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# Experimentation and Robust Design of Engineering Systems

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and
Fig. 2 in Frey, Daniel D., and Wang, Hungjen. "Adaptive One-Factor-at-a-Time Experimentation and Expected Value of Improvement." *Technometrics* 48 (August 2006): 418-431.

**Dan Frey**

**Associate Professor of Mechanical Engineering and Engineering Systems**

# Research Overview

Concept Design

Outreach to K-12

Adaptive Experimentation and Robust Design

$$\Pr\left(\beta_{12} x_1^* x_2^* > 0 \,\middle|\, \beta_{12} > \beta_{ij}\right) > \frac{1}{\pi}\binom{n}{2}\int_0^{\infty}\int_{-x_2}^{\infty} \frac{\left[erf\left(\frac{1}{\sqrt{2}}\frac{x_1}{\sigma_{INT}}\right)\right]^{\binom{n}{2}-1} e^{\frac{-x_1^2}{2\sigma_{INT}^2}+\frac{-x_2^2}{2\left(\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_{\varepsilon}^2\right)}}}{\sigma_{INT}\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_{\varepsilon}^2}}\,dx_2\,dx_1$$

Complex Systems

Methodology Validation

# Outline

- Introduction
  - History
  - Motivation
- Recent research
  - Adaptive experimentation
  - Robust design

"An experiment is simply a question put to nature … The chief requirement is simplicity: only one question should be asked at a time."

Russell, E. J., 1926, "Field experiments: How they are made and what they are," *Journal of the Ministry of Agriculture* **32**:989-1001.

Text removed due to copyright restrictions. Please see Table III in Fisher, R. A. "Studies in Crop Variation. I. An Examination of the Yield of Dressed Grain from Broadbalk." *Journal of Agricultural Science* 11 (1921): 107-135.

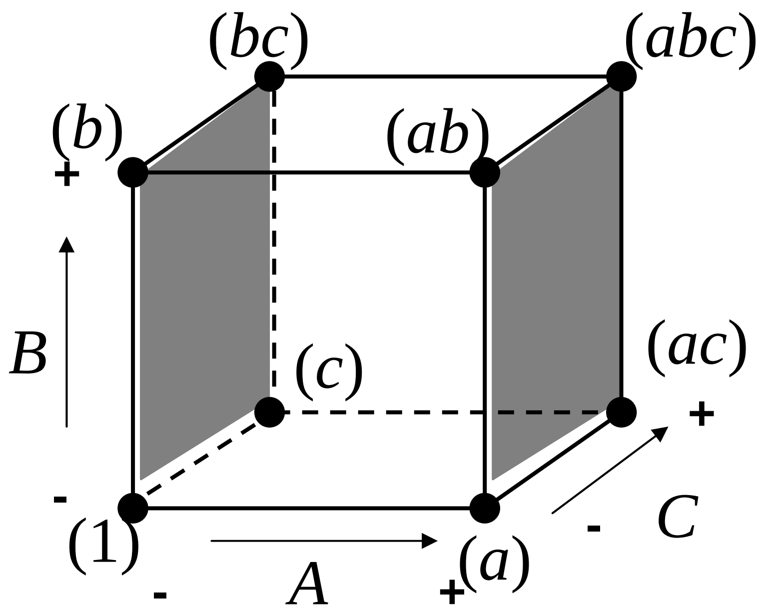
“To call in the statistician after the experiment is done may be no more than asking him to perform a post-mortem examination: he may be able to say what the experiment died of.”

- Fisher, R. A., Indian Statistical Congress, Sankhya, 1938.

Image removed due to copyright restrictions. Please see Fig. 1 in Fisher, R. A. “The Arrangement of Field Experiments.” *Journal of the Ministry of Agriculture of Great Britain* 33 (1926): 503-513.

# Estimation of Factor Effects

Say the independent experimental error of observations ($a$), ($ab$), et cetera is $\sigma_{\varepsilon}$.




We define the main effect estimate $A$ to be

$$A \equiv \frac{1}{4}\left[(abc)+(ab)+(ac)+(a)-(b)-(c)-(bc)-(1)\right]$$

The standard deviation of the estimate is

$$\sigma_A = \frac{1}{4}\sqrt{8}\sigma_{\varepsilon} = \frac{1}{2}\sqrt{2}\sigma_{\varepsilon}$$

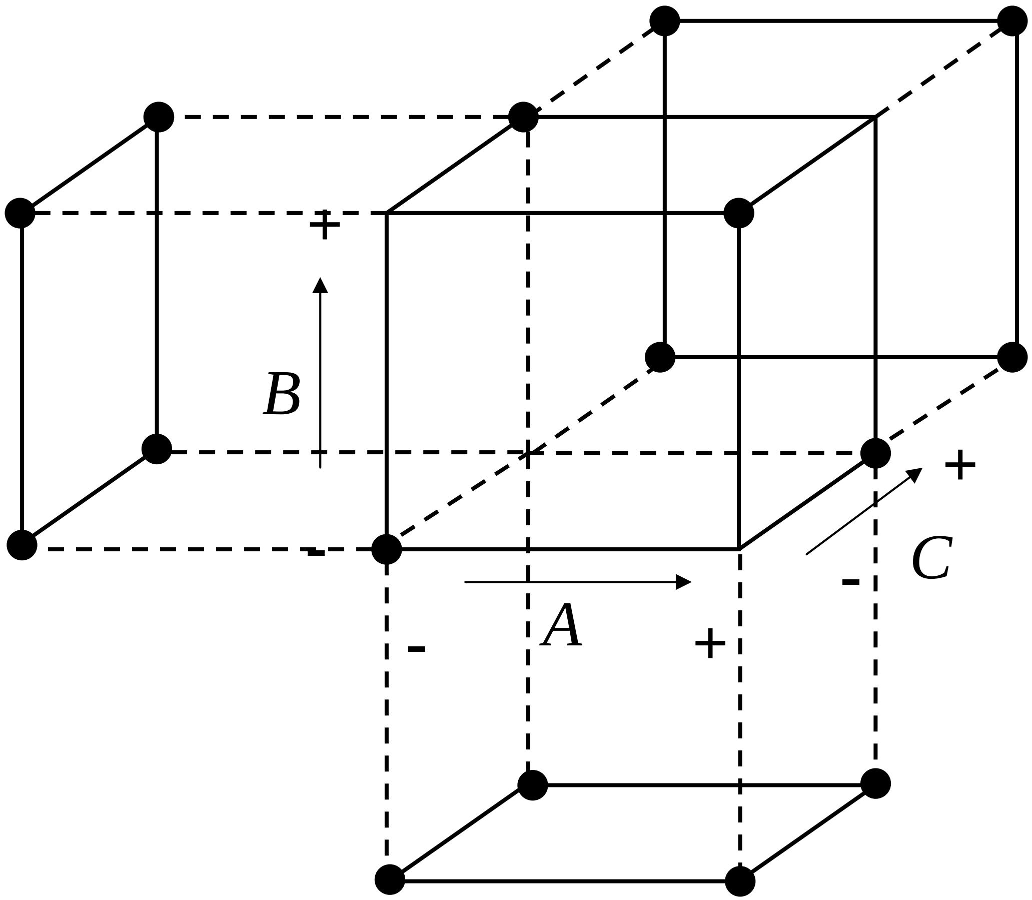
A factor of two improvement in efficiency as compared to "single question methods"

# Fractional Factorial Experiments

"It will sometimes be advantageous deliberately to sacrifice all possibility of obtaining information on some points, these being confidently believed to be unimportant … These comparisons to be sacrificed will be deliberately confounded with certain elements of the soil heterogeneity… Some additional care should, however, be taken…"

Fisher, R. A. "The Arrangement of Field Experiments." *Journal of the Ministry of Agriculture of Great Britain* 33 (1926): 503-513.

# Fractional Factorial Experiments




$$2^{3-1}_{III}$$

# Fractional Factorial Experiments

| Trial | A | B | C | D | E | F | G | FG=-A |
|---|---|---|---|---|---|---|---|---|
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | +1 |
| 2 | -1 | -1 | -1 | +1 | +1 | +1 | +1 | +1 |
| 3 | -1 | +1 | +1 | -1 | -1 | +1 | +1 | +1 |
| 4 | -1 | +1 | +1 | +1 | +1 | -1 | -1 | +1 |
| 5 | +1 | -1 | +1 | -1 | +1 | -1 | +1 | -1 |
| 6 | +1 | -1 | +1 | +1 | -1 | +1 | -1 | -1 |
| 7 | +1 | +1 | -1 | -1 | +1 | +1 | -1 | -1 |
| 8 | +1 | +1 | -1 | +1 | -1 | -1 | +1 | -1 |

$2^{7-4}$ Design (aka “orthogonal array”)

Every factor is at each level an equal number of times (balance).

High replication numbers provide precision in effect estimation.

Resolution III.

# Robust Parameter Design

Robust Parameter Design … is a statistical / engineering methodology that aims at reducing the performance variation of a system (i.e. a product or process) by choosing the setting of its control factors to make it less sensitive to noise variation.

Wu, C. F. J. and M. Hamada, 2000, *Experiments: Planning, Analysis, and Parameter Design Optimization*, John Wiley & Sons, NY.

# Cross (or Product) Arrays

**Noise Factors** $2_{III}^{3-1}$

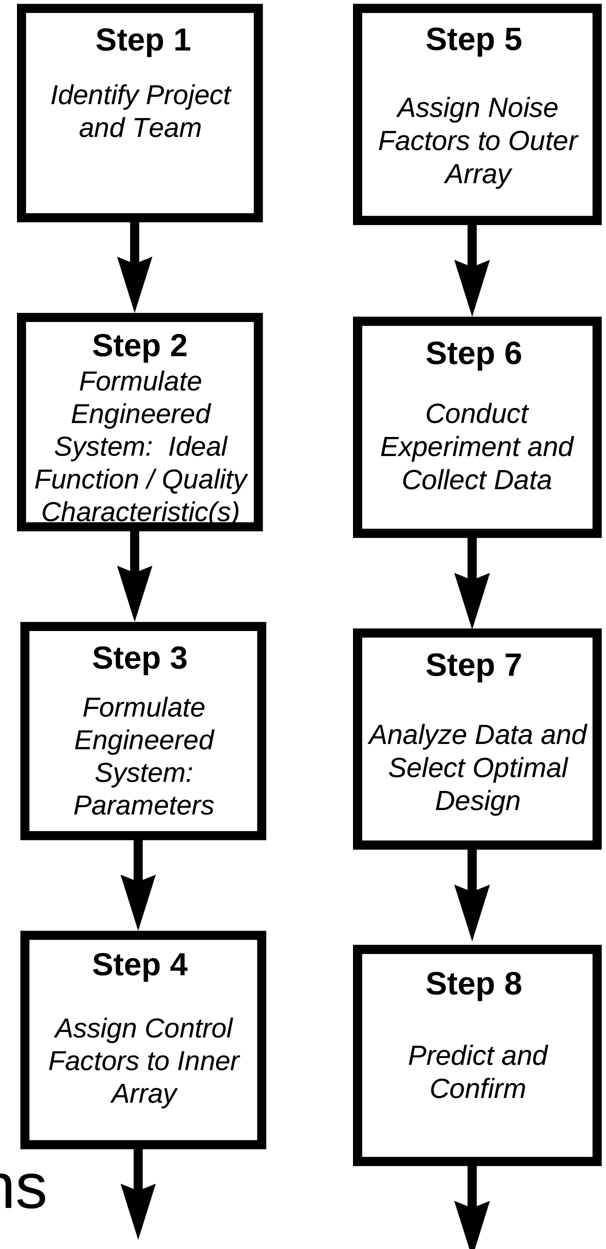
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | **a** | -1 | -1 | +1 | +1 |
| | | | | | | | | **b** | -1 | +1 | -1 | +1 |
| **Control Factors** | | | | | | | | **c** | -1 | +1 | +1 | -1 |
| | **A** | **B** | **C** | **D** | **E** | **F** | **G** | | | | | |
| **1** | -1 | -1 | -1 | -1 | -1 | -1 | -1 | | | | | |
| **2** | -1 | -1 | -1 | +1 | +1 | +1 | +1 | | | | | |
| **3** | -1 | +1 | +1 | -1 | -1 | +1 | +1 | | | | | |
| **4** | -1 | +1 | +1 | +1 | +1 | -1 | -1 | | | | | |
| **5** | +1 | -1 | +1 | -1 | +1 | -1 | +1 | | | | | |
| **6** | +1 | -1 | +1 | +1 | -1 | +1 | -1 | | | | | |
| **7** | +1 | +1 | -1 | -1 | +1 | +1 | -1 | | | | | |
| **8** | +1 | +1 | -1 | +1 | -1 | -1 | +1 | | | | | |

$2_{III}^{7-4}$

$2_{III}^{7-4} \times 2_{III}^{3-1}$

Taguchi, G., 1976, *System of Experimental Design*.

Image removed due to copyright restrictions. Please see "Robust System Design Application." *FAO Reliability Guide – Tools & Methods Module* 18. Dearborn, MI: Ford Motor Company.




## Step 4 Summary:

- Determine control factor levels
- Calculate the DOF
- Determine if there are any interactions
- Select the appropriate orthogonal array

# Majority View on "One at a Time"

One way of thinking of the great advances of the science of experimentation in this century is as **the final demise of the "one factor at a time" method**, although it should be said that there are still organizations which have never heard of factorial experimentation and use up many man hours wandering a crooked path.

Logothetis, N., and Wynn, H.P., 1994, *Quality Through Design: Experimental Design, Off-line Quality Control and Taguchi's Contributions*, Clarendon Press, Oxford.

# My Observations of Industry

- Farming equipent company has reliability problems
- Large blocks of robustness experiments had been planned at outset of the design work
- More than 50% were not finished
- Reasons given
  - Unforeseen changes
  - Resource pressure
  - Satisficing

**“Well, in the third experiment, we found a solution that met all our needs, so we cancelled the rest of the experiments and moved on to other tasks…”**

# Minority Views on "One at a Time"

"…the factorial design has certain deficiencies … It devotes observations to exploring regions that may be of no interest…These deficiencies … suggest that an efficient design for the present purpose ought to be sequential; that is, ought to adjust the experimental program at each stage in light of the results of prior stages."

Friedman, Milton, and L. J. Savage, 1947, "Planning Experiments Seeking Maxima", in *Techniques of Statistical Analysis*, pp. 365-372.

"Some scientists do their experimental work in single steps. They hope to learn something from each run … they see and react to data more rapidly …If he has in fact found out a good deal by his methods, it must be true that the effects are at least three or four times his average random error per trial."

Cuthbert Daniel, 1973, "One-at-a-Time Plans", *Journal of the American Statistical Association*, vol. 68, no. 342, pp. 353-360.

# Adaptive OFAT Experimentation

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Frey, D. D., F. Engelhardt, and E. Greitzer, 2003, "A Role for One Factor at a Time Experimentation in Parameter Design", *Research in Engineering Design* 14(2): 65-74.

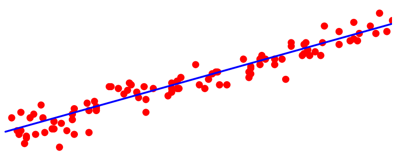
# Empirical Evaluation of Adaptive OFAT Experimentation

- Meta-analysis of 66 responses from published, full factorial data sets
- When experimental error is <25% of the combined factor effects OR interactions are >25% of the combined factor effects, adaptive OFAT provides more improvement on average than fractional factorial DOE.

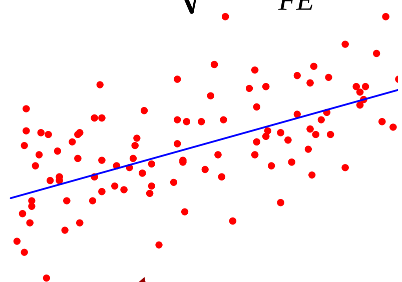
Frey, D. D., F. Engelhardt, and E. Greitzer, 2003, “A Role for One Factor at a Time Experimentation in Parameter Design”, *Research in Engineering Design* 14(2): 65-74.

# Detailed Results

$\sigma = 0.1\sqrt{MS_{FE}}$



$\sigma = 0.4\sqrt{MS_{FE}}$



OFAT/FF

Gray if OFAT>FF

| Interaction Strength | | Strength of Experimental Error | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **0** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **1** |
| | **Mild** | 100/99 | 99/98 | 98/98 | 96/96 | 94/94 | 89/92 | 86/88 | 81/86 | 77/82 | 73/79 | 69/75 |
| | **Moderate** | 96/90 | 95/90 | 93/89 | 90/88 | 86/86 | 83/84 | 80/81 | 76/81 | 72/77 | 69/74 | 64/70 |
| | **Strong** | 86/67 | 85/64 | 82/62 | 79/63 | 77/63 | 72/64 | 71/63 | 67/61 | 64/58 | 62/55 | 56/50 |
| | **Dominant** | 80/39 | 79/36 | 77/34 | 75/37 | 72/37 | 70/35 | 69/35 | 64/34 | 63/31 | 61/35 | 59/35 |

# A Mathematical Model of Adaptive OFAT

initial observation → $O_0 = y(\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n)$

observation with first factor toggled → $O_1 = y(-\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n)$

first factor set → $x_1^* = \tilde{x}_1 sign\{O_0 - O_1\}$

repeat for all remaining factors

$$\text{for } i = 2 \ldots n$$

$$O_i = y(x_1^*, \ldots x_{i-1}^*, -\tilde{x}_i, \tilde{x}_{i+1}, \ldots \tilde{x}_n)$$

$$x_i^* = \tilde{x}_i sign\{\max(O_0, O_1, \ldots O_{i-1}) - O_i\}$$

process ends after $n$+1 observations with $E\left[y\left(x_1^*, x_2^*, \ldots x_n^*\right)\right]$

Frey, D. D., and H. Wang, 2006, "Adaptive One-Factor-at-a-Time Experimentation and Expected Value of Improvement", *Technometrics* 48(3):418-31.

# A Mathematical Model of a Population of Engineering Systems

$$y(x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} x_i x_j + \varepsilon_k$$

system response

$$\beta_i \sim \mathrm{N}\left(0, \sigma_{ME}{}^2\right)$$

main effects

$$\beta_{ij} \sim \mathrm{N}\left(0, \sigma_{INT}{}^2\right)$$

two-factor interactions

$$\varepsilon_k \sim \mathrm{N}\left(0, \sigma_\varepsilon{}^2\right)$$

experimental error

$y_{\max} \equiv$ the largest response within the space of discrete, coded, two-level factors $x_i \in \{-1, +1\}$

Model adapted from Chipman, H., M. Hamada, and C. F. J. Wu, 2001, “A Bayesian Variable Selection Approach for Analyzing Designed Experiments with Complex Aliasing”, *Technometrics* 39(4)372-381.

# Probability of Exploiting an Effect

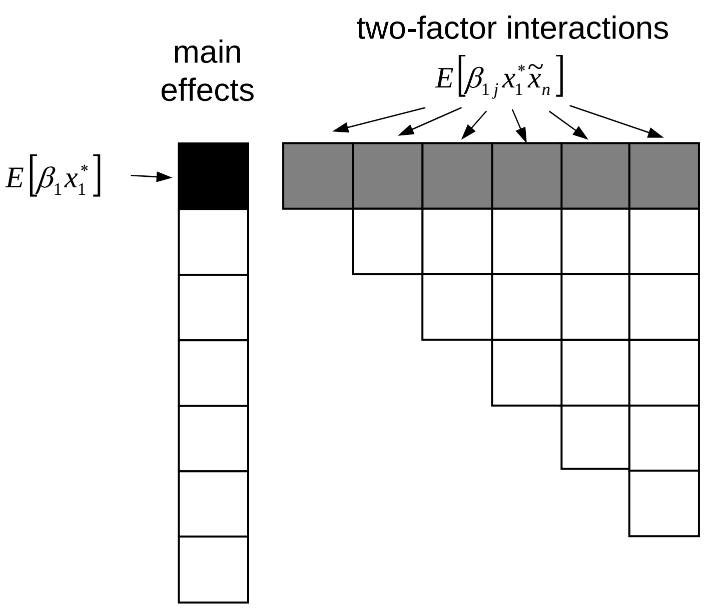
- The $i^{\text{th}}$ main effect is said to be "exploited" if $\boxed{\beta_i x_i^* > 0}$
- The two-factor interaction between the $i^{\text{th}}$ and $j^{\text{th}}$ factors is said to be "exploited" if $\boxed{\beta_{ij} x_i^* x_j^* > 0}$
- The probabilities and conditional probabilities of exploiting effects provide insight into the mechanisms by which a method provides improvements

# The Expected Value of the Response after the *First* Step

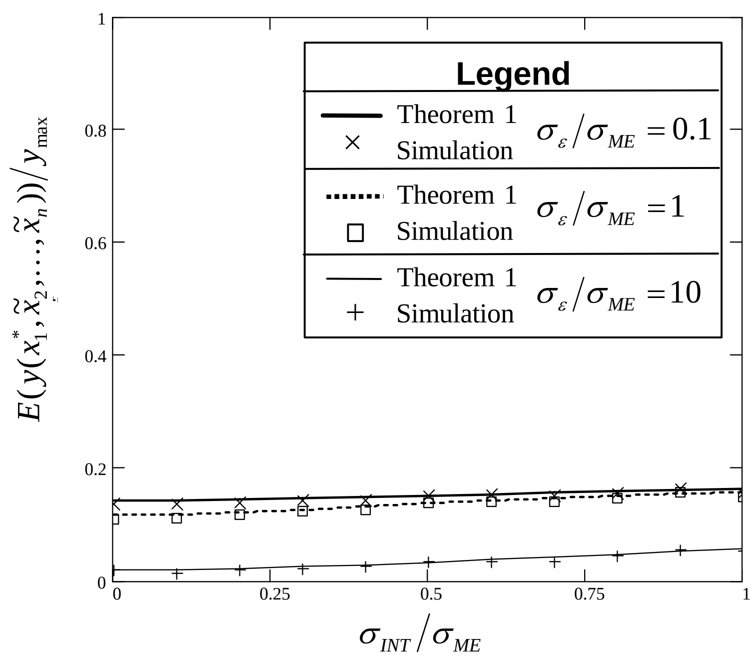
$$E(y(x_1^*,\tilde{x}_2,\ldots,\tilde{x}_n)) = E\left[\beta_1 x_1^*\right] + (n-1)E\left[\beta_{1j}x_1^*\tilde{x}_j\right]$$

$$E\left[\beta_1 x_1^*\right] = \sqrt{\frac{2}{\pi}}\,\frac{\sigma_{ME}^2}{\sqrt{\sigma_{ME}^2+(n-1)\sigma_{INT}^2+\frac{1}{2}\sigma_{\varepsilon}^2}}$$

$$E\left[\beta_{1j}x_1^*\tilde{x}_n\right] = \sqrt{\frac{2}{\pi}}\,\frac{\sigma_{INT}^2}{\sqrt{\sigma_{ME}^2+(n-1)\sigma_{INT}^2+\frac{1}{2}\sigma_{\varepsilon}^2}}$$



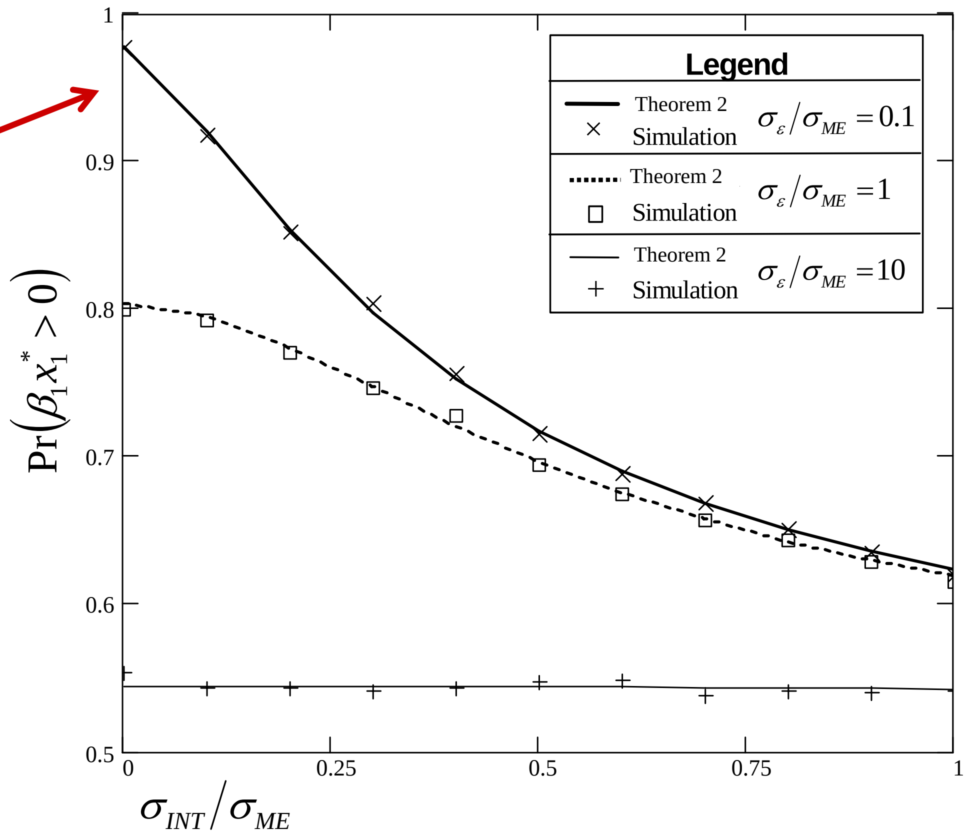

$n$=7



# Probability of Exploiting the First Main Effect

$$\Pr\left(\beta_1 x_1^* > 0\right) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\frac{\sigma_{ME}}{\sqrt{\sigma_{ME}^{\;2} + (n-1)\sigma_{INT}^{\;2} + \frac{1}{2}\sigma_{\varepsilon}^{\;2}}}$$

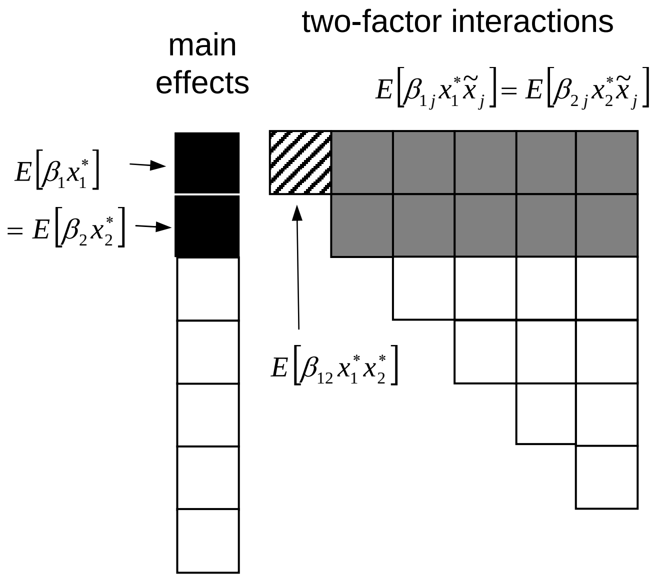
If interactions are small and error is not too large, OFAT will tend to exploit main effects

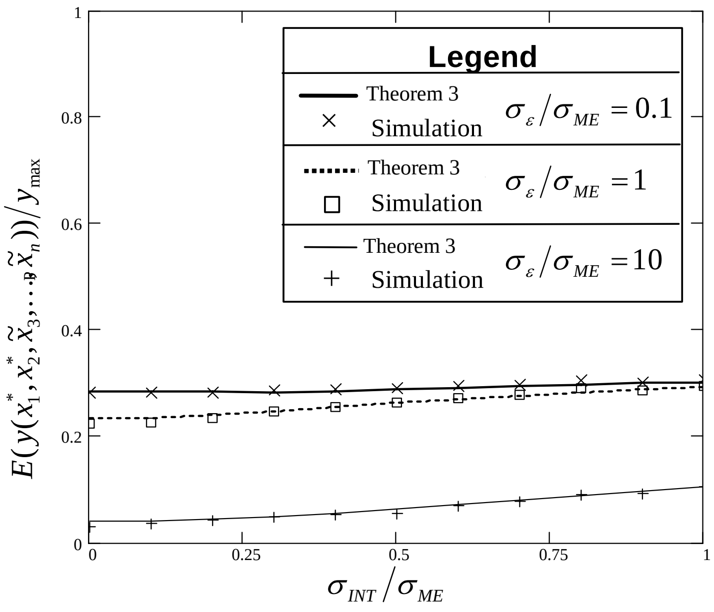


# The Expected Value of the Response After the *Second* Step

$$E(y(x_1^*, x_2^*, \tilde{x}_3, \ldots, \tilde{x}_n)) = 2E\left[\beta_1 x_1^*\right] + 2(n-2)E\left[\beta_{1j} x_1^*\right] + E\left[\beta_{12} x_1^* x_2^*\right]$$

$$E\left[\beta_{12} x_1^* x_2^*\right] = \sqrt{\frac{2}{\pi}} \left[ \frac{\sigma_{INT}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \frac{\sigma_\varepsilon^2}{2}}} \right]$$



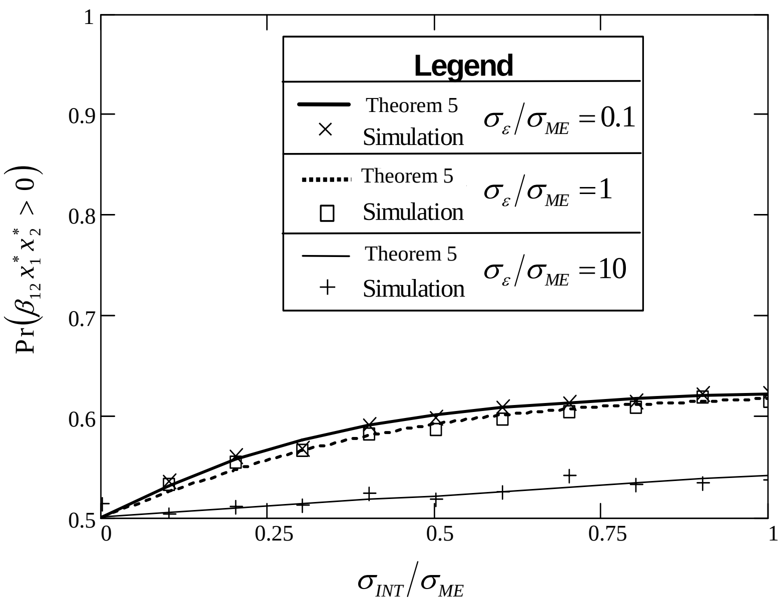



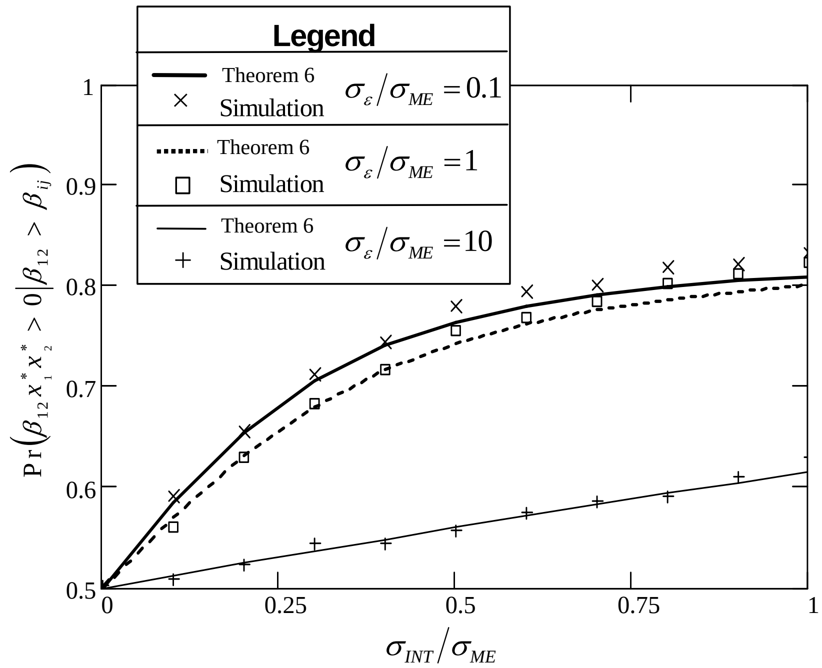
# Probability of Exploiting the First Interaction

$$\Pr\left(\beta_{12}x_1^* x_2^* > 0\right) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{\sigma_{INT}}{\sqrt{\sigma_{ME}^2 + (n-2)\sigma_{INT}^2 + \frac{1}{2}\sigma_\varepsilon^2}}$$

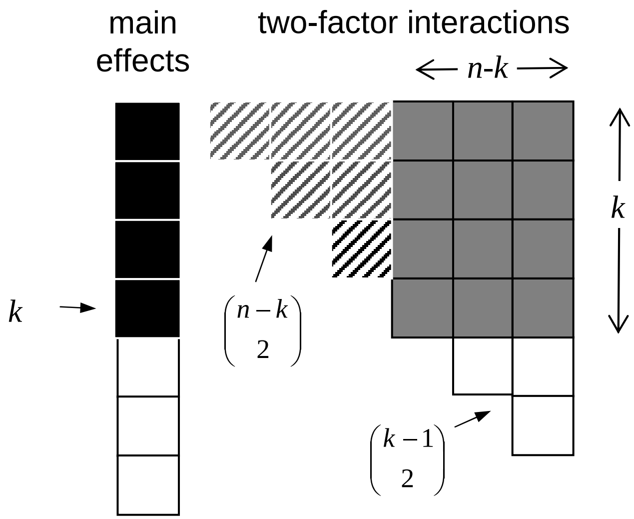
$$\Pr\left(\beta_{12}x_1^* x_2^* > 0 \middle| \beta_{12} > \beta_{ij}\right) >$$

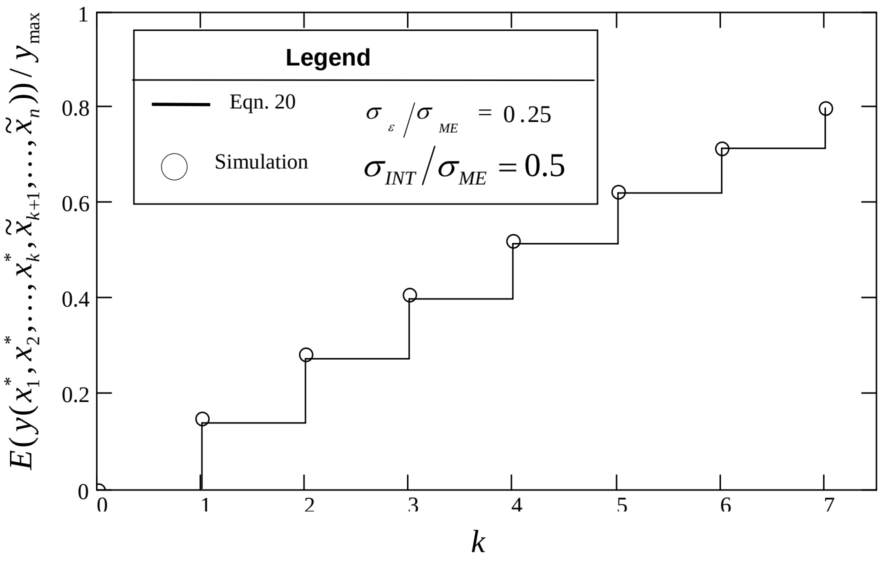
$$\frac{1}{\pi}\binom{n}{2}\int_0^\infty \int_{-x_2}^\infty \frac{\left[erf\left(\frac{1}{\sqrt{2}}\frac{x_1}{\sigma_{INT}}\right)\right]^{\binom{n}{2}-1} e^{\frac{-x_1^2}{2\sigma_{INT}^2}+\frac{-x_2^2}{2\left(\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2\right)}}}{\sigma_{INT}\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2}}\, dx_2 dx_1$$



# And it Continues

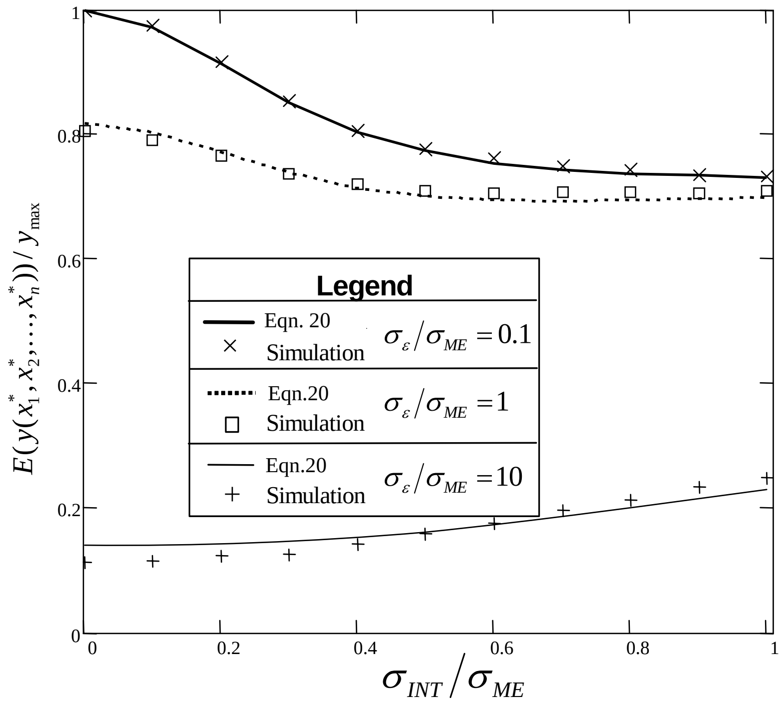


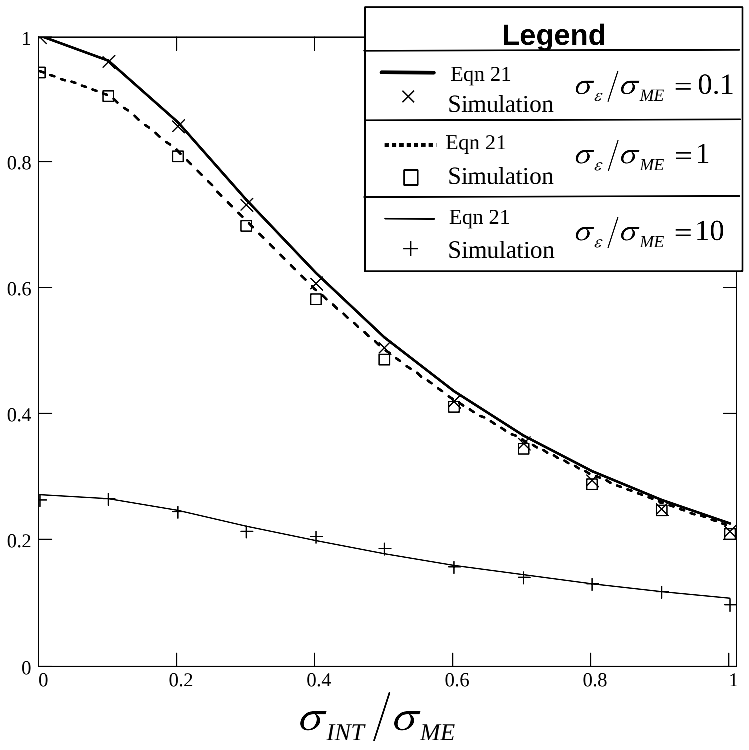

$$\Pr\left(\beta_{ij}x_i^*x_j^*>0\right)\geq\Pr\left(\beta_{12}x_1^*x_2^*>0\right)$$

We can prove that the probability of exploiting interactions is sustained. Further we can now prove exploitation probability is a function of *j* only and increases monotonically.

# Final Outcome

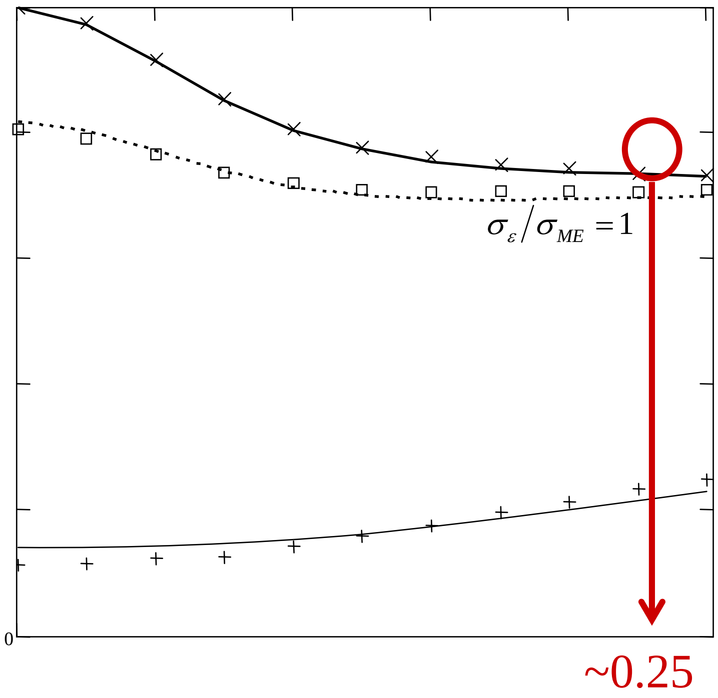



Adaptive OFAT

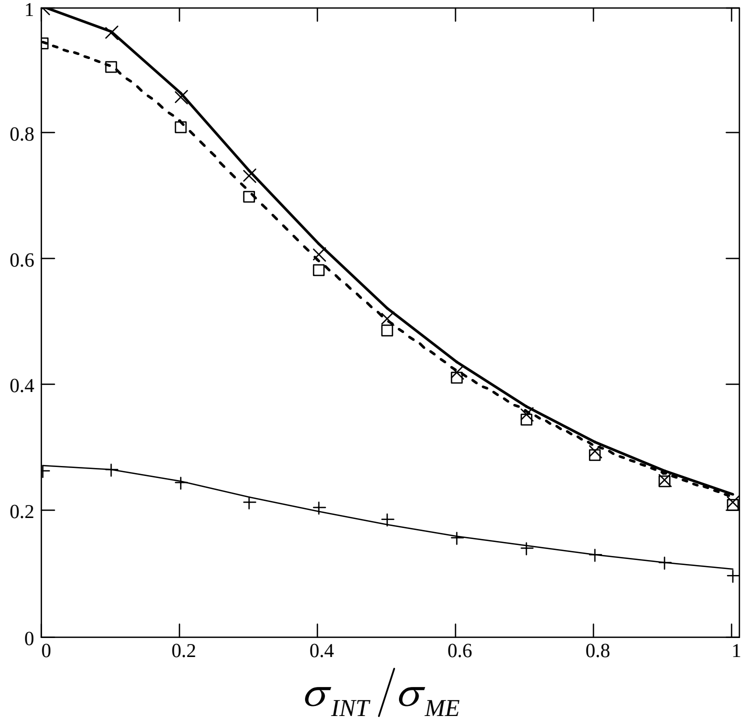



Resolution III Design

# Final Outcome




Adaptive OFAT




Resolution III Design

# Adaptive "One Factor at a Time" for Robust Design

Run a resolution III on noise factors

Change one factor

Again, run a resolution III on noise factors. If there is an improvement, in transmitted variance, retain the change

If the response gets worse, go back to the previous state

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Stop after you've changed every factor once

# Sheet Metal Spinning

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# Paper Airplane

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# Results Across Four Case studies

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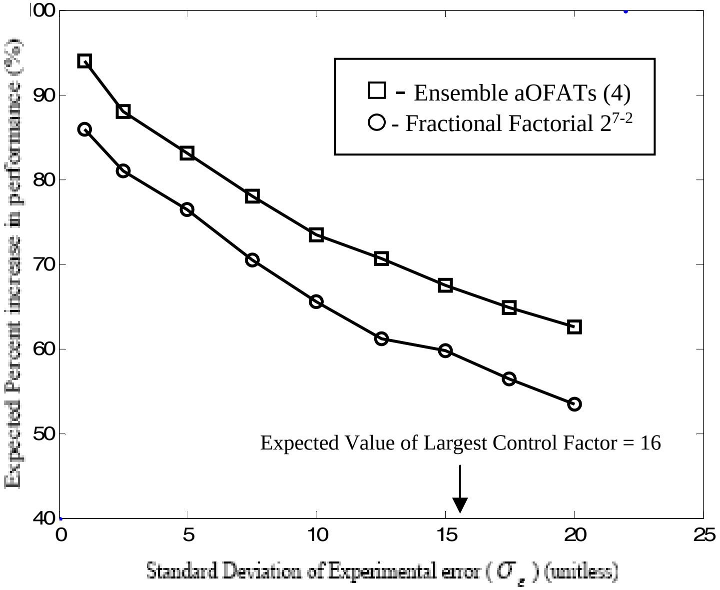
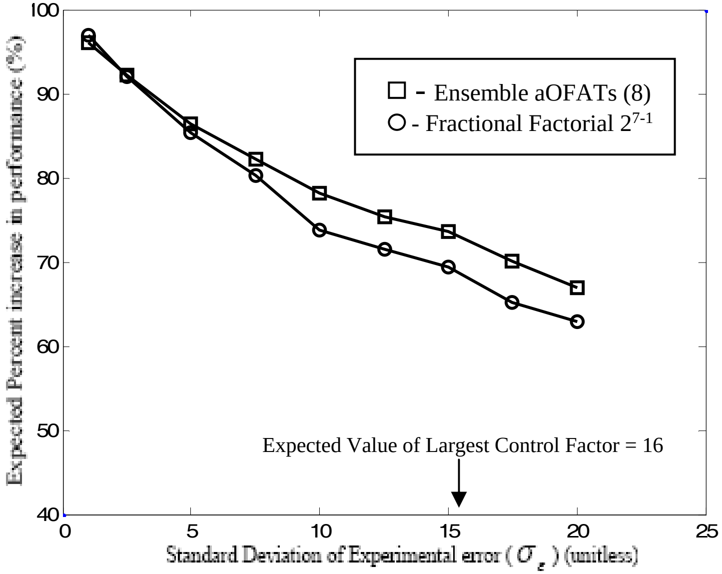
# Ensembles of *a*OFATs




**Comparing an Ensemble of 4 *a*OFATs with a $2^{7-2}$ Fractional Factorial array using the HPM**




**Comparing an Ensemble of 8 *a*OFATs with a $2^{7-1}$ Fractional Factorial array using the HPM**

Courtesy of Nandan Sudarsanam. Used with permission.

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# Conclusions

- A new model and theorems show that
  - Adaptive OFAT plans exploit two-factor interactions especially when they are large
  - Adaptive OFAT plans provide around 80% of the benefits achievable via parameter design
- Adaptive OFAT can be “crossed” with factorial designs which proves to be highly effective

Frey, D. D., and N. Sudarsanam, 2007, “An Adaptive One-factor-at-a-time Method for Robust Parameter Design: Comparison with Crossed Arrays via Case Studies,” accepted to *ASME Journal of Mechanical Design*.

# Questions?