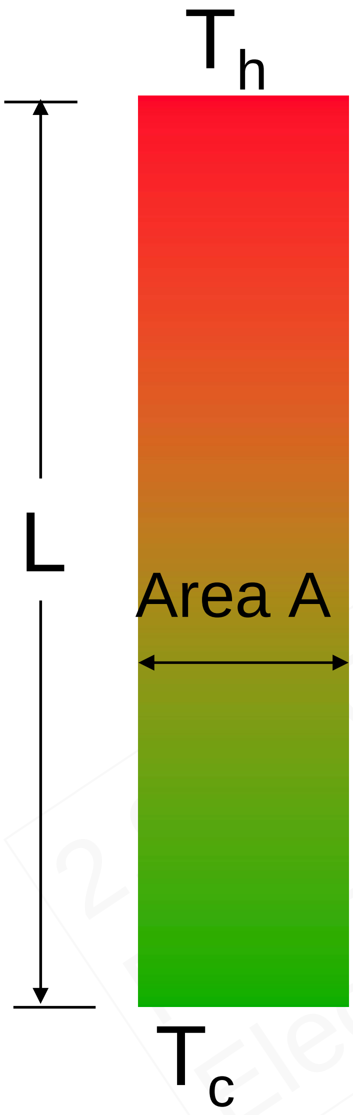






# Performance of Thermoelectric Devices

# Other Basic Relations: Heat Conduction



- Fourier Law for heat conduction

$$\mathbf{q} = -\underset{\uparrow}{k}\nabla T$$

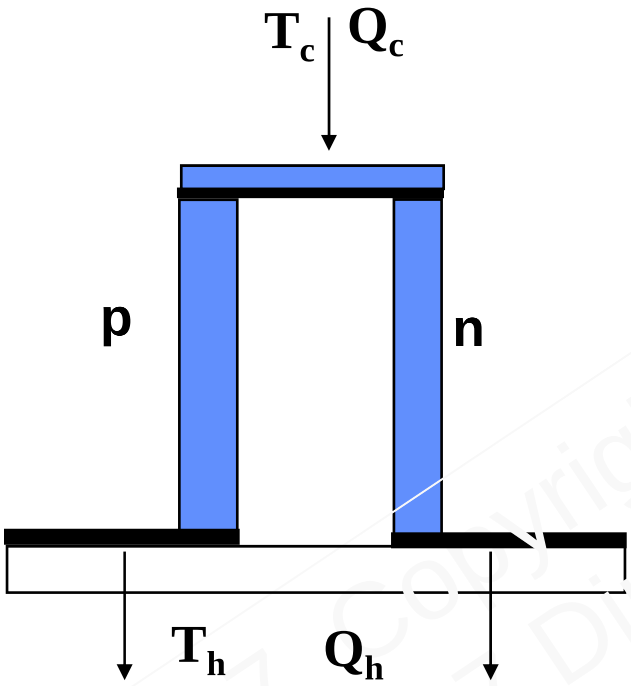
Heat Flux [W/m$^2$] **Thermal Conductivity [W/m.K]**

- One-dimensional heat conduction

$$Q = Ak\frac{T_h - T_c}{L} = K\Delta T$$

$$\text{Thermal Conductance}: K = \frac{kA}{L}$$

# Device Analysis: Cooling



- **Ideal Devices**

**No Joule Heating,**
**No Heat Conduction**

$$Q_c = (\Pi_p - \Pi_n) \bullet I$$

- **Real Devices:**

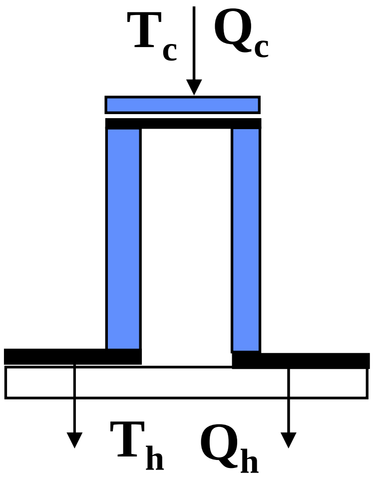
**Joule Heating & Heat Conduction**

$$Q_c = (\Pi_p - \Pi_n) \bullet I - I^2 R/2 - K\,(T_h - T_c)$$

**Electrical Resistance** **Thermal Conductance**

$$R = \frac{L_p \rho_p}{A_p} + \frac{L_n \rho_n}{A_n} \qquad K = \frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}$$

# Refrigerator Performance



**Voltage Drop:** $V = IR + (S_p - S_n)(T_h - T_c)$

**Coefficient of Performance:**

$$\phi = \frac{Q_C}{W} = \frac{\left(S_p - S_n\right) I T_C - K\left(T_H - T_C\right) - \frac{1}{2} I^2 R}{\left(S_p - S_n\right) I \left(T_H - T_C\right) + I^2 R}$$

**Optimize Current:**

$$\phi_{max} = \frac{T_C}{\left(T_H - T_C\right)} \cdot \frac{\sqrt{1 + Z T_M} - \frac{T_H}{T_C}}{\sqrt{1 + Z T_M} + 1}$$

$$T_M = 0.5(T_h + T_c)$$

$$Z = \frac{\left(S_p - S_n\right)^2}{KR} = \frac{\left(S_p - S_n\right)^2}{\left(\frac{L_p \rho_p}{A_p} + \frac{L_n \rho_n}{A_n}\right)\left(\frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}\right)}$$

# Figure of Merit Z

$$KR=\left(\frac{L_p\rho_p}{A_p}+\frac{L_n\rho_n}{A_n}\right)\left(\frac{k_pA_p}{L_p}+\frac{k_nA_n}{L_n}\right)$$

$$(KR)_{min}=\left(\sqrt{k_p\rho_p}+\sqrt{k_n\rho_n}\right)^2$$ **when** $$\frac{L_nA_p}{L_pA_n}=\left(\frac{\rho_pk_n}{\rho_nk_p}\right)^{1/2}$$

$$Z_{max}=\frac{\left(S_p-S_n\right)^2}{\left(\sqrt{k_p\rho_p}+\sqrt{k_n\rho_n}\right)^2}$$

**For a single material:** $Z=\frac{S^2}{\rho k}=\frac{\sigma S^2}{k}$

**In a device, pn pairs are used:**

**(1) Areas of each type of legs need to be optimized**
**(2) Two types of legs should have comparable properties**
**(3) Current input to the device needs to be optimized**

# Typical Number

- $Bi_2Te_3$-based materials ~300 K

**S=220 μV/K**
**$\sigma=10^5$ $Sm^{-1}$**
**k=1.5 W/mK**

**Power Factor: $S^2\sigma$=48 μW/cm-K**
**Figure of Merit: Z=$3.2\times10^{-3}$ 1/K**
**ZT=1**

- **Device Leg: 1 mm x 1 mm x 2 mm**

$$R_1=\frac{L}{\sigma A}=\frac{2\times10^{-3}}{10^5\times10^{-6}}=0.02\quad\Omega$$

**Legs are electrically in series but thermally in parallel**

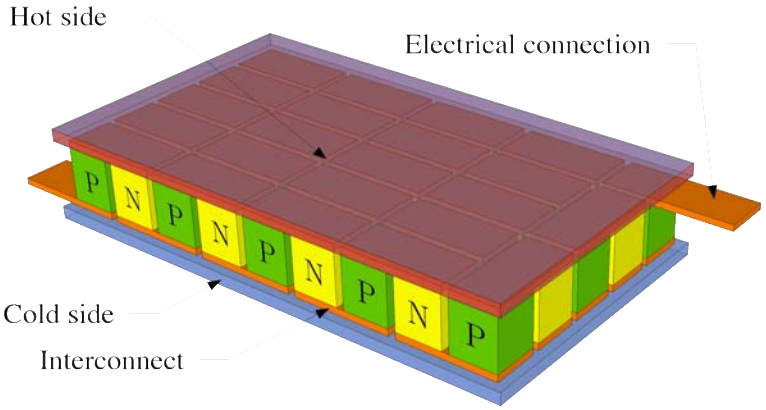
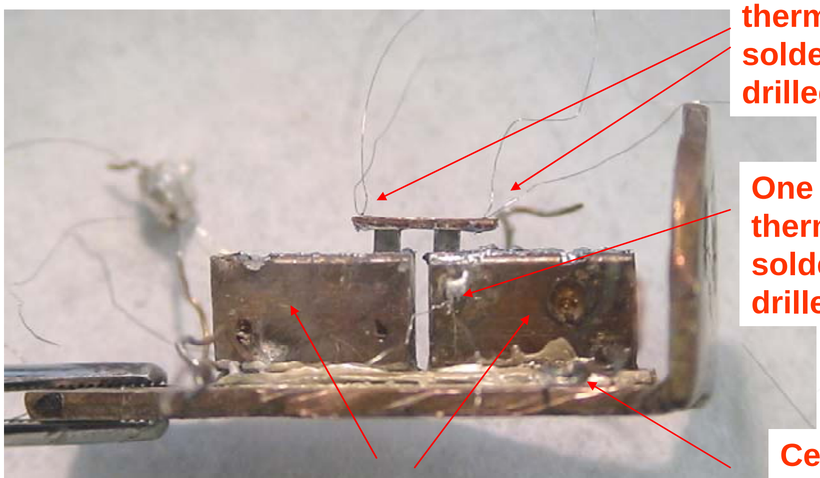
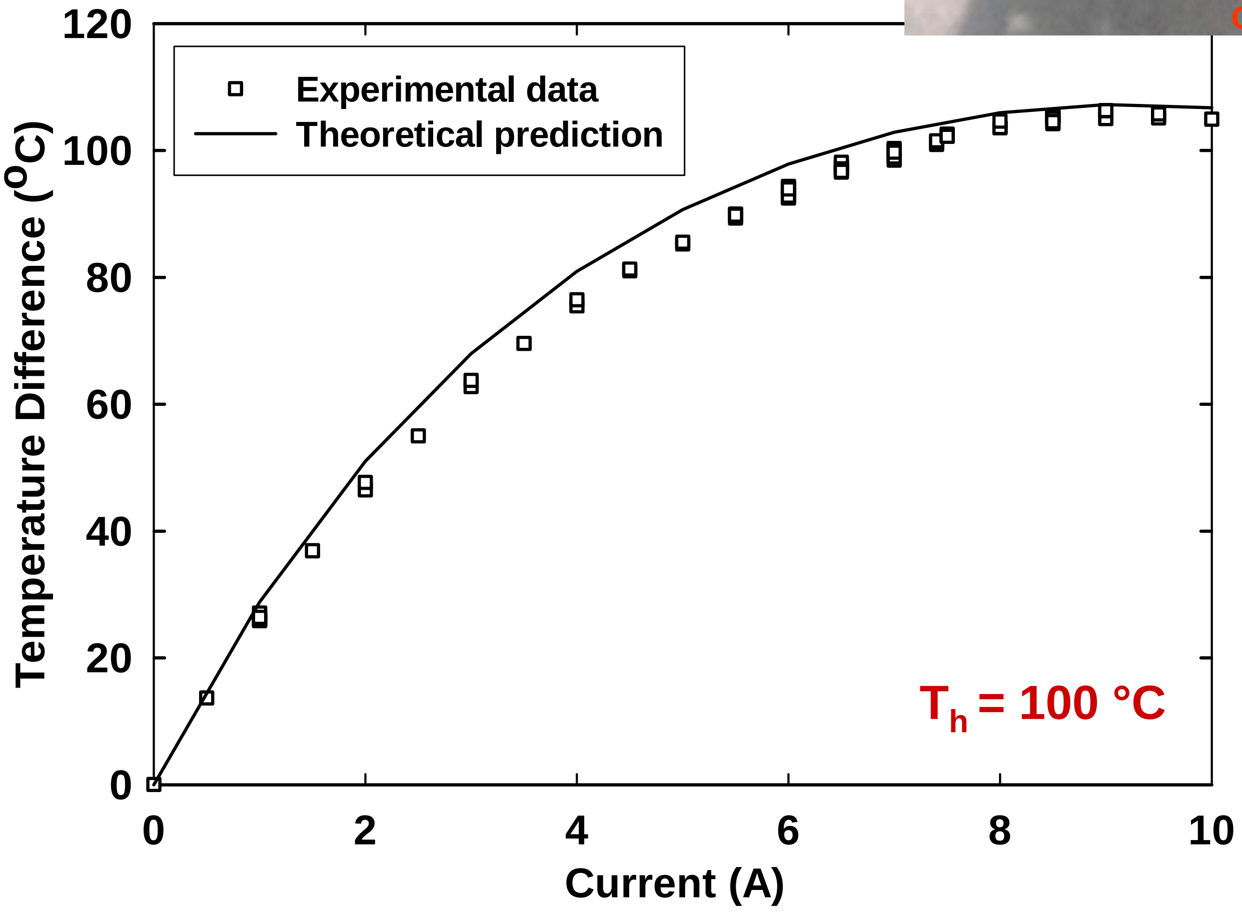


Image by michbich at Wikipedia.

# An Example





- Match two leg size
- Minimize contact resistance
- Optimize current

# Temperature Dependence of Properties

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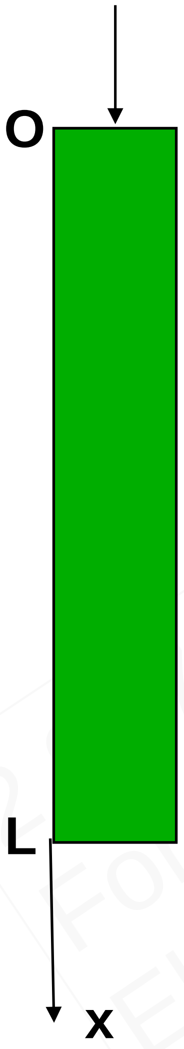
Please see Fig. 2a,b,d,e in Poudel, Bed, et al.
"High-Thermoelectric Performance of Nanostructured
Bismuth Antimony Telluride Bulk Alloys." *Science* 320 (May 2, 2008): 634-638.

# Differential Analysis



$$\frac{d}{dx^2}\left(k\frac{dT}{dx}\right)-JT\frac{dS}{dT}\frac{dT}{dx}+J^2\rho=0$$

**Thomson Effect, Usually Neglected** **Joule Heating**

**Boundary Conditions (Cooling):**

**x=0:** **$T_c$ given or** $q_c=-k\frac{dT}{dx}+\Pi J=-k\frac{dT}{dx}+ST_cJ$

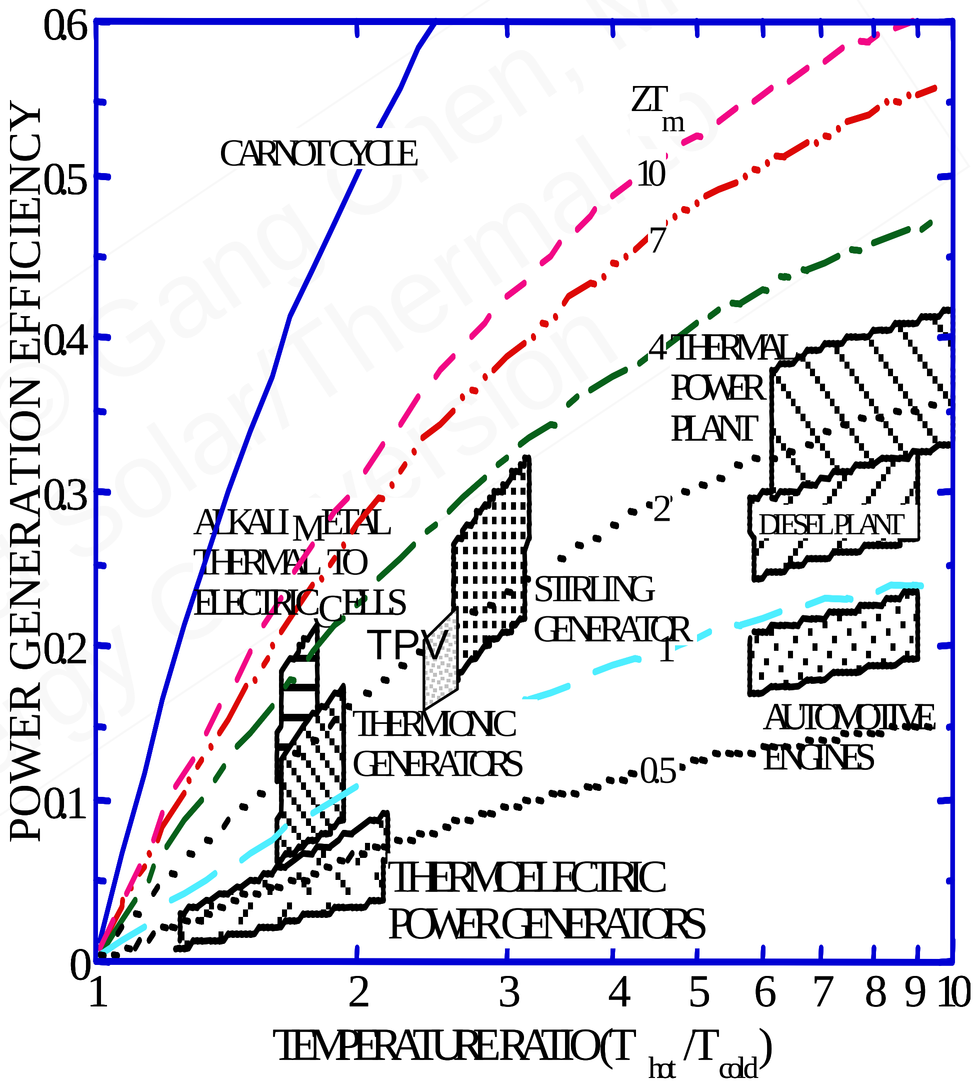
# Thermoelectric Power Generation

Efficiency

$$\eta = \left(1-\frac{T_c}{T_h}\right)\frac{\sqrt{1+ZT_{ave}}-1}{\sqrt{1+ZT_{ave}}+\frac{T_c}{T_h}}$$

$$T_{ave}=\frac{T_c+T_h}{2}$$

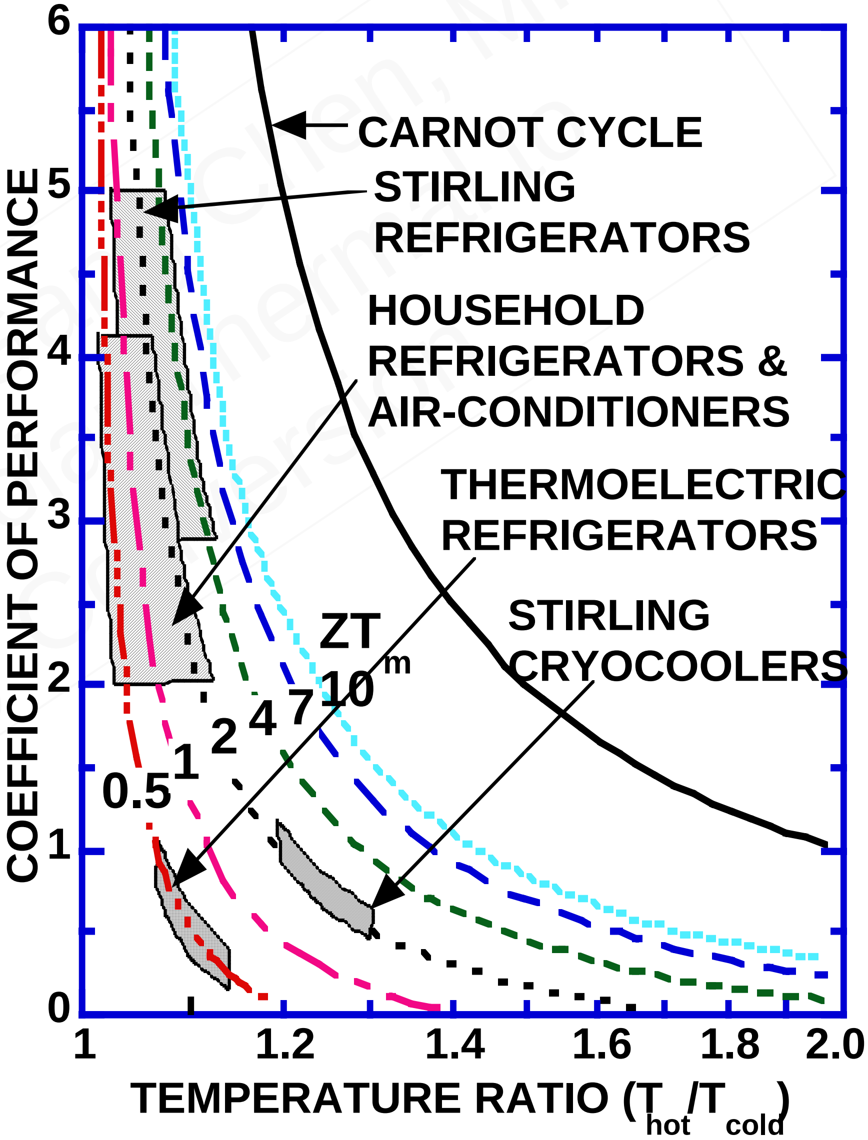
Constant Properties



# Thermoelectric Refrigeration

Coefficient of Performance

$$COP = \frac{T_c}{T_h - T_c} \frac{\sqrt{1+ZT_{ave}} - T_h/T_c}{\sqrt{1+ZT_{ave}} + 1}$$



# Current and Potential Applications

# Commercial Thermoelectric Devices

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Please see http://www.hi-z.com/index.php
http://www.marlow.com/thermoelectric-modules/

**Power Generators from Hi-Z**

**Coolers from Marlow Industries**

# Current Applications in Refrigeration

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Please see: http://www.roadtrucker.com/12-volt-coolers-accessories/12-volt-coolers-products/igloo-40-quart-kool-mate-40-12-volt-thermo-electric-cooler-6402.jpg
http://image.made-in-china.com/2f0j00kvZEKWVPgtlu/Refrigerator-BC-65A-.jpg
http://www.newdavincis.com/images/wc-1682%2016%20bottles.jpg
http://www.rmtltd.ru/datasheets/TO812.4MD04116xx.pdf
http://www.medsystechnology.com/images/gem4000_w32a.jpg
http://amerigon.com/ccs_works.php

# Current Applications in Power Generation

http://thermoelectrics.caltech.edu/images/mhw-rtg.gif

http://globalte.com/pdf/teg_5120_spec.pdf

http://www.roachman.com/thermic/thermic1.jpg

http://www.research.philips.com/newscenter/pictures/downloads/misc-sustainability_05-0_h.jpg

# System Consideration

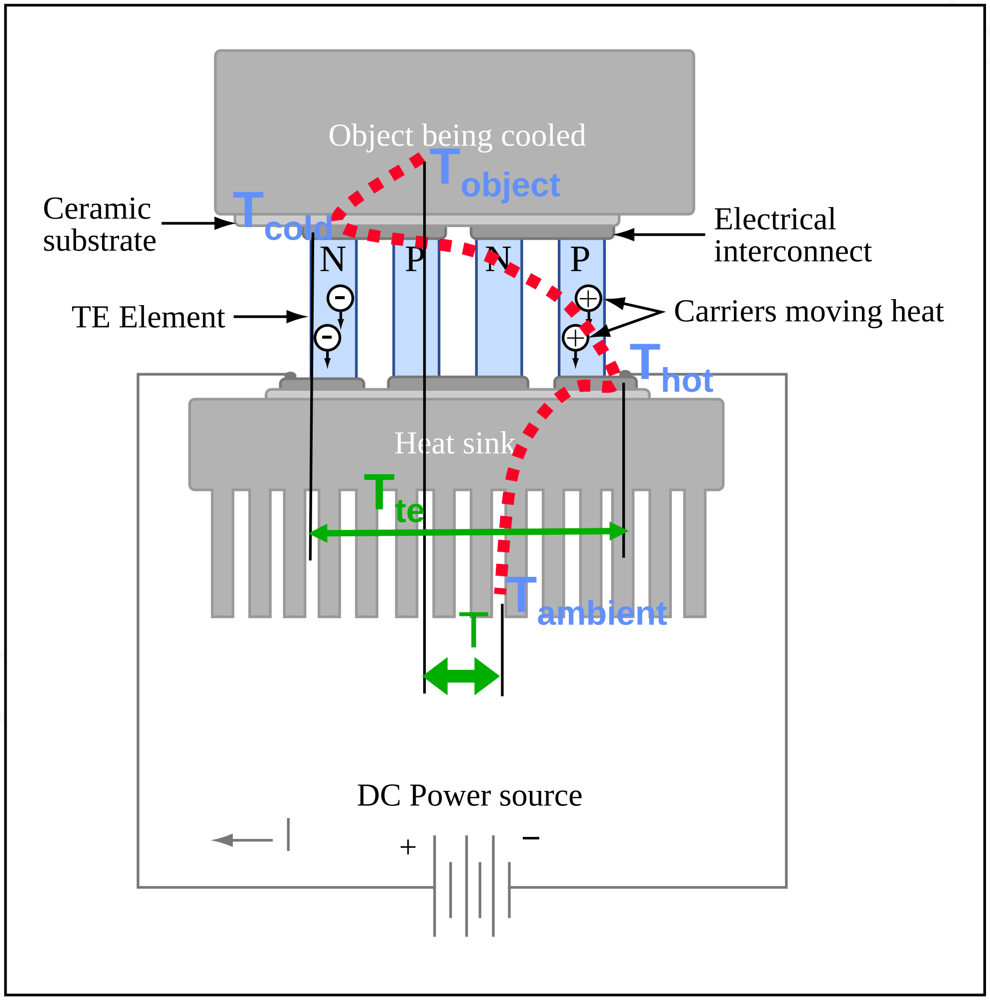
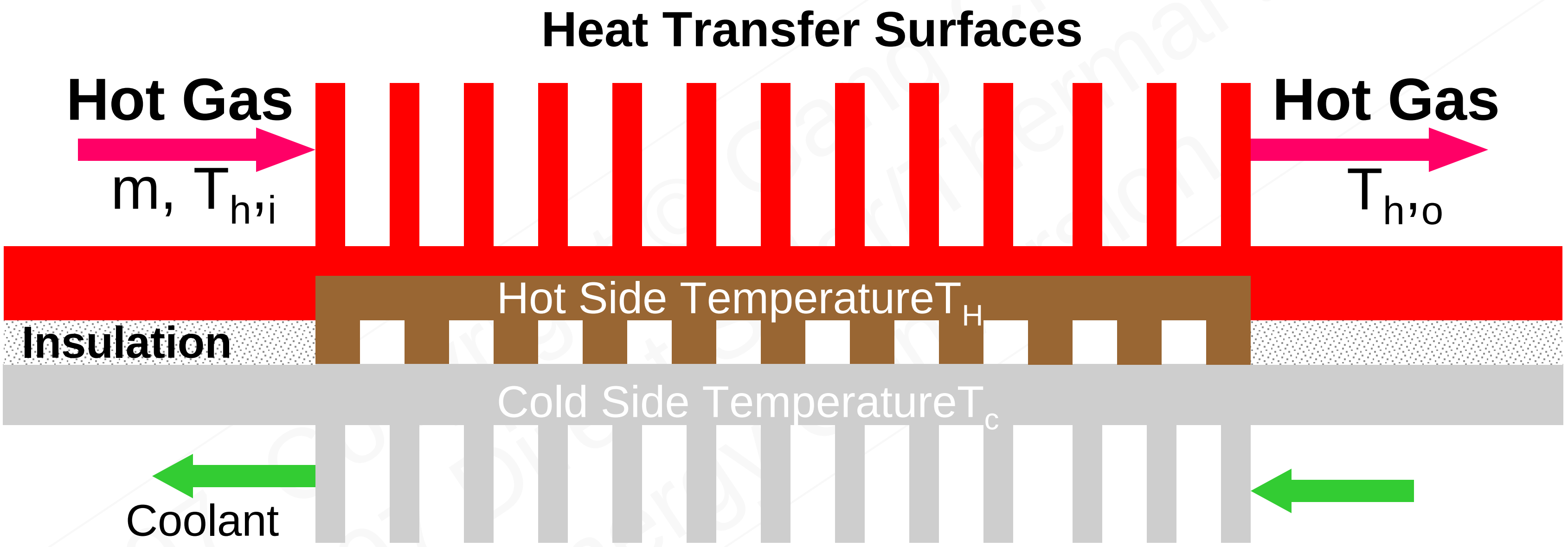


Figure by MIT OpenCourseWare.

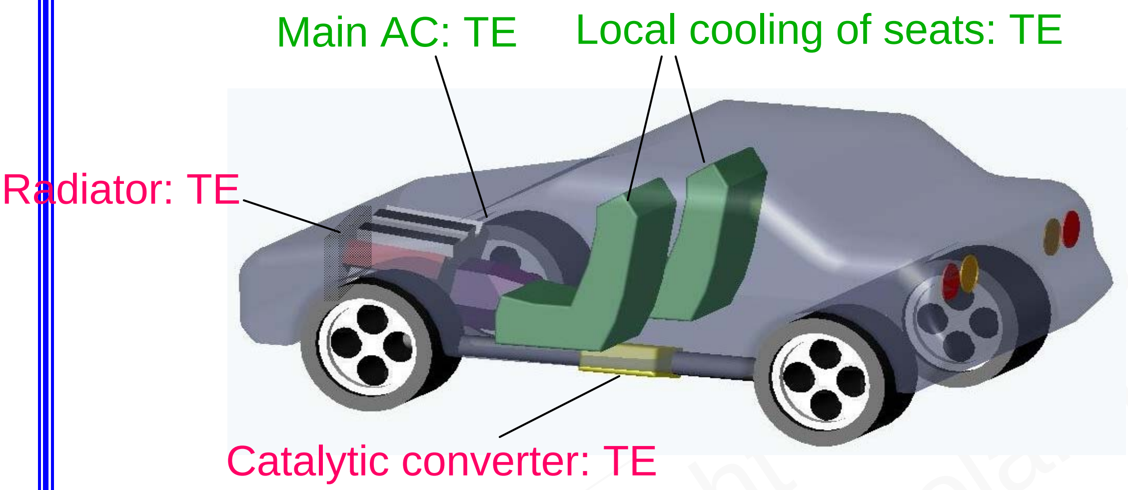
Sometimes, thermal systems more expansive

# Heat to Electricity Recovery from Gas Stream

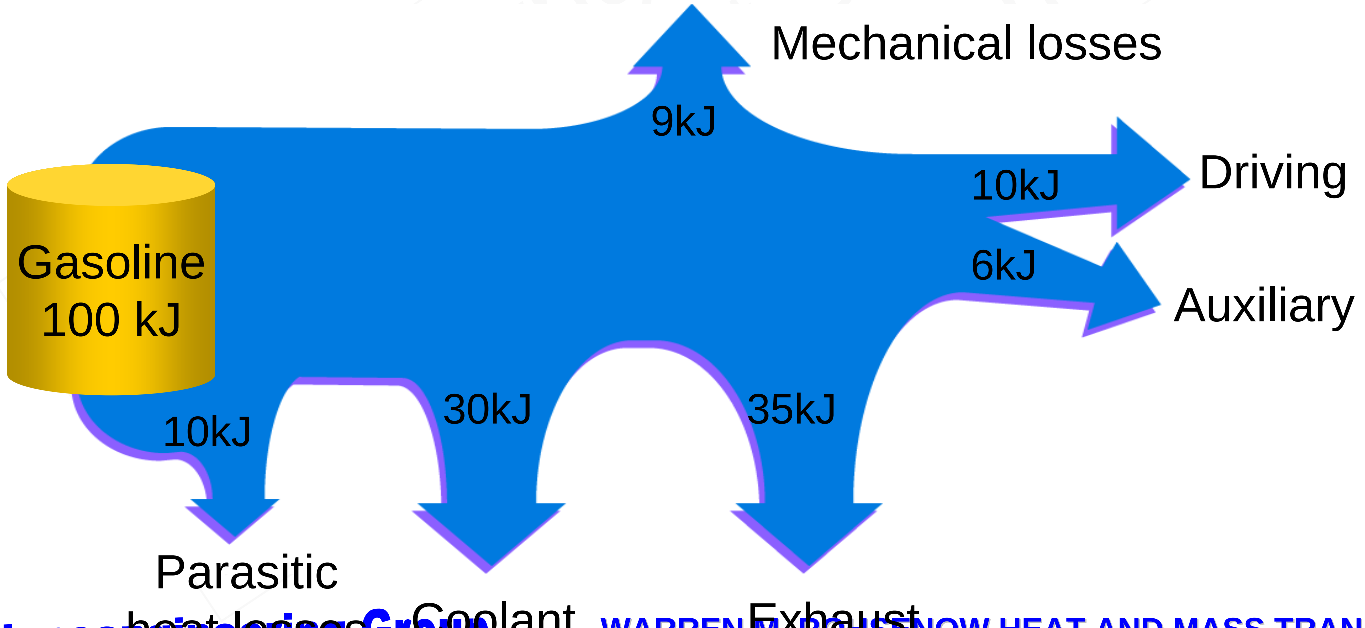


- For thermoelectric devices, $T_H$ higher is better
- However, maximum heat intercepted from hot gas stream, $mc_p(T_{h,i}-T_H)$, decreases with $T_H$

# Vehicle Systems



**In US, transportation uses ~26% of total energy.**



# Prototypes

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http://cdn-www.greencar.com/images/waste-exhaust-heat-generates-electricity-cars-efficient.php/bmw-teg-1.jpg

http://cache.gawkerassets.com/assets/images/12/2009/03/BMW_TEG.jpg

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2.997 Direct Solar/Thermal to Electrical Energy Conversion Technologies
Fall 2009