2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

Control of Manufacturing Processes

Subject 2.830/6.780/ESD.63

Spring 2008

Lecture #4

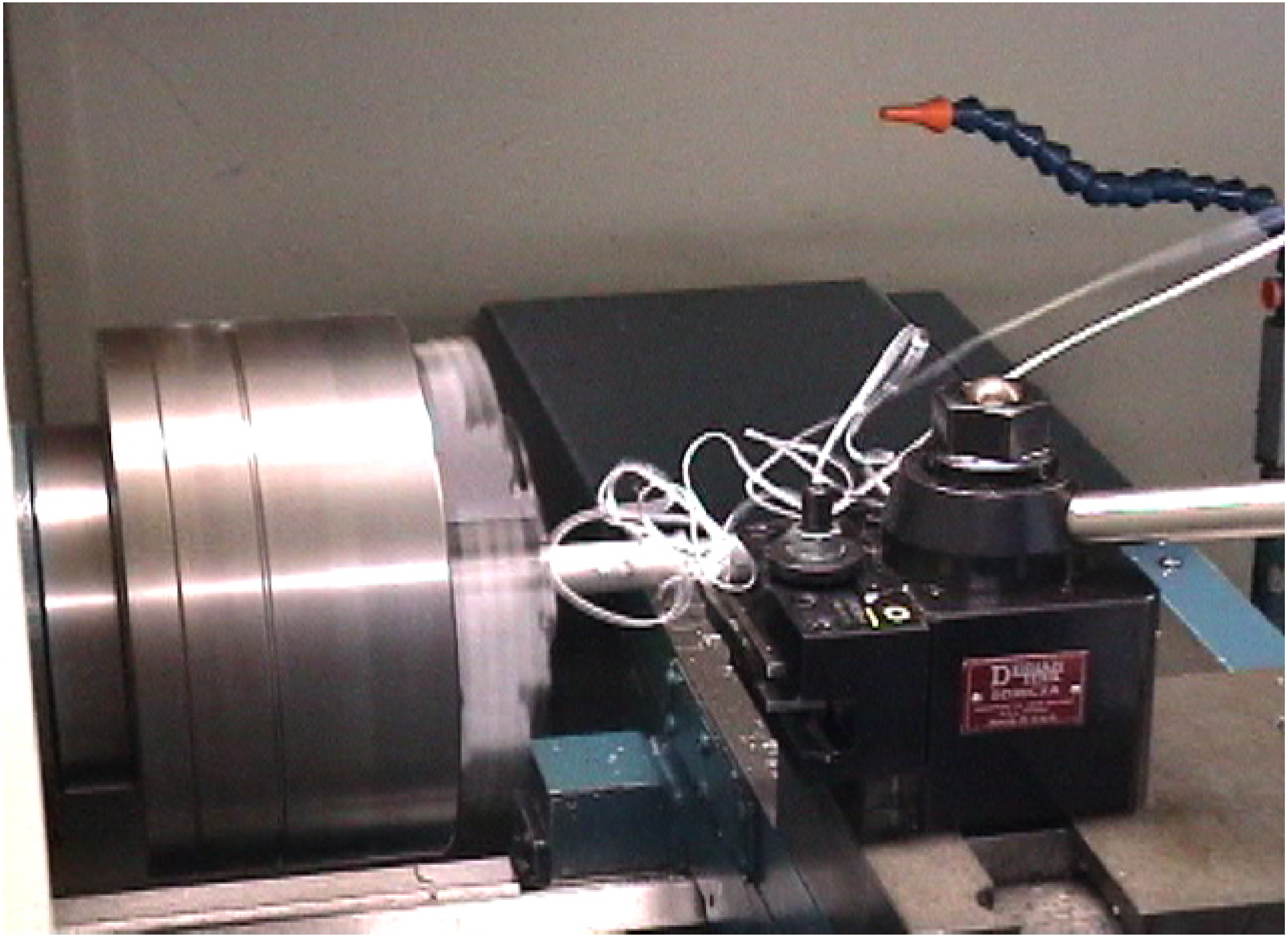
Probability Models of Manufacturing Processes

February 14, 2008

Note: Reading Assignment

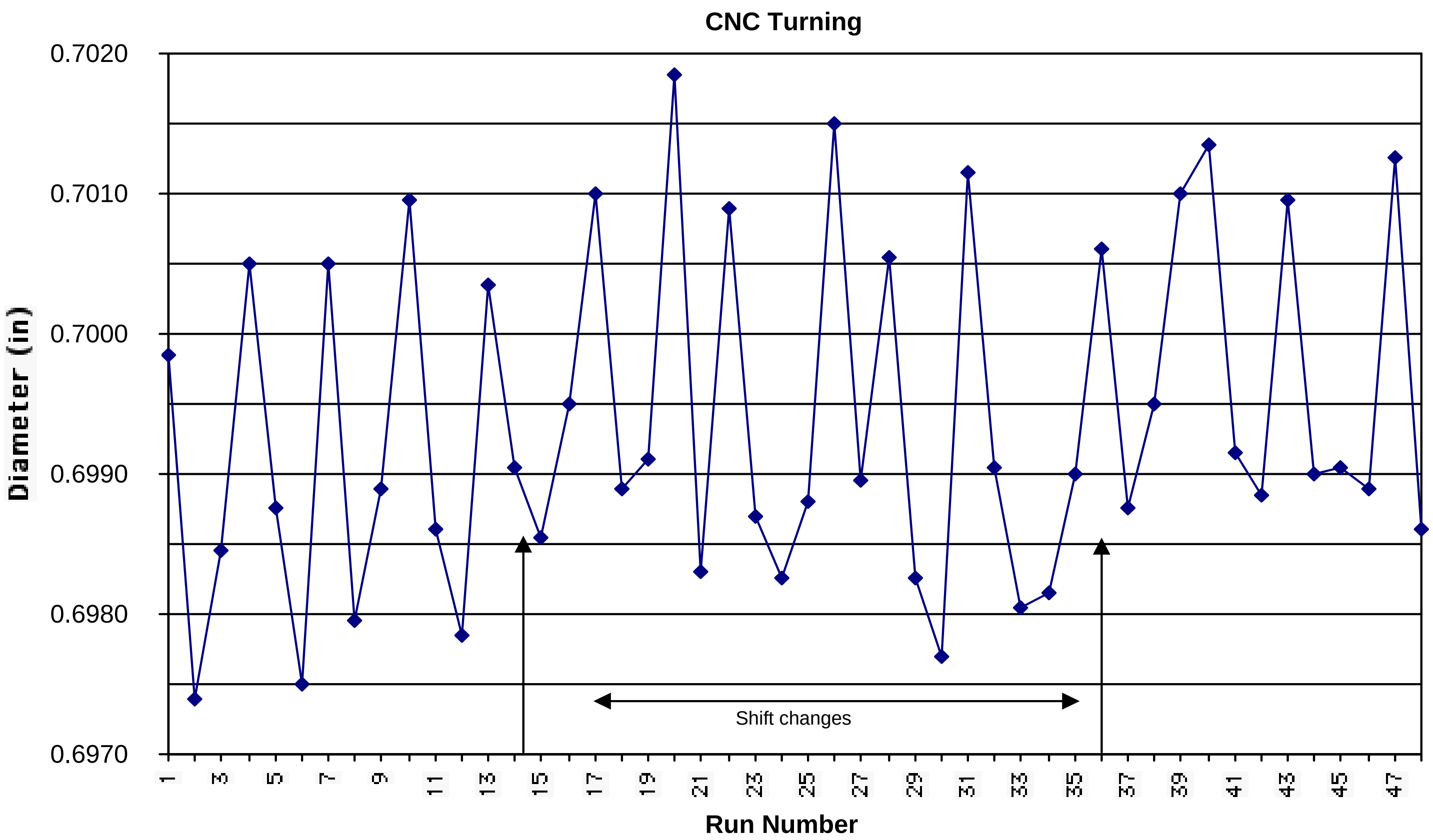
- May & Spanos
  - Read Chapter 4
- Montgomery
  - Skim/consult Chapters 2 & 3 if need additional explanations or examples beyond May & Spanos

Turning Process



Observations from Experiments

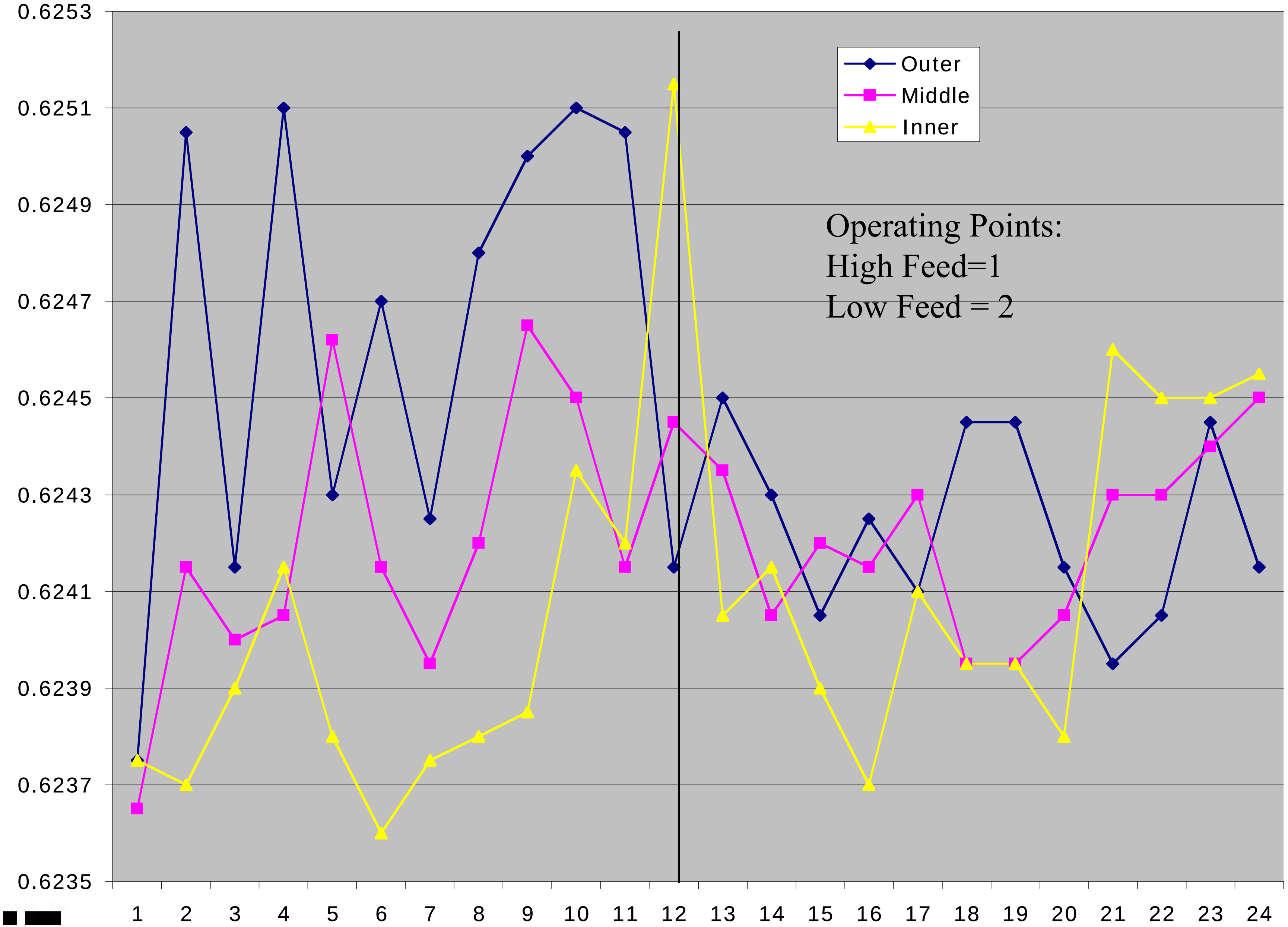
- Randomness + Deterministic Changes



random or unknown

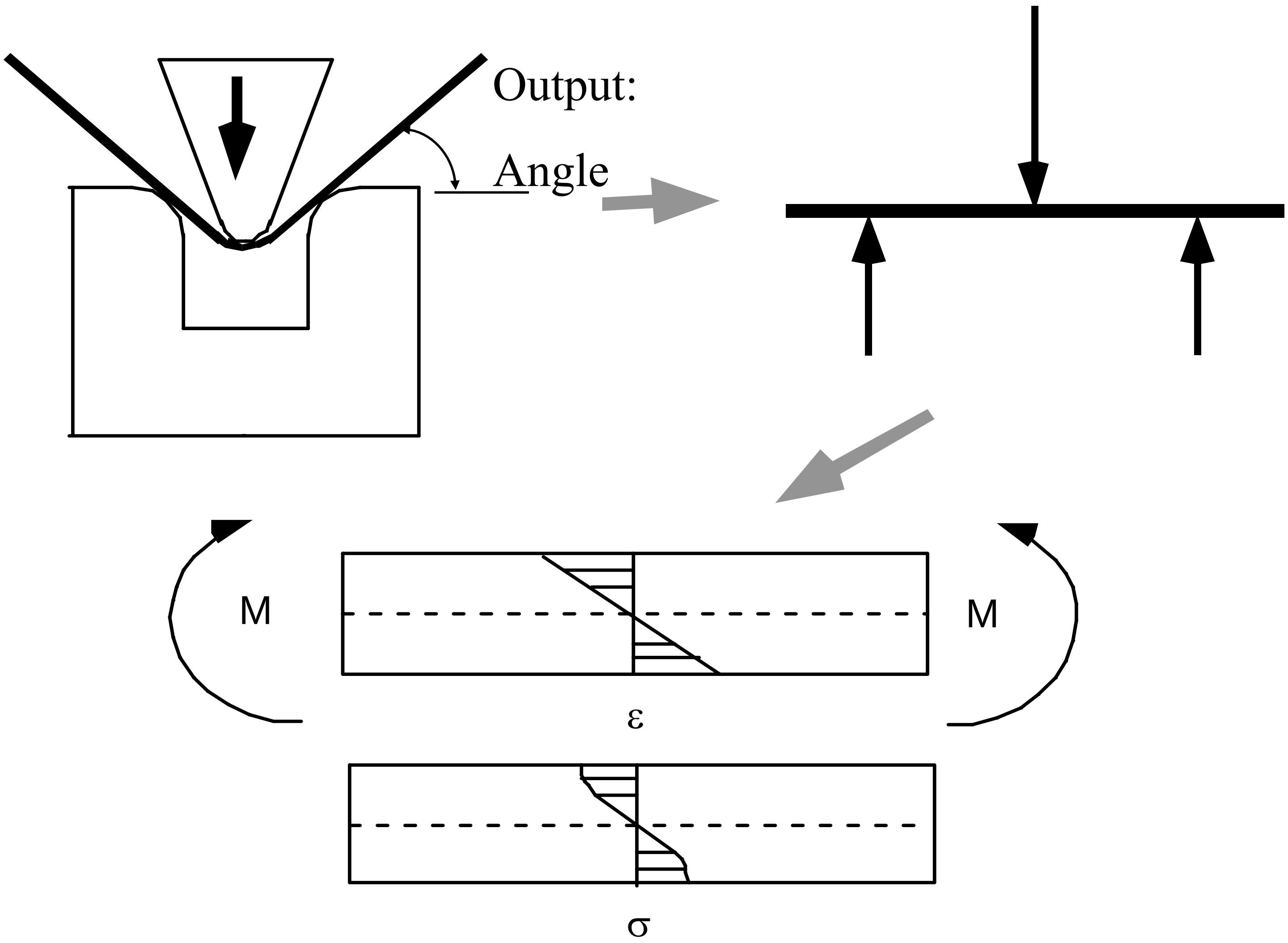
$\Delta\alpha$

CNC Data

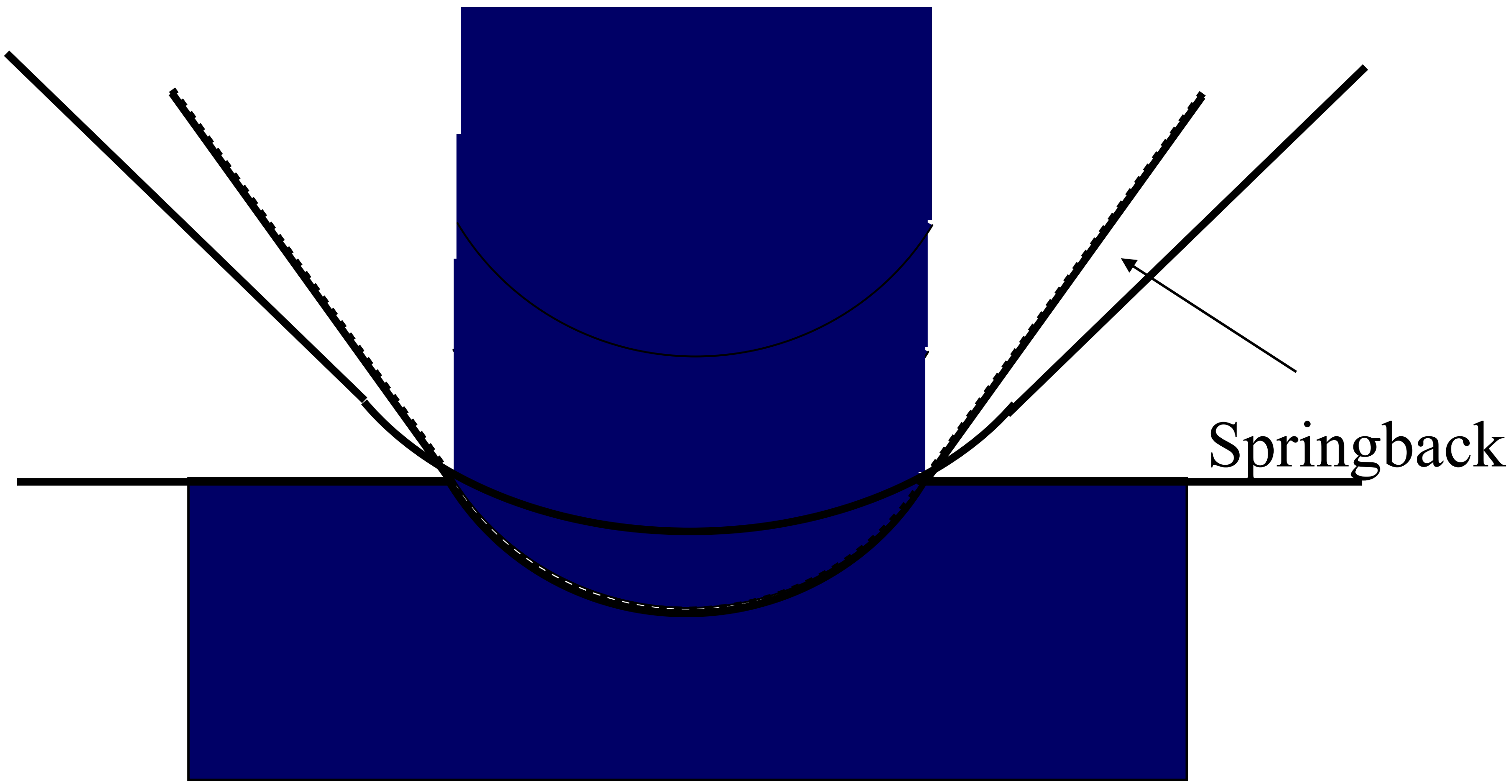


0.6253
0.6251
0.6249
0.6247
0.6245
0.6243
0.6241
0.6239
0.6237
0.6235
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Outer
Middle
Inner
Operating Points:
High Feed=1
Low Feed = 2

Brake Bending of Sheet

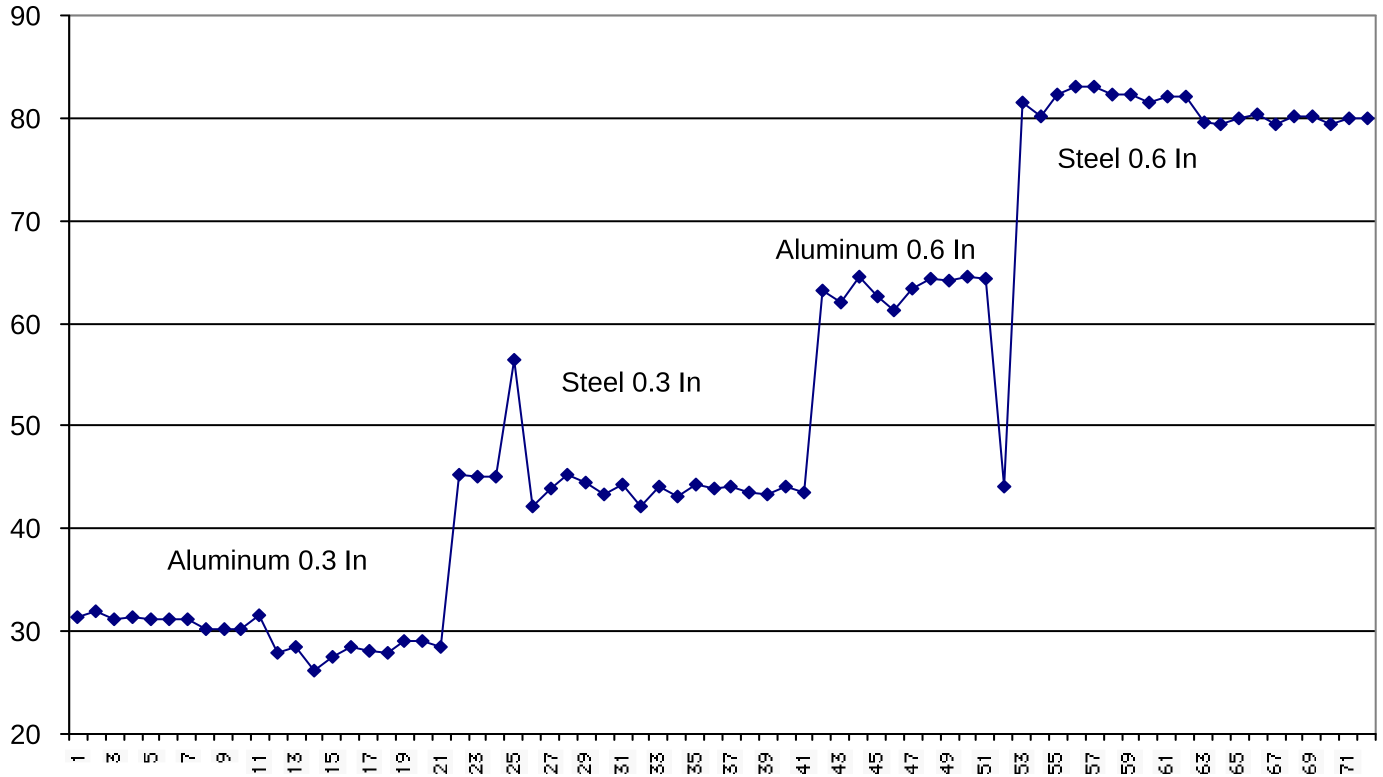


Bending Process



Observations from Bending Process

- Clear Input-Output Effects (Deterministic)
- Also Randomness as well



Angle changes with depth

$\Delta Y \Rightarrow \Delta u$

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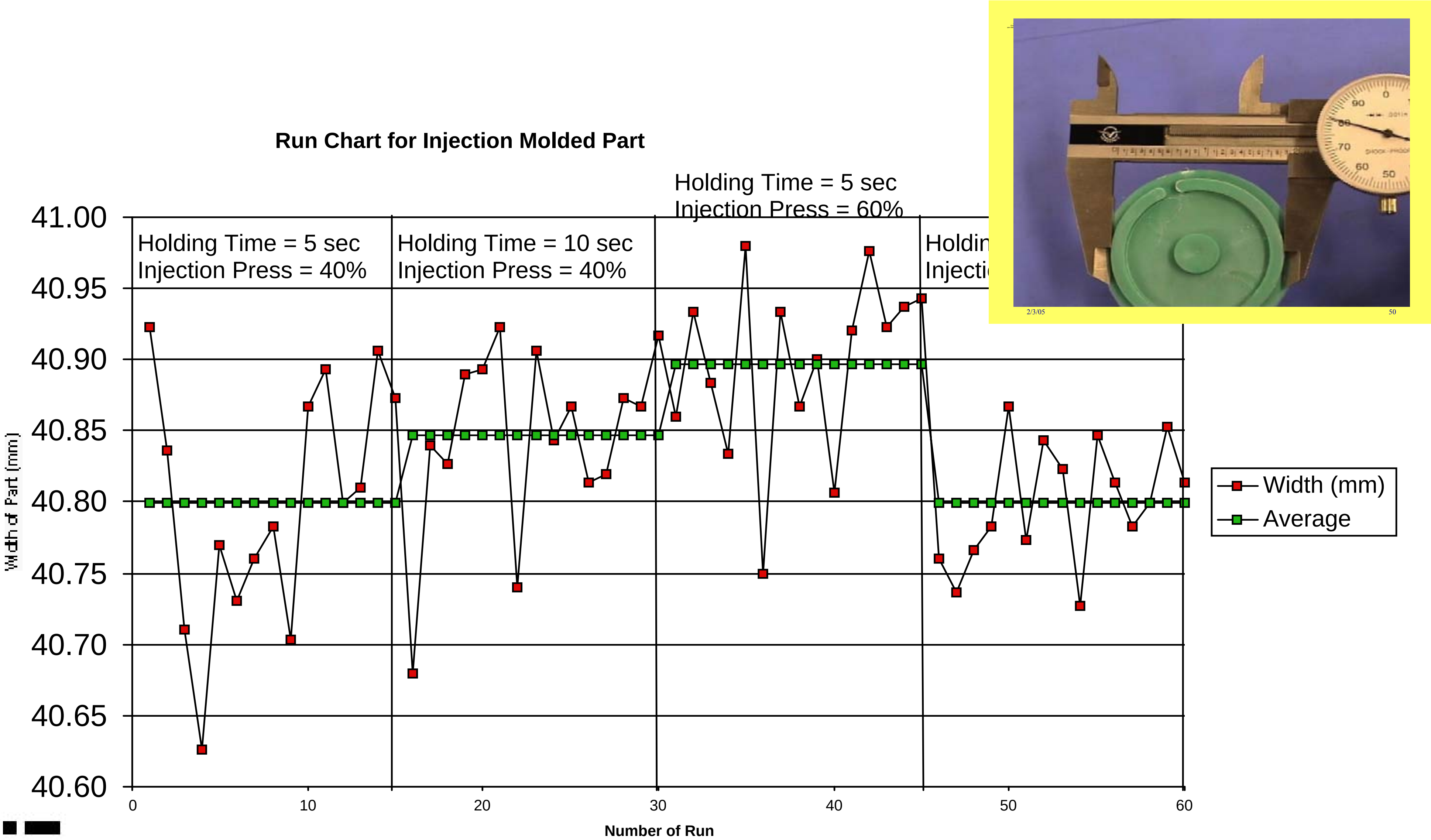
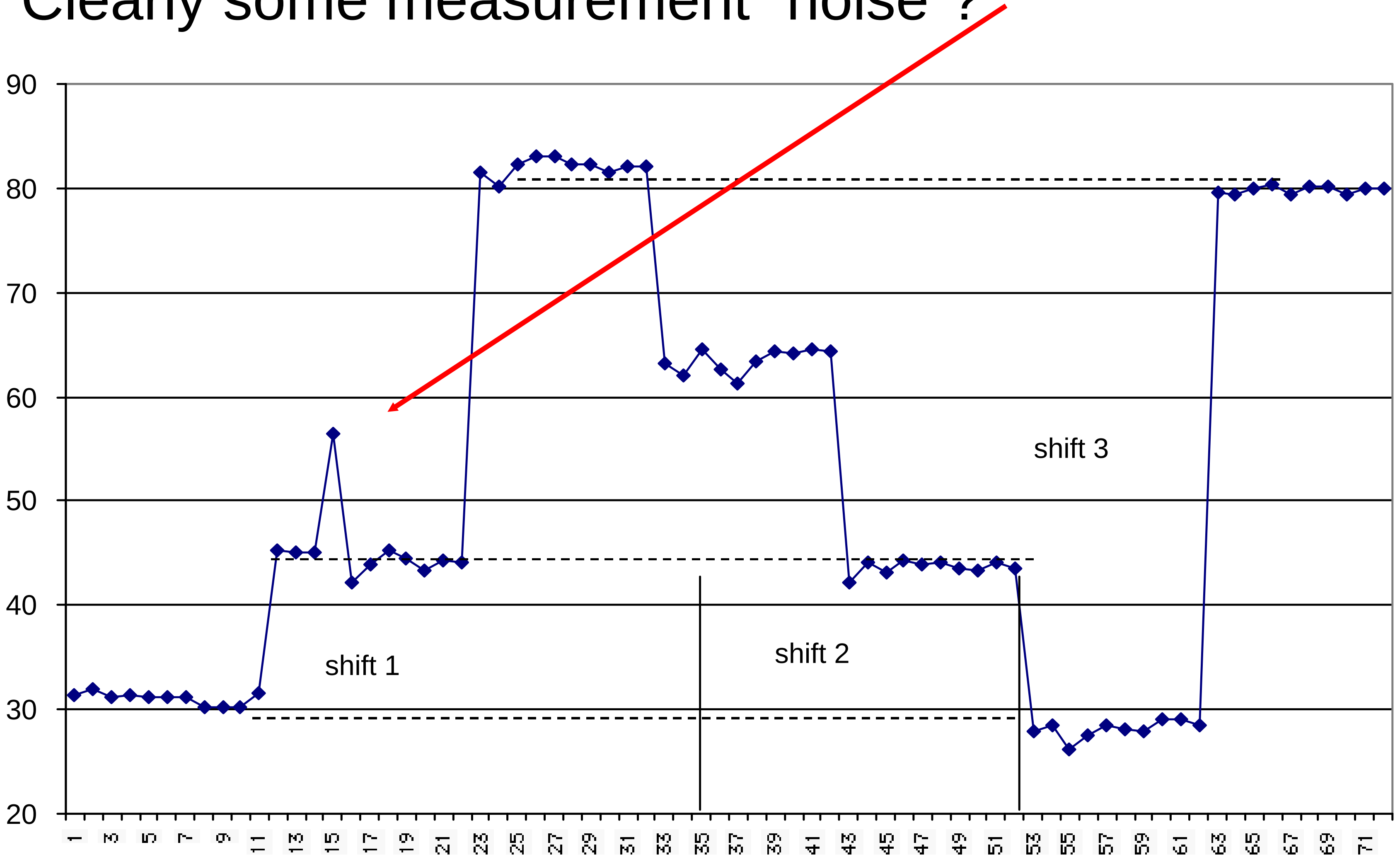
Observations from Injection Molding



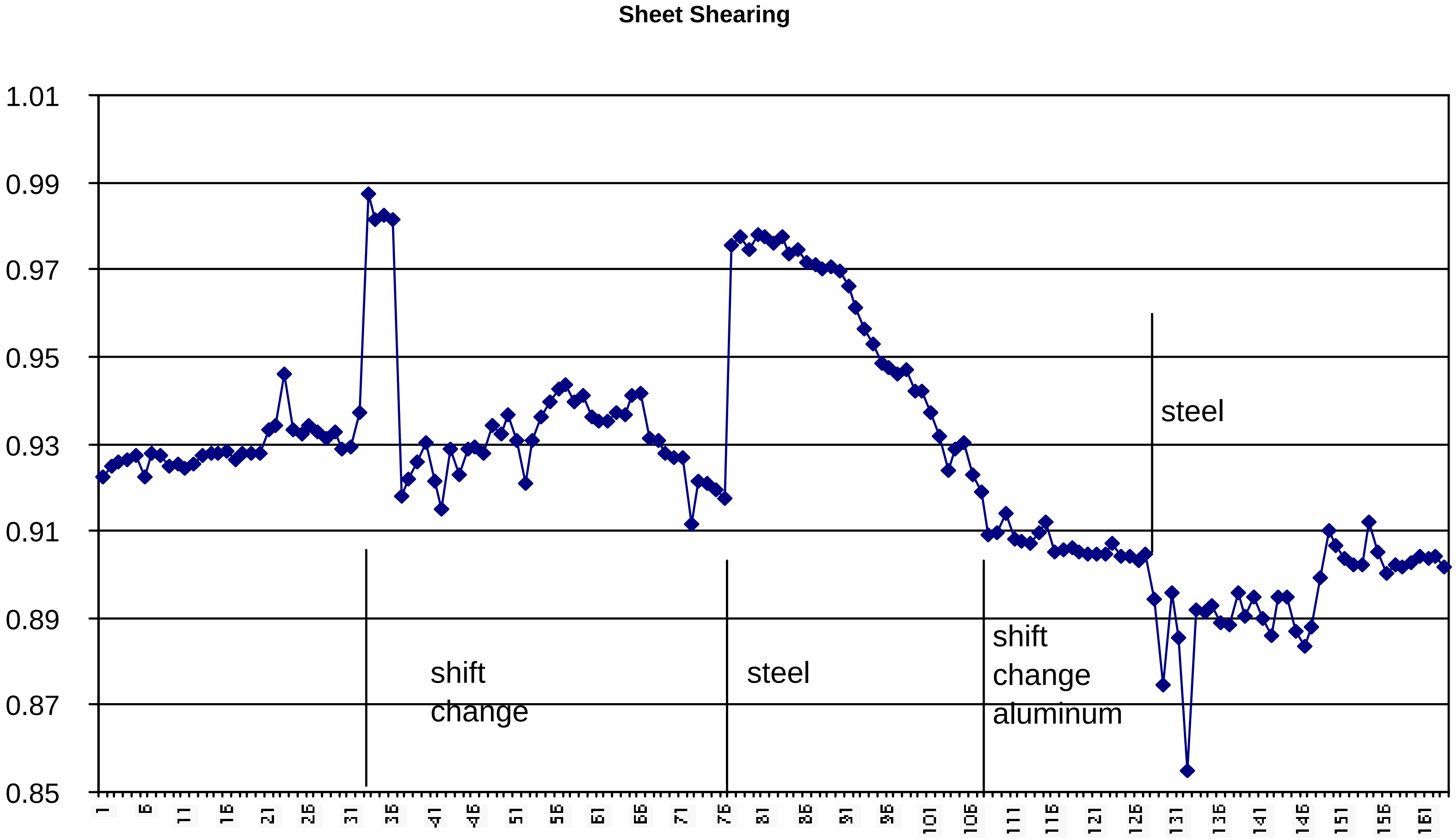
Observations from Data

- Clearly some measurement "noise"?

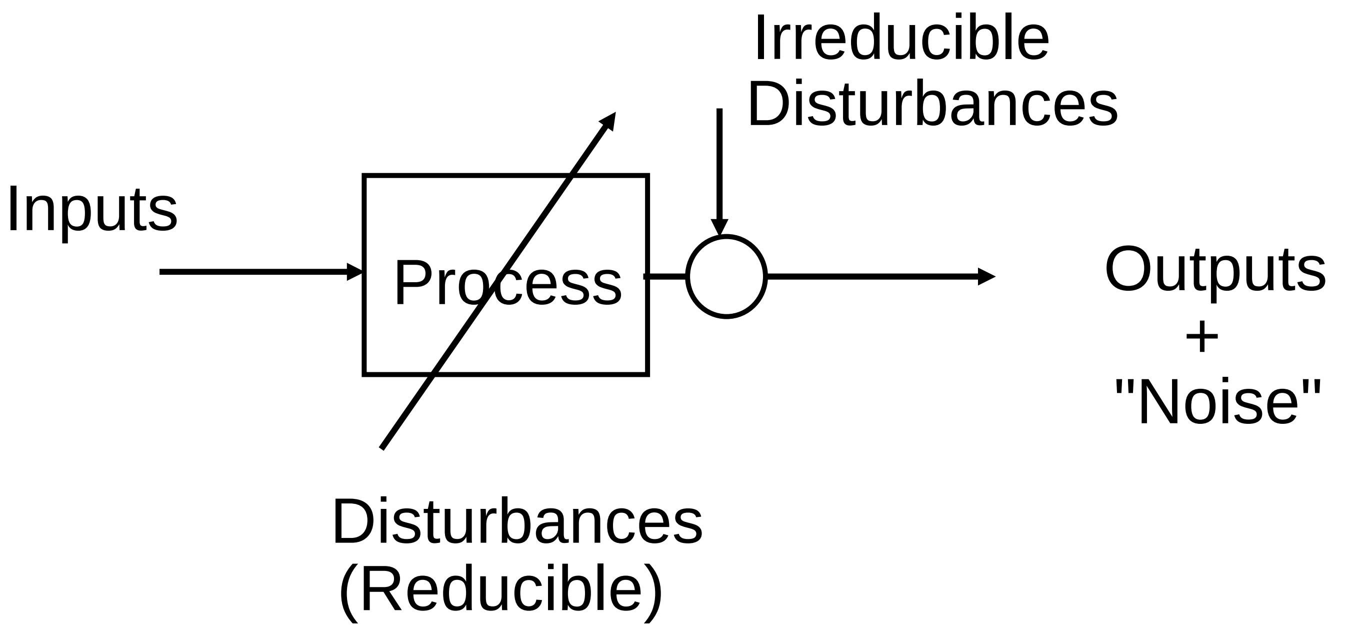


Observations from Data

- Systematic/traceable "operator error"



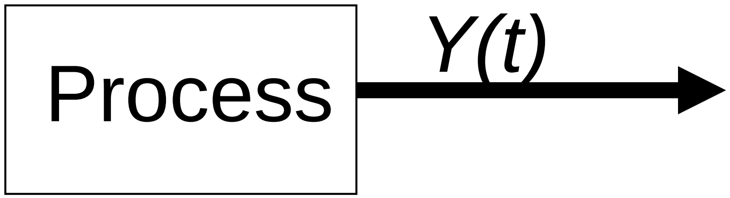
How Model to Distinguish these Effects?



A Random Process + A Deterministic Process

Random Processes

- Consider the Output-only, “Black Box” view of the Run Chart

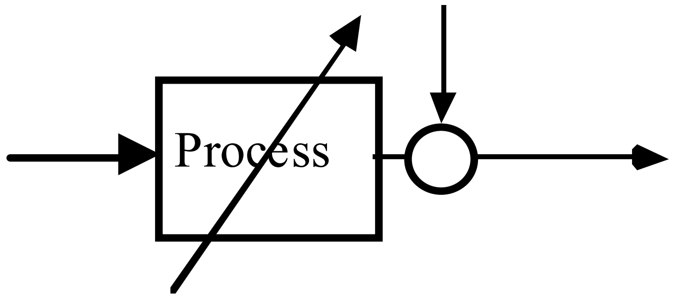


- How do we characterize the process?
  - Using $Y(t)$ only

- WHY do we characterize the process
  - Using $Y(t)$ only?

The Why

- Did output really change?
- Did the input cause the change?
- If not, why did the output vary?
- How confident are we of these answers?



- Can we model the randomness?

Background Needed

- Theory of Random Processes and Random Variables
- Use of Sample Statistics Based on Measurements
  - SPC basis
  - DOE: use of experimental I/O data
  - Feedback control with random disturbances

How to Describe Randomness?

- Look at a Frequency Histogram of the Data
- Estimates likelihood of certain ranges occurring:
  - $\Pr(y_1 < Y < y_2)$
  - "Probability that a random variable Y falls between the limits y_1 and y_2"

Example: Thermoforming Histogram (2000 data)

How to Describe *Continuous* Randomness

- Process outputs *Y* are continuous variables
- The *Probability* of *Y(t)* taking on any specific value for a continuum

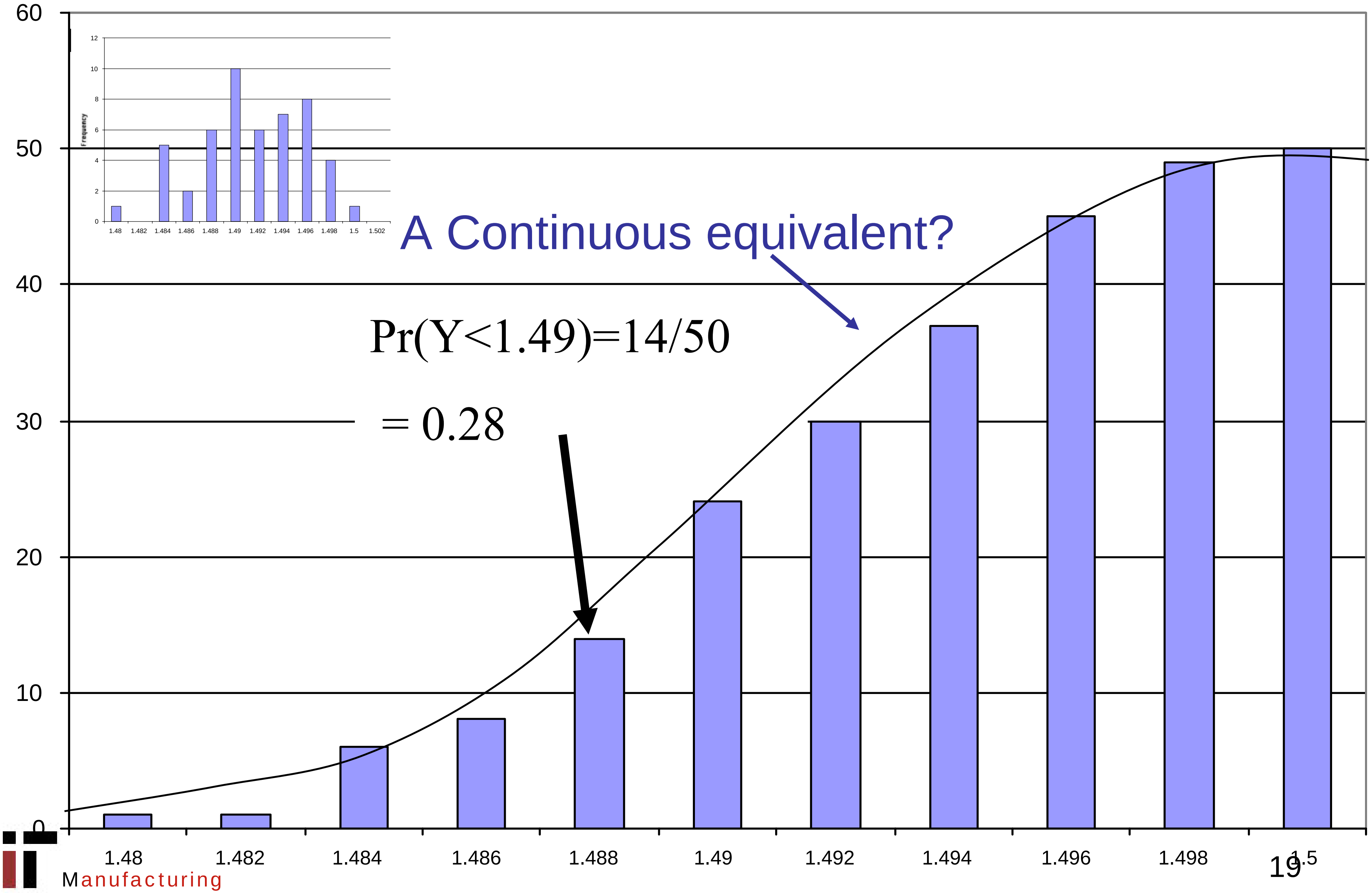
$$\text{Prob}(Y(t) = y^*) = \underline{0}$$

- Must use instead a **Cumulative** *Probability Function*

$$Pr(Y(t) < y^*)$$

  – Look *at* Cumulative Frequency

Cumulative Frequency



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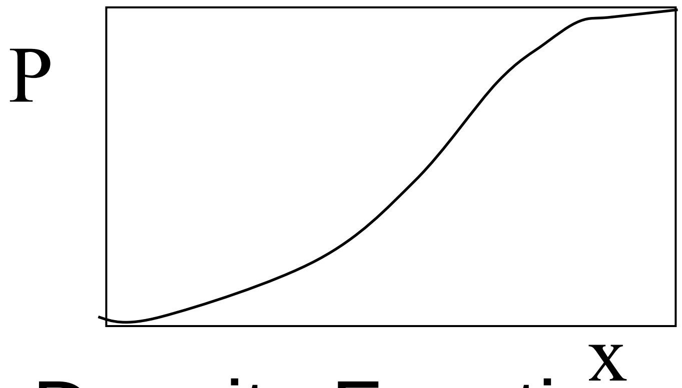
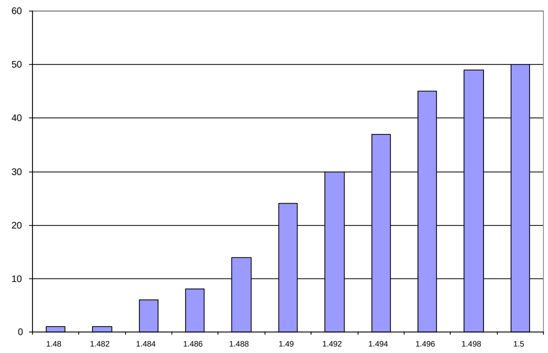
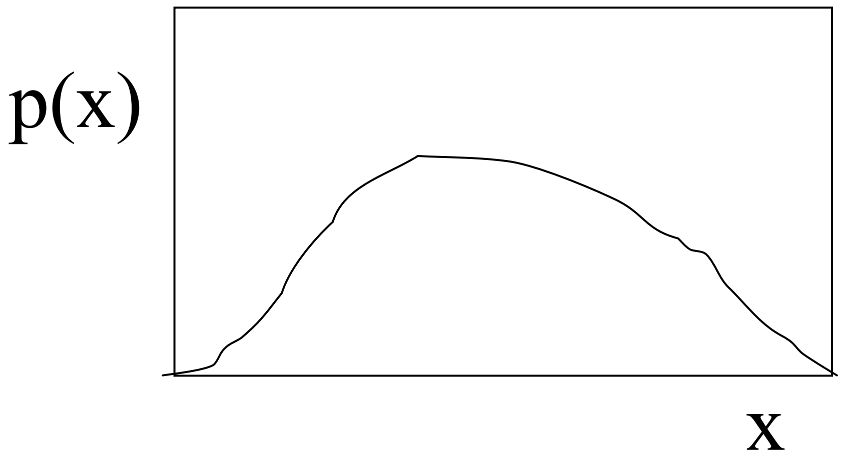
Continuous Equivalents

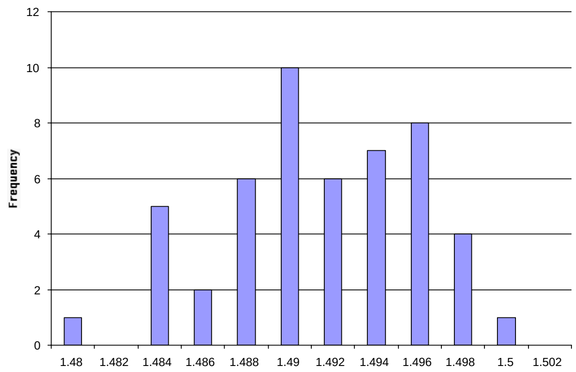
- Probability Function: (P(x))





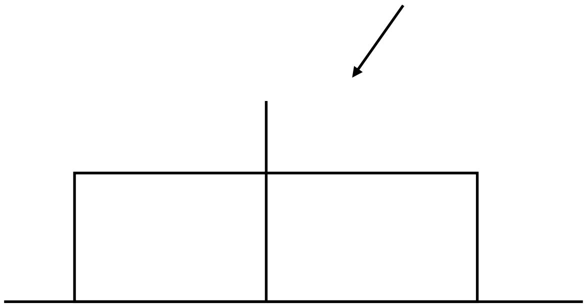
- Probability Density Function *pdf*(*x*) = *dP*/*dx*

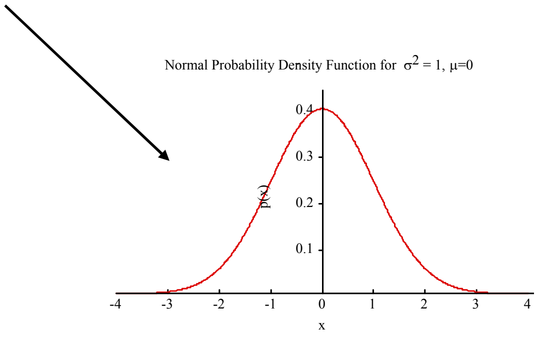




Process Outputs as a Random Variable

- The Histogram suggests a *pdf*
  - Parent or underlying behavior “sampled” by the process
- Standard Forms (There are Many)
  - e.g. The Uniform and Normal pdf’s

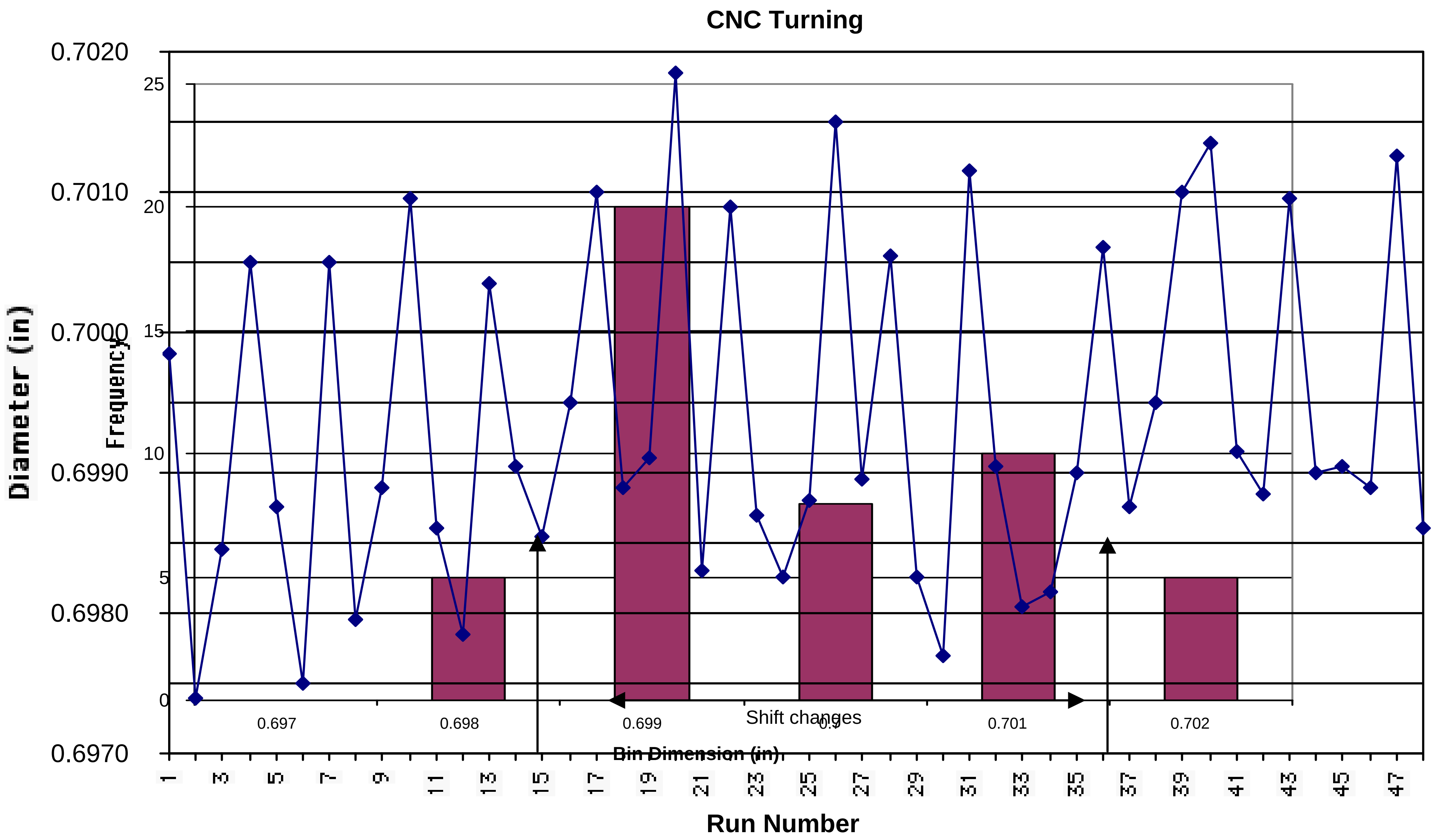




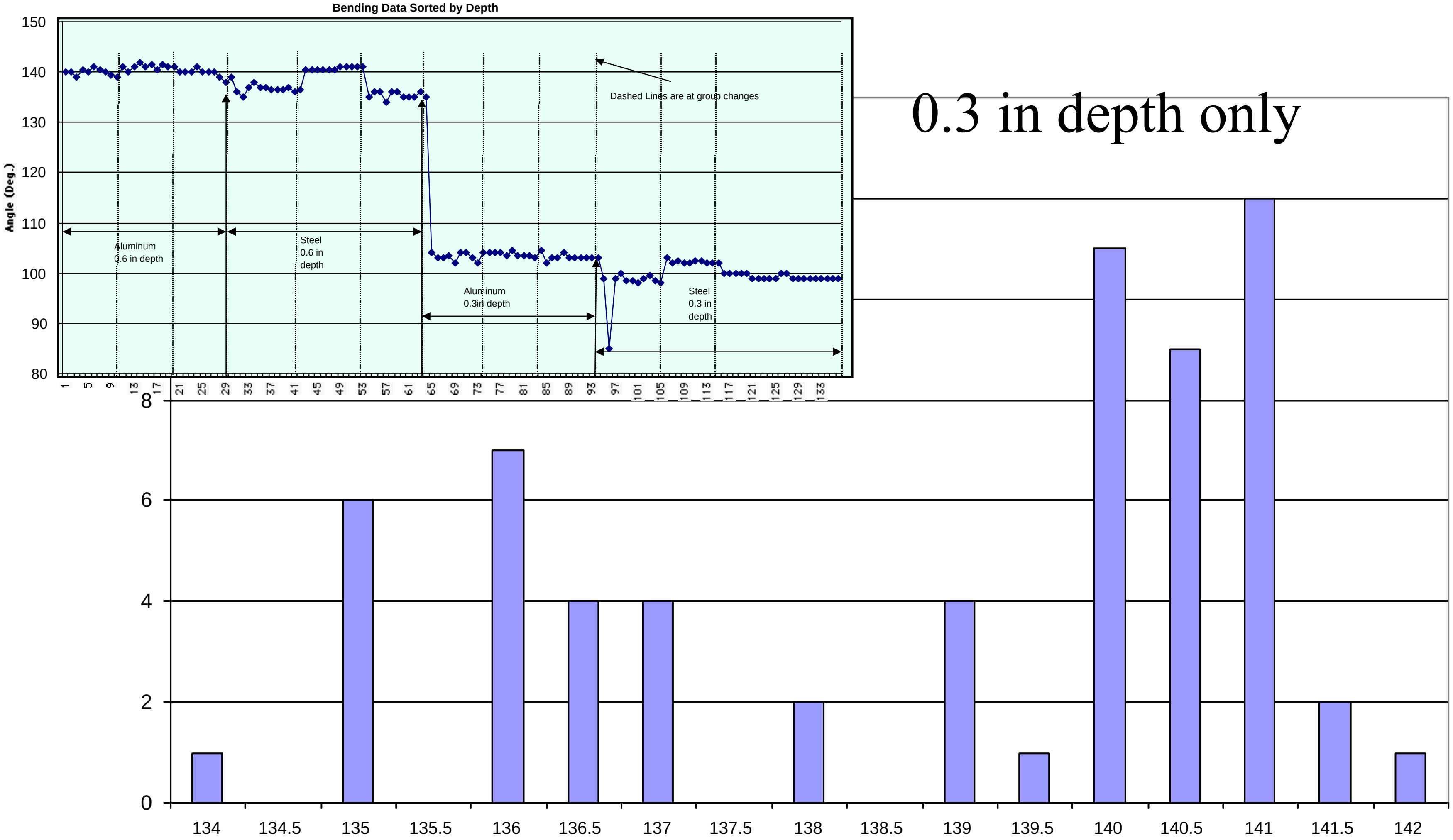
Analysis of Histograms

- Is there a consistent pattern?
- Is an underlying “parent” distribution suggested?

Histogram for CNC Turning



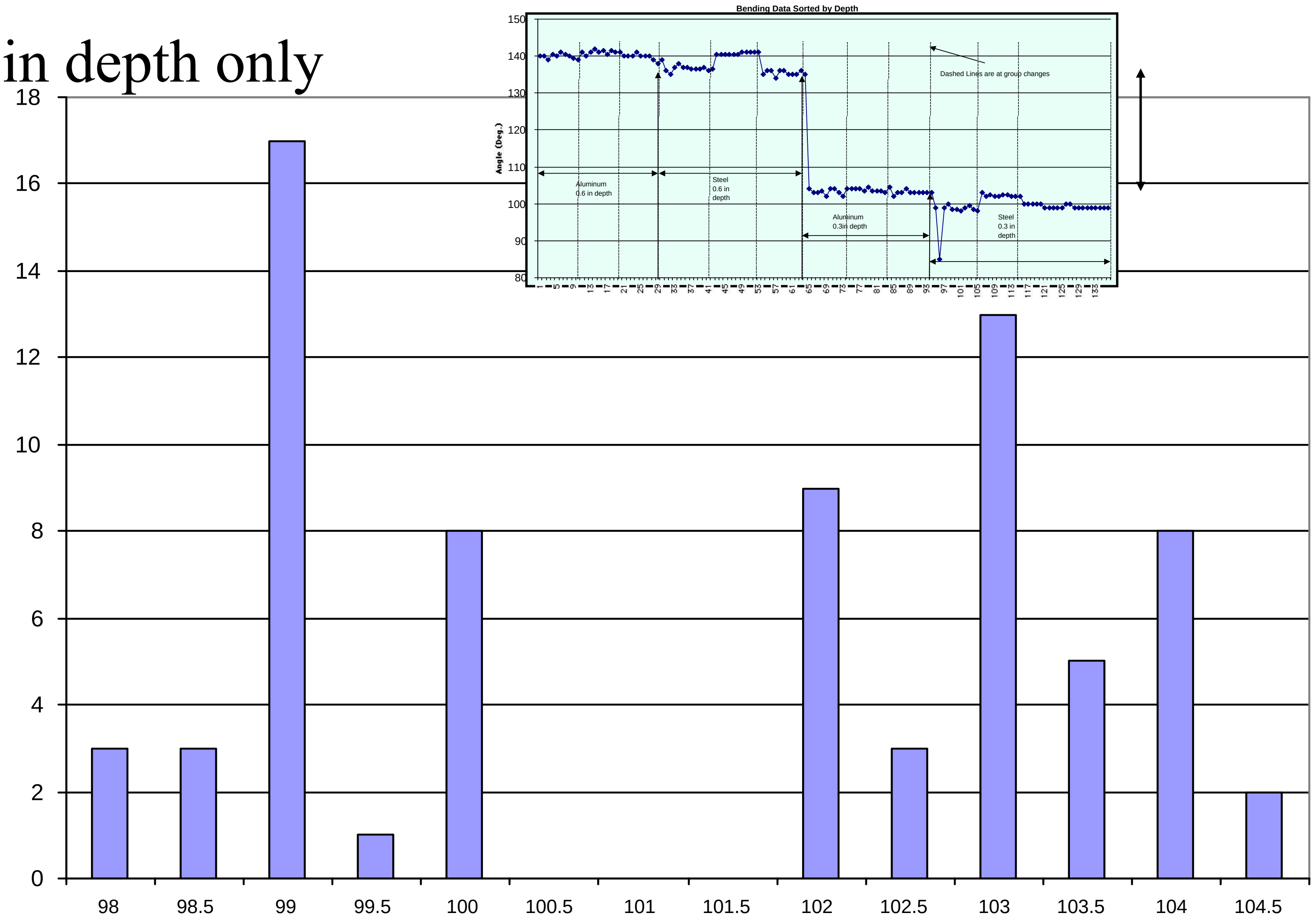
Histogram for Bending
(MIT 2002 data)



Histogram for Bending

(MIT 2002 data)

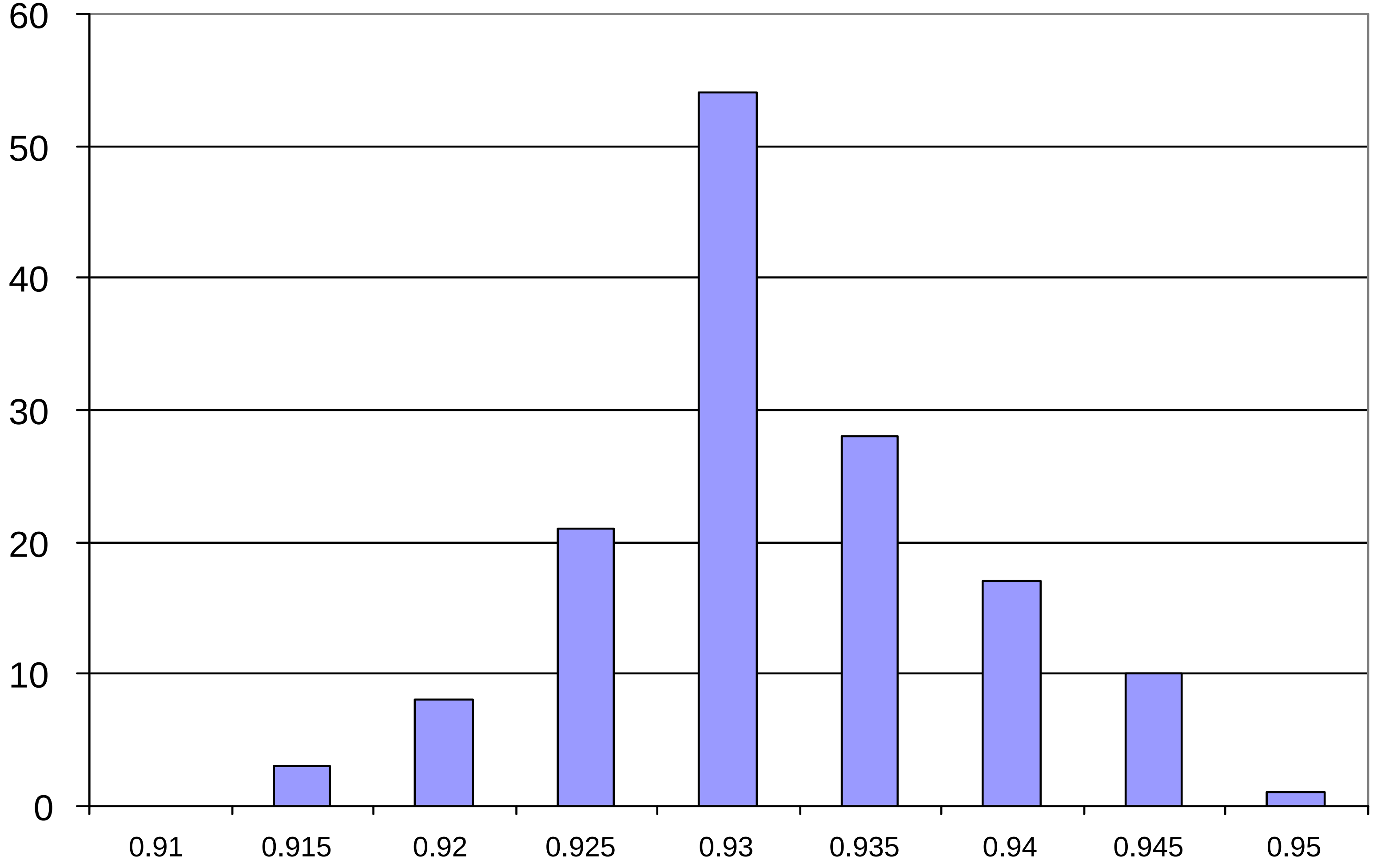
0.6 in depth only



Mean Shift?

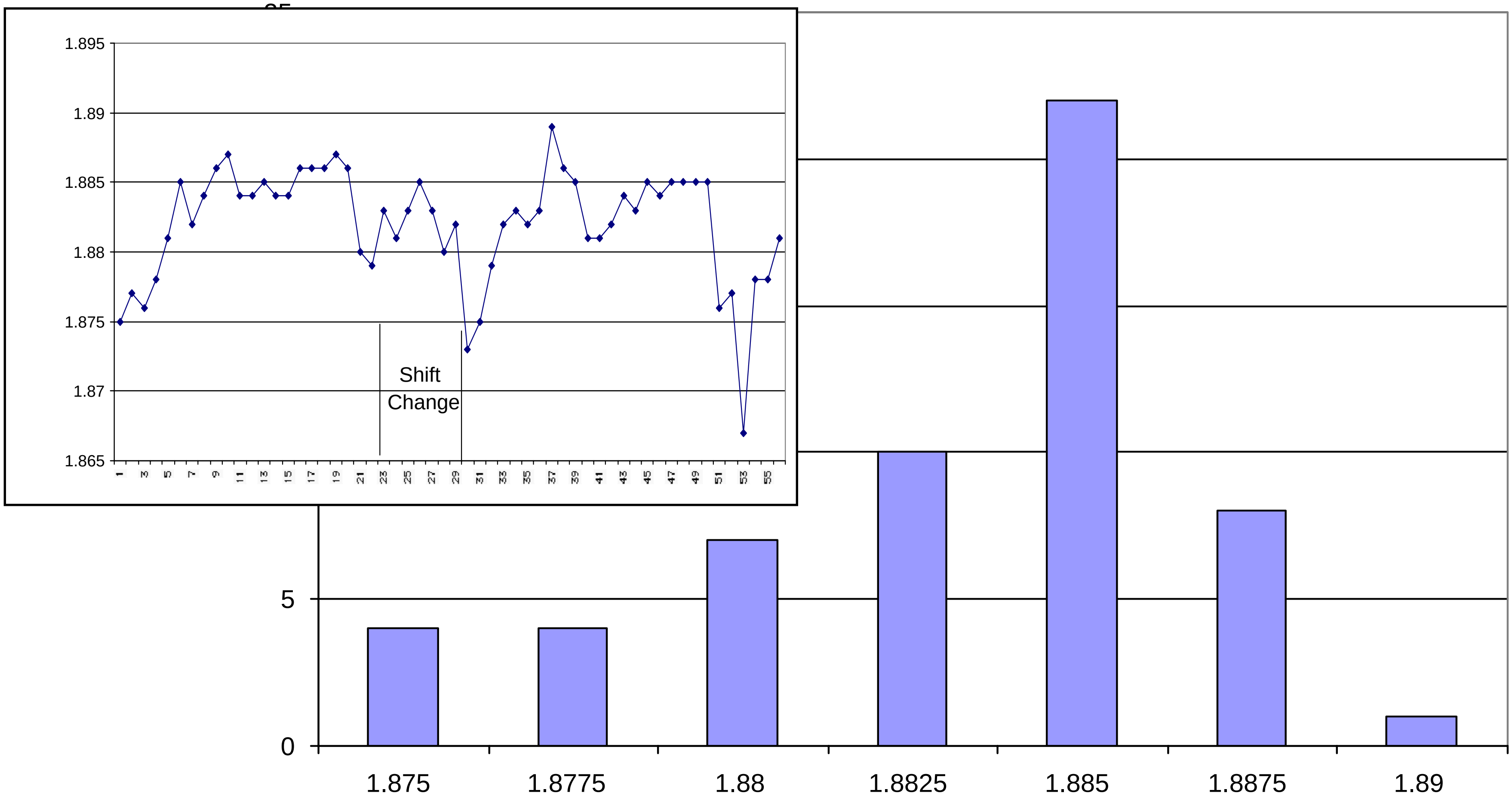
Consider: No Intentional Changes (Δu = 0)

- Shearing during shift 1('02) , aluminum only

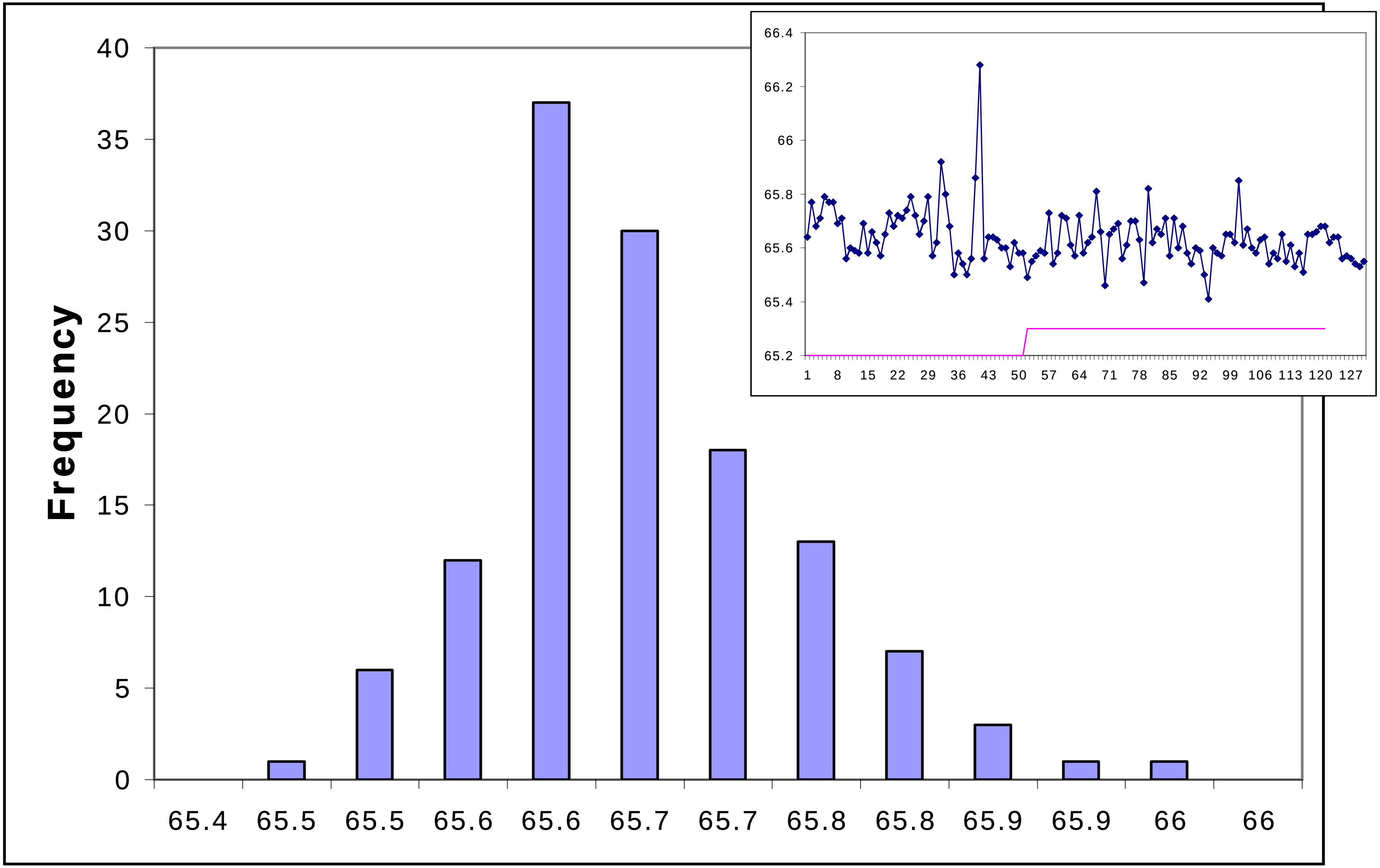


Consider: No Effective Changes ($\partial Y / \partial u = 0$)

- Injection Molding Entire MIT Run (2002)



Injection Molding (S'2003)



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Conclusion?

- When there are no input effect (no Δu *or* $\partial Y / \partial u$) a consistent histogram pattern *can* emerge
- How do we use knowledge of this pattern?
  - Predict behavior
  - Set limits on "normal" behavior
- *Define analytical probability density functions*

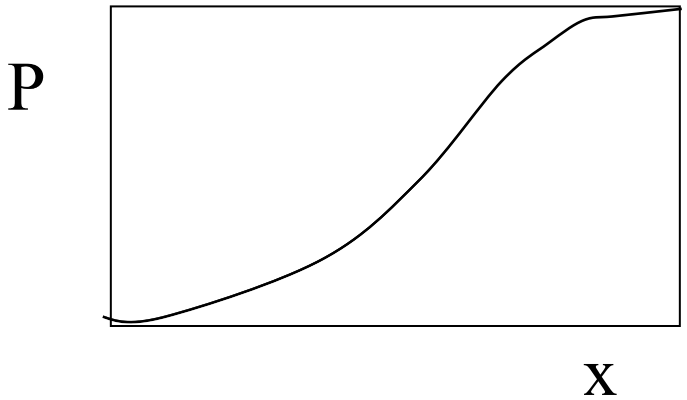
Underlying or “Parent” Probability

- A model of the “true”, continuous behavior of the random process
- Can be thought of as a continuous random variable obeying a set of rules (the *probability function*)
- We can only glimpse into these rules by sampling the random variable (i.e. the process output)
- Underlying process can have
  - Continuous values (e.g. geometry)
  - Discrete values (e.g. defect occurrence)

Continuous Probability Functions

- Recall Probability Function (Cumulative)

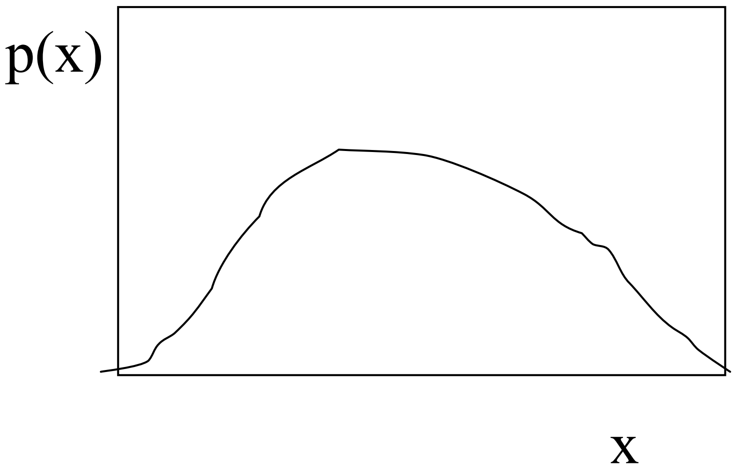
  P(x) = Prob(*Y(t)*<*x*)



- Define *pdf* = *p(x)* = dP/dx

Thus :

$$P(x) = \int_{-\infty}^{x} pdf\,(x)dx$$



Use of the *pdf* : Expectation

$$E\{x(t)\} = \text{ expected value of x(t)}$$

$$E\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, pdf(x,t)\, dx$$

$$\mu(t) = E\{\mathrm{x(t)}\} \quad \text{mean value of } x$$

Note that *pdf* and μ *(or any other expected value)* can be functions of time.

In general, they may be non-stationary.

Stationary Processes

$$pdf(x,t) = pdf(x) = p(x) \quad : stationary\ pdf$$

$$E\{x\} = \int_{-\infty}^{\infty} x \ \ p(x)dx = \mu_x$$

μ_x : theoretical or "true" mean

- For a stationary process μ_x is a constant

Stationary Processes

$$E\{(x-\mu_x)^2\} = \int_{-\infty}^{\infty} (x-\mu_x)^2 p(x)dx = \sigma_x^2$$

$$= \text{"true" variance}$$

- For a <u>stationary</u> process σ_x is a <u>constant</u>

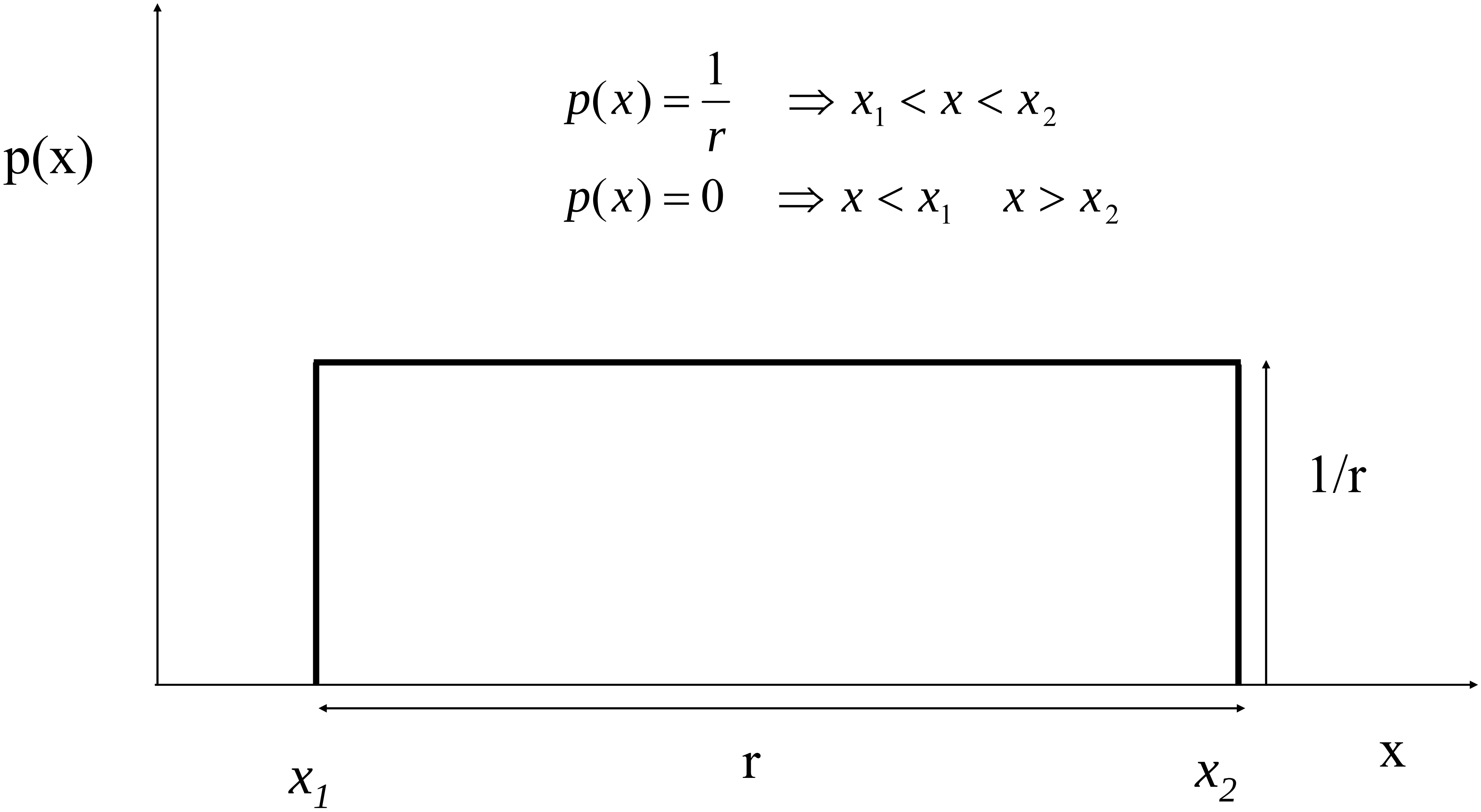
$$\sigma_x^2 = E\{x^2\} - \mu_x^2$$

$$= \text{mean square - square of mean}$$

The Uniform Distribution

$$p(x) = \frac{1}{r} \quad \Rightarrow x_1 < x < x_2$$

$$p(x) = 0 \quad \Rightarrow x < x_1 \quad x > x_2$$



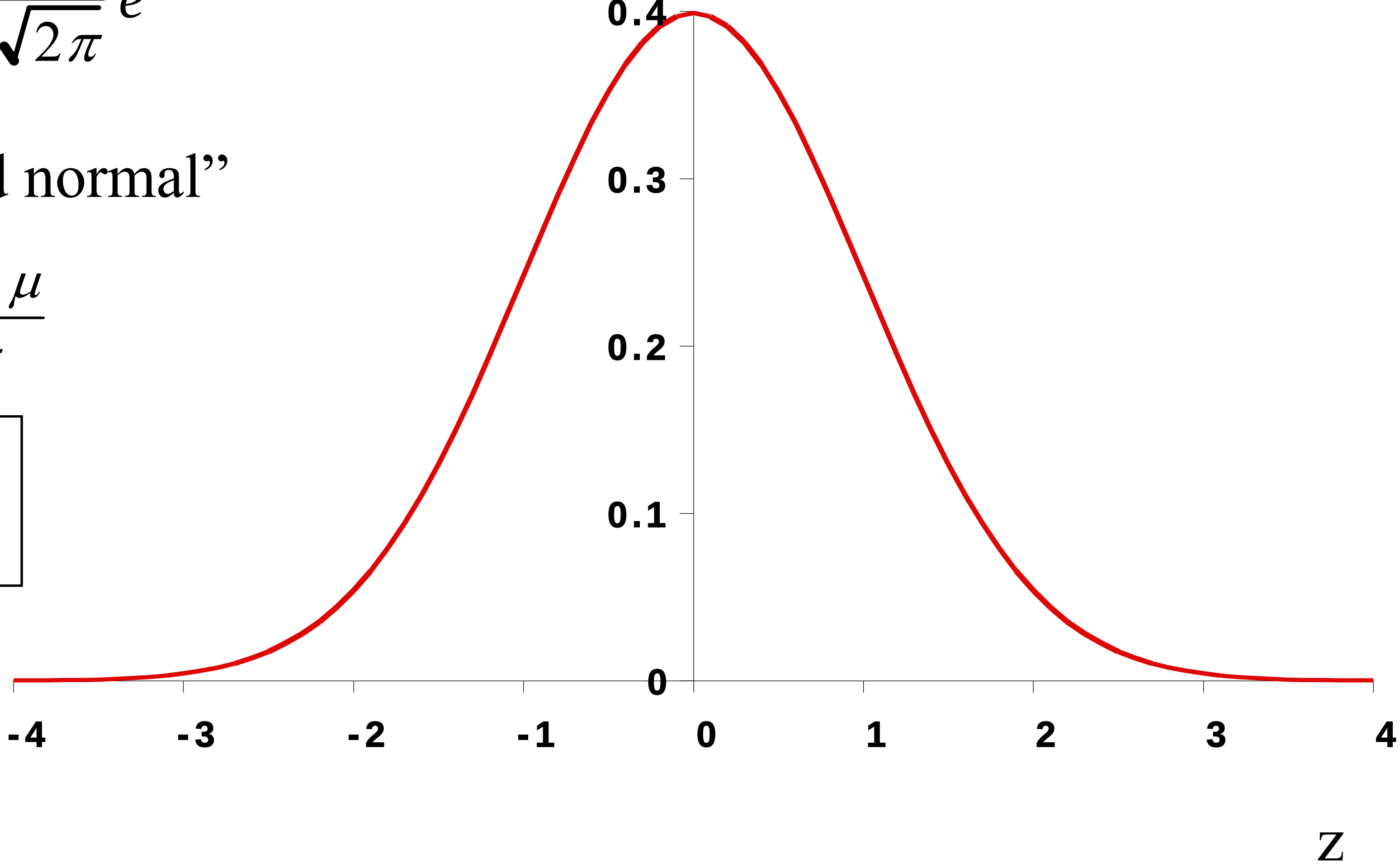
The Normal Distribution

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

"Standard normal"

$$z = \frac{x-\mu}{\sigma}$$

$\mu_z=0$
$\sigma_z=1$



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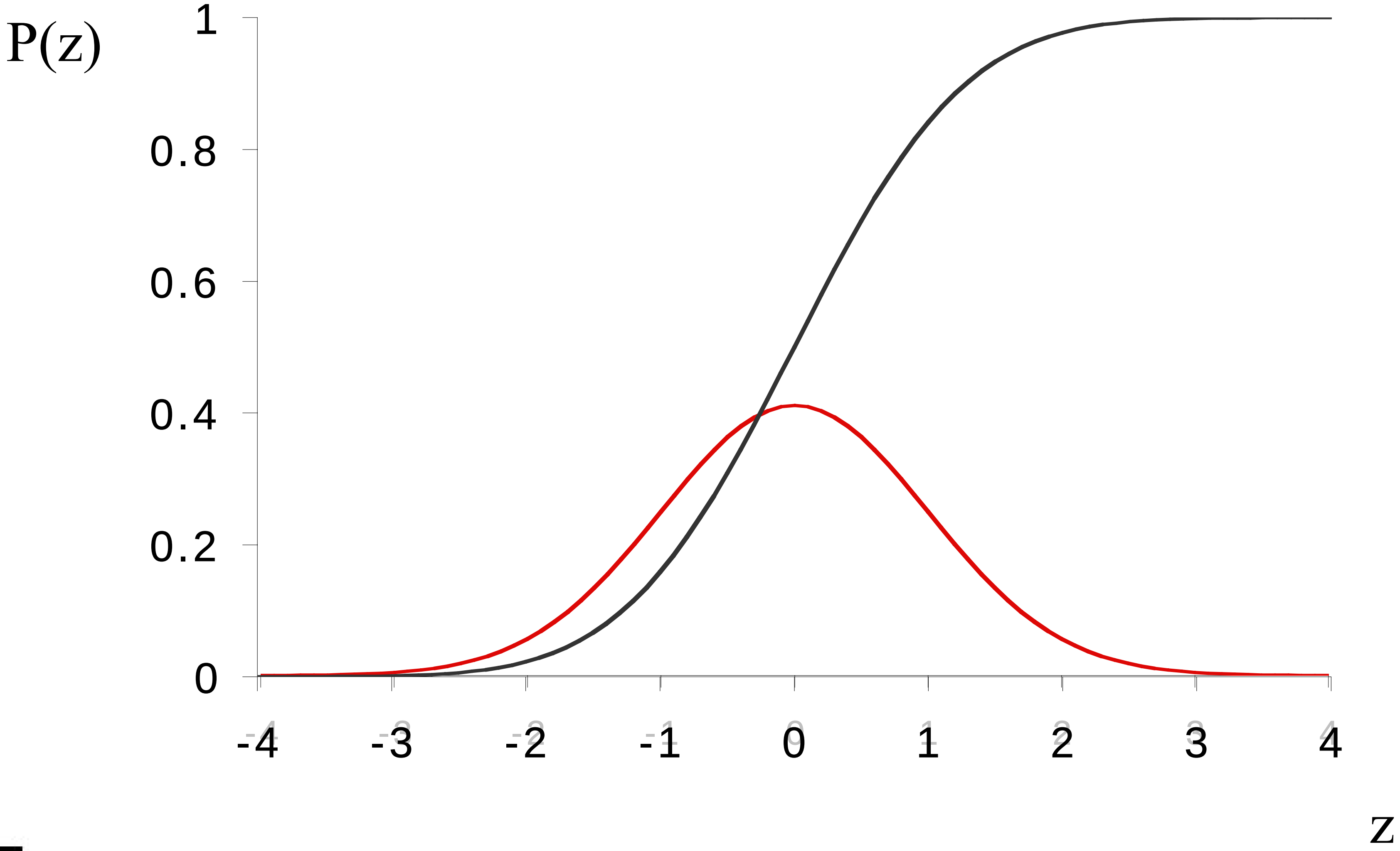
Cumulative Distribution



Properties of the Normal pdf

- Symmetric about mean
- Only two parameters:

  μ and σ

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

- Superposition Applies:
  – sum of normal random variables has a normal distribution

Superposition of Random Variables

If we define a variable

$$y = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + \ldots$$

- c_i are constants
- x_i are independent random variables

$$\mu_y = c_1\mu_1 + c_2\mu_2 + c_3\mu_3 + c_4\mu_4 + \ldots$$

$$\sigma_y^2 = c_1^2\sigma_\iota^2 + c_2^2\sigma_2^2 + c_3^2\sigma_3^2 + c_4^2\sigma_4^2$$

From expectation operation, for any *pdf*.

Use of the PDF: Confidence Intervals

- How likely are certain values of the random variable?
- For a "Standard Normal" Distribution:

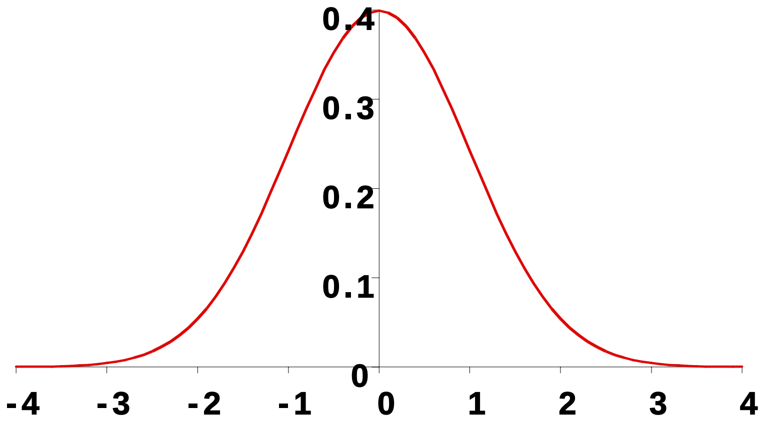
$$z = \frac{(x-\mu)}{\sigma}$$

N(0,1) $\mu = 0$ $\sigma = 1$

$$z = 1 \quad \Rightarrow \quad x = 1\sigma$$

$$z = 2 \quad \Rightarrow \quad x = 2\sigma$$

$$z = 3 \quad \Rightarrow \quad x = 3\sigma$$



Confidence Intervals

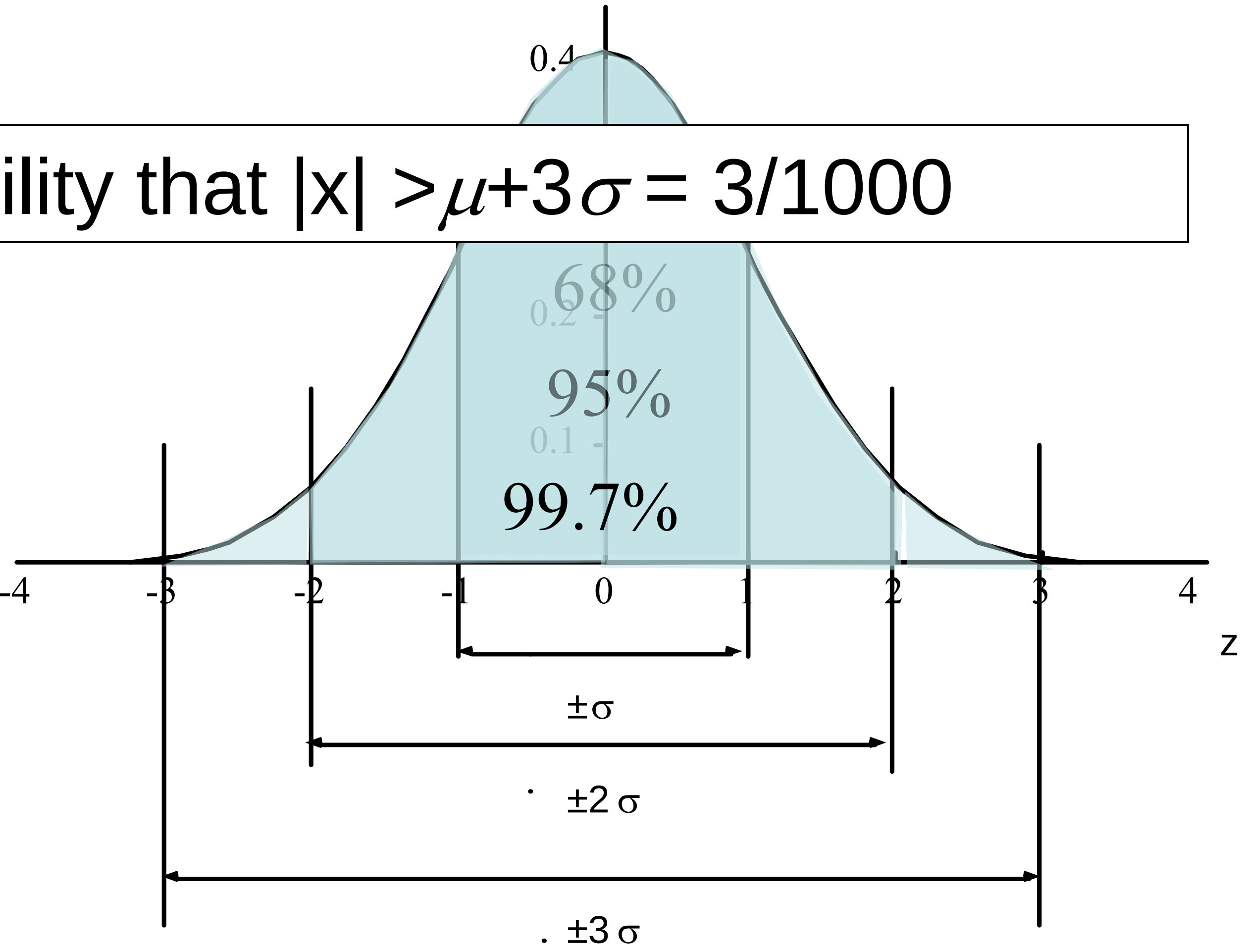
$P(-1 \leq z \leq 1) = P(z \leq 1) - P(z \leq -1) = 0.841 - (1-0.841) = \mathbf{0.682}$
($\pm$ 1σ)

$P(-2 \leq z \leq 2) = P(z \leq 2) - P(z \leq -2) = 0.977 - (1-0.977) = \mathbf{0.954}$
($\pm$ 2σ)

$P(-3 \leq z \leq 3) = P(z \leq 3) - P(z \leq -3) = 0.998 - (1-0.998) = \mathbf{0.997}$
($\pm$ 3σ)

P(z) tabulated (e.g. p. 752 of Montgomery)

Confidence Intervals



- Probability that |x| > $\mu + 3\sigma$ = 3/1000

Is the Process “Normal” ?

- Is the underlying distribution really normal?
  – Look at histogram
  – Look at curve fit to histogram
  – Look at % of data in 1, 2 and 3 σ bands
    - Confidence Intervals
  – Probability (or qq) plots (see Mont. 3-3.7)
  – Look at “kurtosis”
    - Measure of deviation from normal

Kurtosis: Deviation from Normal

$$k = \frac{E(x-\mu_x)^4}{\sigma^4}$$

k=1 - normal

k>1 more "peaked"

k<1 more "flat"

Or for sampled data:

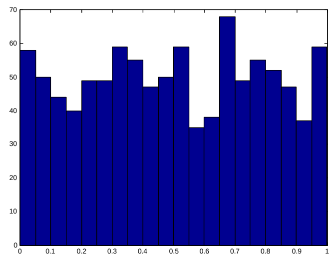
$$k = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum \left(\frac{x_i - \bar{x}}{s} \right)^4 \right] - \frac{3(n-1)^2}{(n-2)(n-3)}$$

The Central Limit Theorem

- If $x_1, x_2, x_3 \ldots x_N \ldots$ are N <u>independent</u> observations of a random variable with "moments" μ_x and σ^2_x,

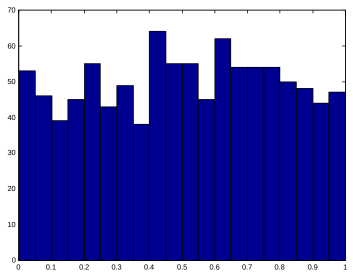
- The distribution of the ***sum*** of all the samples will tend toward normal.

Example: Uniformly Distributed Data



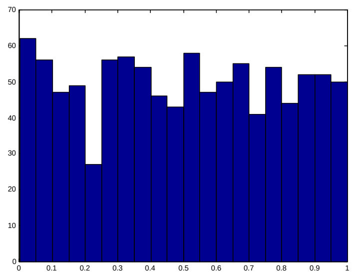
x_1

+



x_2

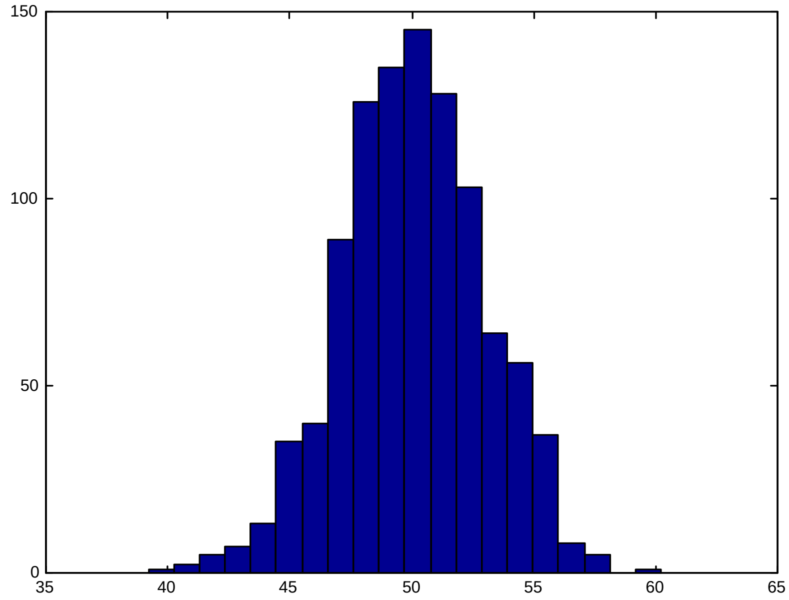
+ . . .



x_{100}

Sum of 100 sets of 1000 points each

$$y = \sum_{i=1}^{100} x_i$$



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Sampling: Using Measurements (Data) to Model the Random Process

- In general *p(x)* is unknown
- Data can suggest form of *p(x)*
  - e.g.. uniform, normal, weibull, etc.
- Data can be used to estimate parameters of distributions
  - e.g. μ and σ for normal distribution - $p(x) = p(x, \mu, \sigma)$

- How to Estimate
  - Sample Statistics
- Uncertainty in Estimates
  - Sample Statistic pdf's

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Sample Statistics

$$x(j) = \text{samples of } x(t) \text{ taken } n \text{ times}$$

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x(j) : \quad \text{Average or Sample Mean}$$

$$S^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x(j) - \bar{x})^2 : \quad \text{Sample Variance}$$

$$S = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x(j) - \bar{x})^2} : \quad \text{Sample Std.Dev.}$$

Sample Mean Uncertainty

- If all x_i come from a distribution with μ_x and σ^2_x, *and we divide the sum by n:*

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = c_1x_1 + c_2x_2 + c_3x_3 + \ldots c_nx_n$$

$$c_i = \frac{1}{n}$$

Then: $\mu_{\bar{x}} = \mu_x$ and $\sigma^2_{\bar{x}} = \frac{1}{n}\sigma^2_x$ or $\sigma_{\bar{x}} = \frac{1}{\sqrt{n}}\sigma_x$

Conclusions

- All Physical Processes Have a Degree of Natural Randomness
- We can Model this Behavior using Probability Distribution Functions
- We can Calibrate and Evaluate the Quality of this Model from Measurement Data