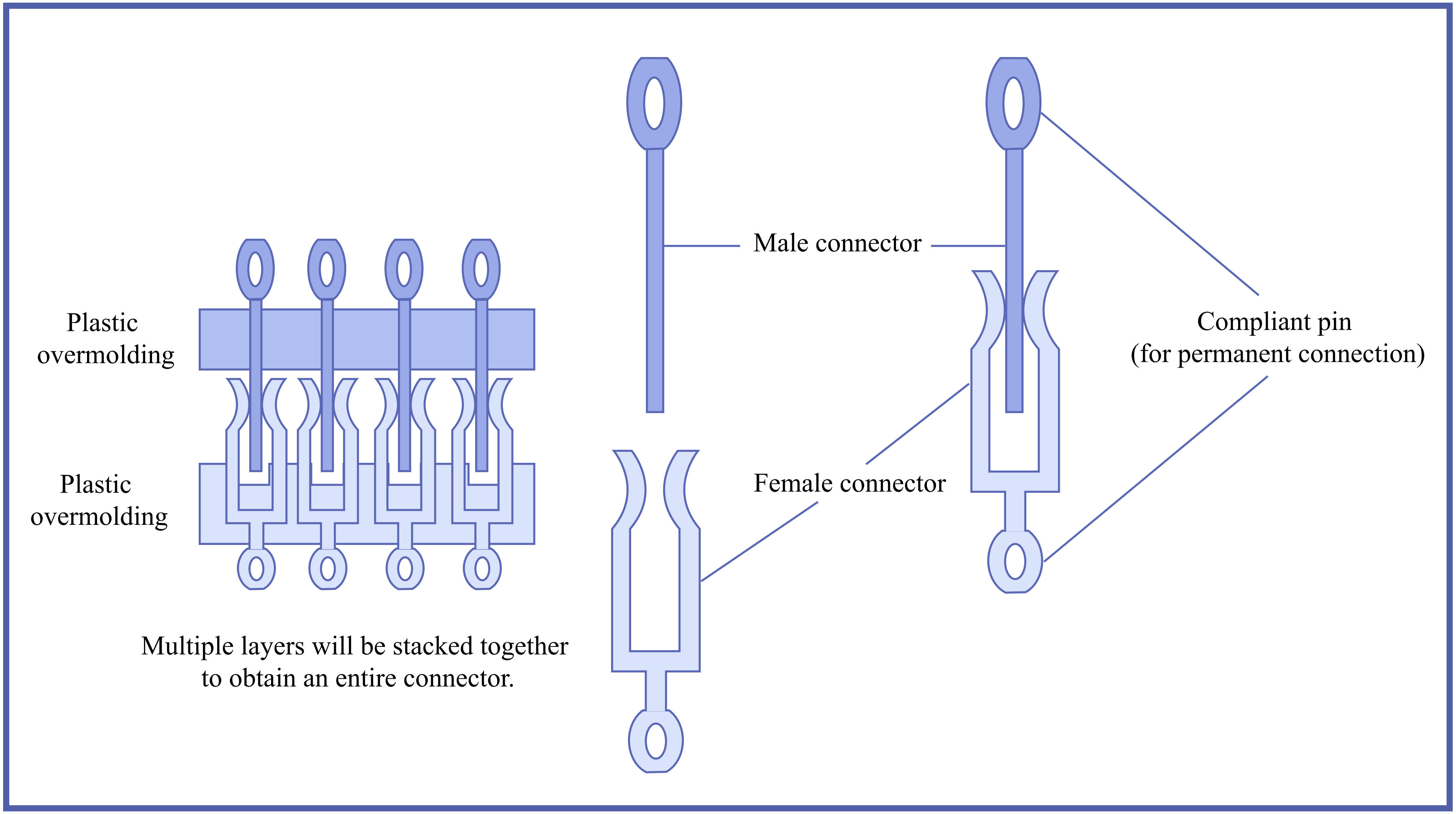
Chapter 10

Introduction to Axiomatic Design

This presentation draws extensively on materials from [Suh 2001]:
Suh, N. P. *Axiomatic Design: Advances and Applications*. New York:
Oxford University Press, 2001. ISBN: 0195134664.

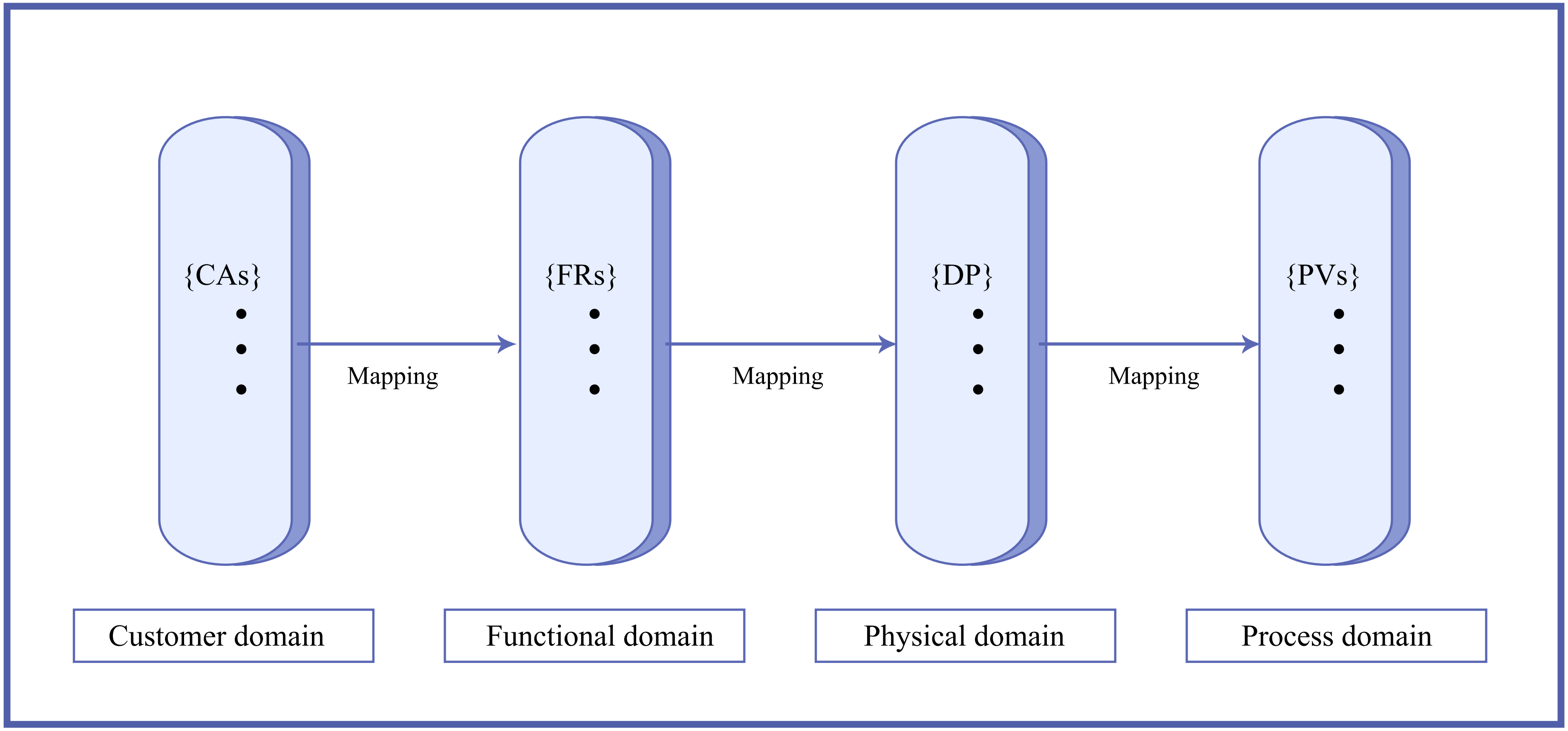
Example: Electrical Connector



Multiple layers will be stacked together to obtain an entire connector.

Figure by MIT OCW.

Axiomatic Design Framework
The Concept of Domains



Four Domains of the Design World.
The {x} are characteristic vectors of each domain.

Figure by MIT OCW. After Figure 1.2 in [Suh 2001].

Characteristics of the four domains of the design world

Domains Character Vectors	Customer Domain {CAs}	Functional Domain {FRs}	Physical Domain {DPs}	Process Domain {PVs}
Manufacturing	Attributes which consumers desire	Functional requirements specified for the product	Physical variables which can satisfy the functional requirements	Process variables that can control design parameters (DPs)
Materials	Desired performance	Required Properties	Micro-structure	Processes
Software	Attributes desired in the software	Output Spec of Program codes	Input Variables or Algorithms Modules Program codes	Sub-routines machine codes compilers modules
Organization	Customer satisfaction	Functions of the organization	Programs or Offices or Activities	People and other resources that can support the programs
Systems	Attribute desired of the overall system	Functional requirements of the system	Machines or components, sub-components	Resources (human, financial, materials, etc.)
Business	ROI	Business goals	Business structure	Human and financial resource

Table by MIT OCW. After Table 1.1 in [Suh 2001].

Definitions

□ ***Axiom:***

Self-evident truth or fundamental truth for which there is no counter examples or exceptions. It cannot be derived from other laws of nature or principles.

Corollary:

Inference derived from axioms or propositions that follow from axioms or other propositions that have been proven.

Definitions - cont’d

Functional Requirement:

Functional requirements (FRs) are a minimum set of independent requirements that completely characterizes the functional needs of the product (or software, organizations, systems, etc.) in the functional domain. By definition, each FR is independent of every other FR at the time the FRs are established.

Constraint:

Constraints (Cs) are bounds on acceptable solutions. There are two kinds of constraints: input constraints and system constraints. Input constraints are imposed as part of the design specifications. System constraints are constraints imposed by the system in which the design solution must function.

Definitions - cont’d

Design parameter:

Design parameters (DPs) are the key physical (or other equivalent terms in the case of software design, etc.) variables in the physical domain that characterize the design that satisfies the specified FRs.

Process variable:

Process variables (PVs) are the key variables (or other equivalent term in the case of software design, etc.) in the process domain that characterizes the process that can generate the specified DPs.

The Design Axioms

Axiom 1: The Independence Axiom

Maintain the independence of the functional requirements (FRs).

Axiom 2: The Information Axiom

Minimize the information content of the design.

Example: Beverage Can Design

Consider an aluminum beverage can that contains carbonated drinks.

How many functional requirements must the can satisfy?

See Example 1.3 in [Suh 2001].

How many physical parts does it have?

What are the design parameters (DPs)? How many DPs are there?

Design Matrix

The relationship between {FRs} and {DPs} can be written as

$$\{FRs\}=[A]\ \{DPs\}$$

When the above equation is written in a differential form as

$$\{dFRs\}=[A]\ \{dDPs\}$$

[A] is defined as the Design Matrix given by elements :

$$Aij = \partial FRi/\partial DPi$$

Example

For a matrix A:

$$[A] = \begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix}$$

Equation (1.1) may be written as

$$\begin{aligned} FR1 &= A11\ DP1 + A12\ DP2 + A13\ DP3 \\ FR2 &= A21\ DP1 + A22\ DP2 + A23\ DP3 \\ FR3 &= A31\ DP1 + A32\ DP2 + A33\ DP3 \end{aligned} \tag{1.3}$$

Uncoupled, Decoupled, and Coupled Design

Uncoupled Design

$$[A] = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix} \qquad (1.4)$$

Decoupled Design

$$[A] = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix} \qquad (1.5)$$

Coupled Design

All other design matrices

Design of Processes

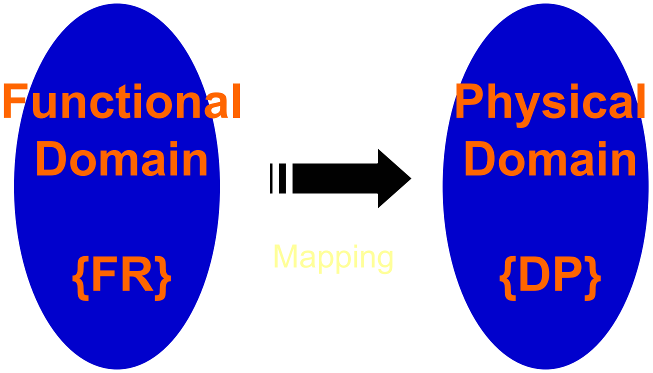
$$\{DPs\}=[B]\ \{PVs\}$$

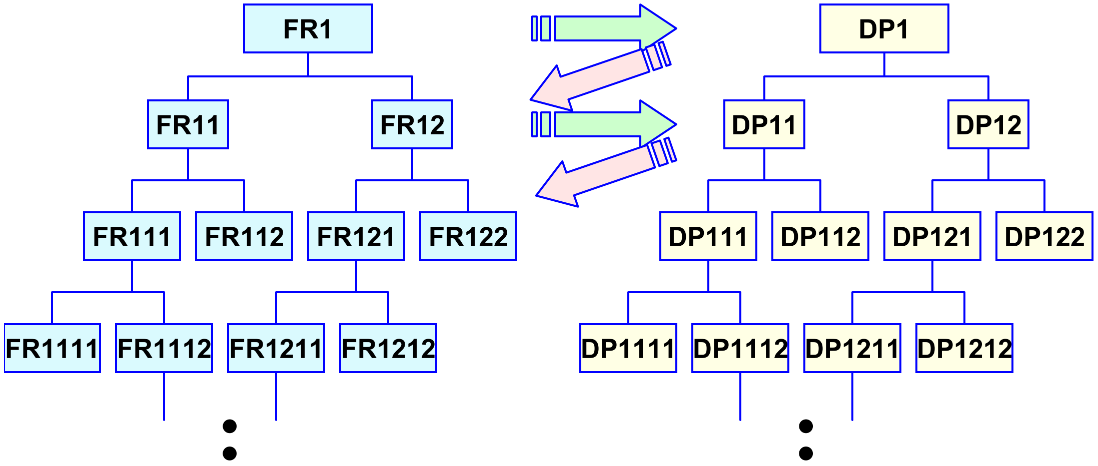
[B] is the design matrix that defines the characteristics of the process design and is similar in form to [A].

Axiomatic Design Theory

Functional Requirement (FR) – '*What*' we want to achieve
A minimum set of requirements a system must satisfy

Design Parameter (DP) – '*How*' FRs will be achieved
Key physical variables that characterize design solution





Decomposition – 'Zigzagging'

Process of developing detailed requirements and concepts by moving between functional and physical domain

Hierarchical FR-DP structure

Independence Axiom
Maintain the independence of FRs

Information Axiom
Minimize the information content

Design Axioms

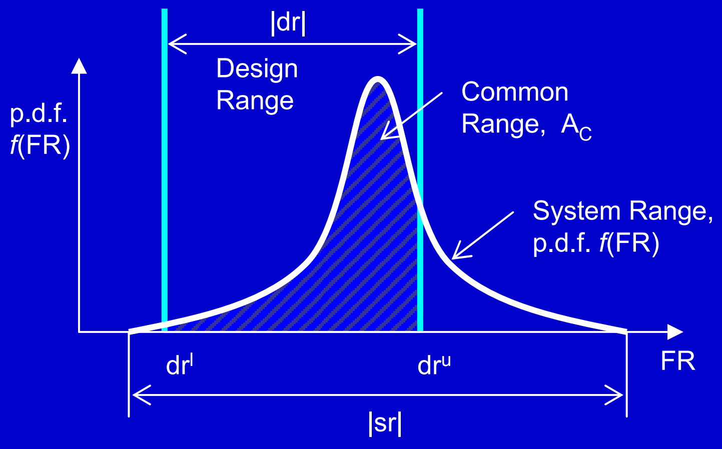
Independence Axiom: Maintain the independence of FRs

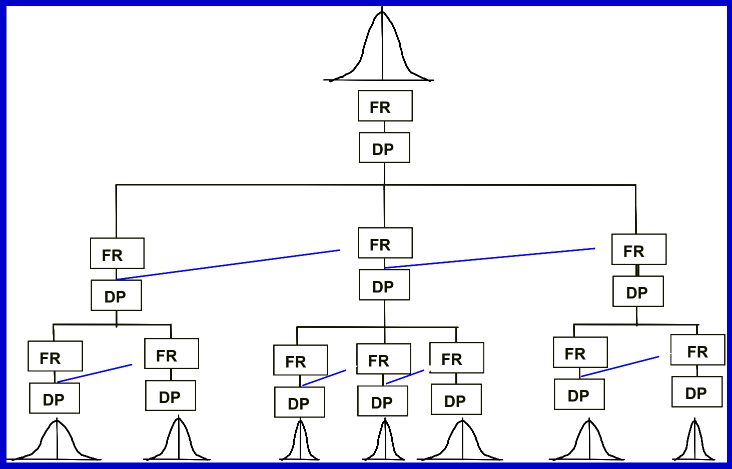
$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} X & O \\ O & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix} \qquad \begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} X & O \\ X & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix} \qquad \begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} X & X \\ X & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Uncoupled **Decoupled** **Coupled**

Information Axiom: Minimize the information content

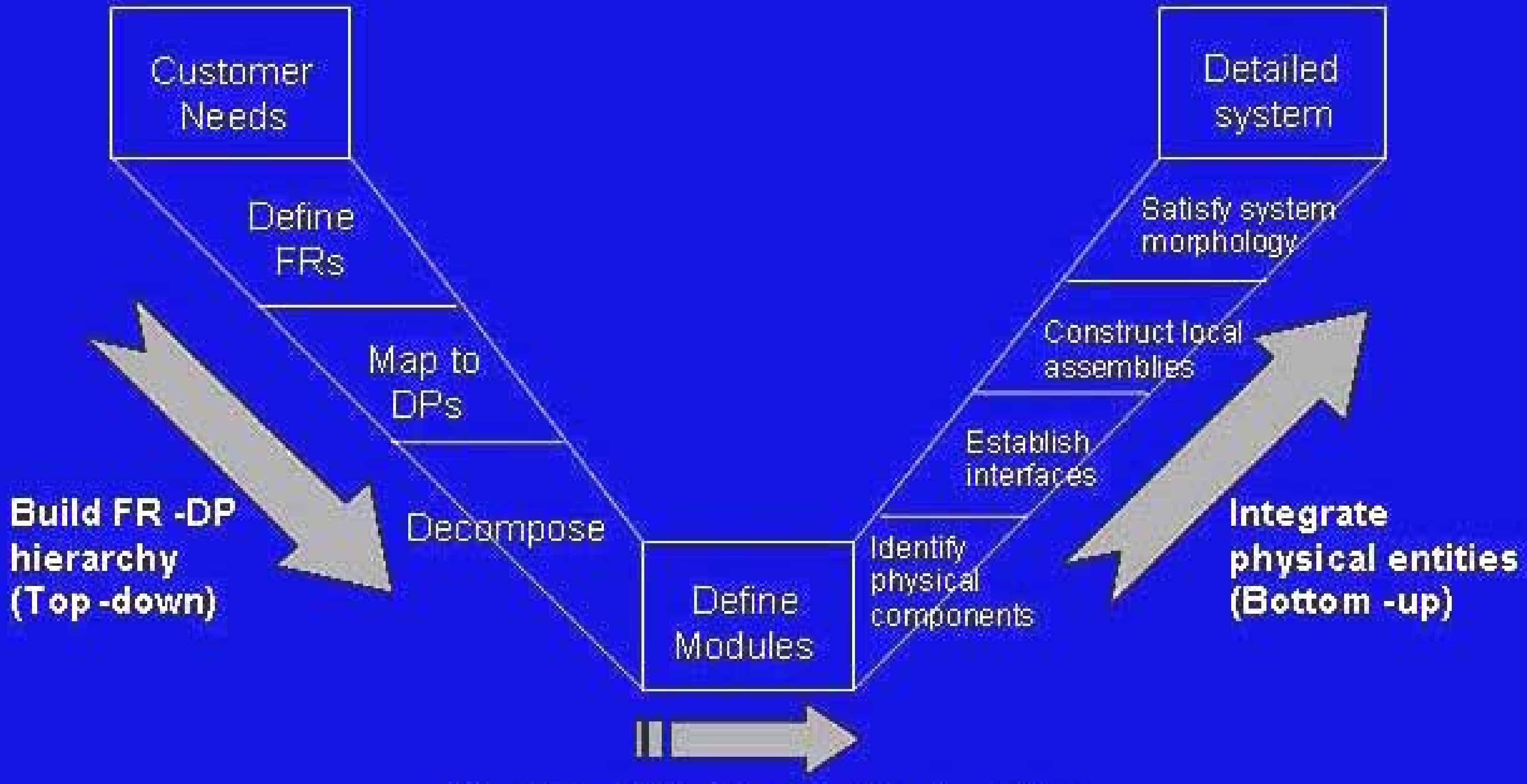
Information content for functional requirement *i* = - $\log_2 P_i$





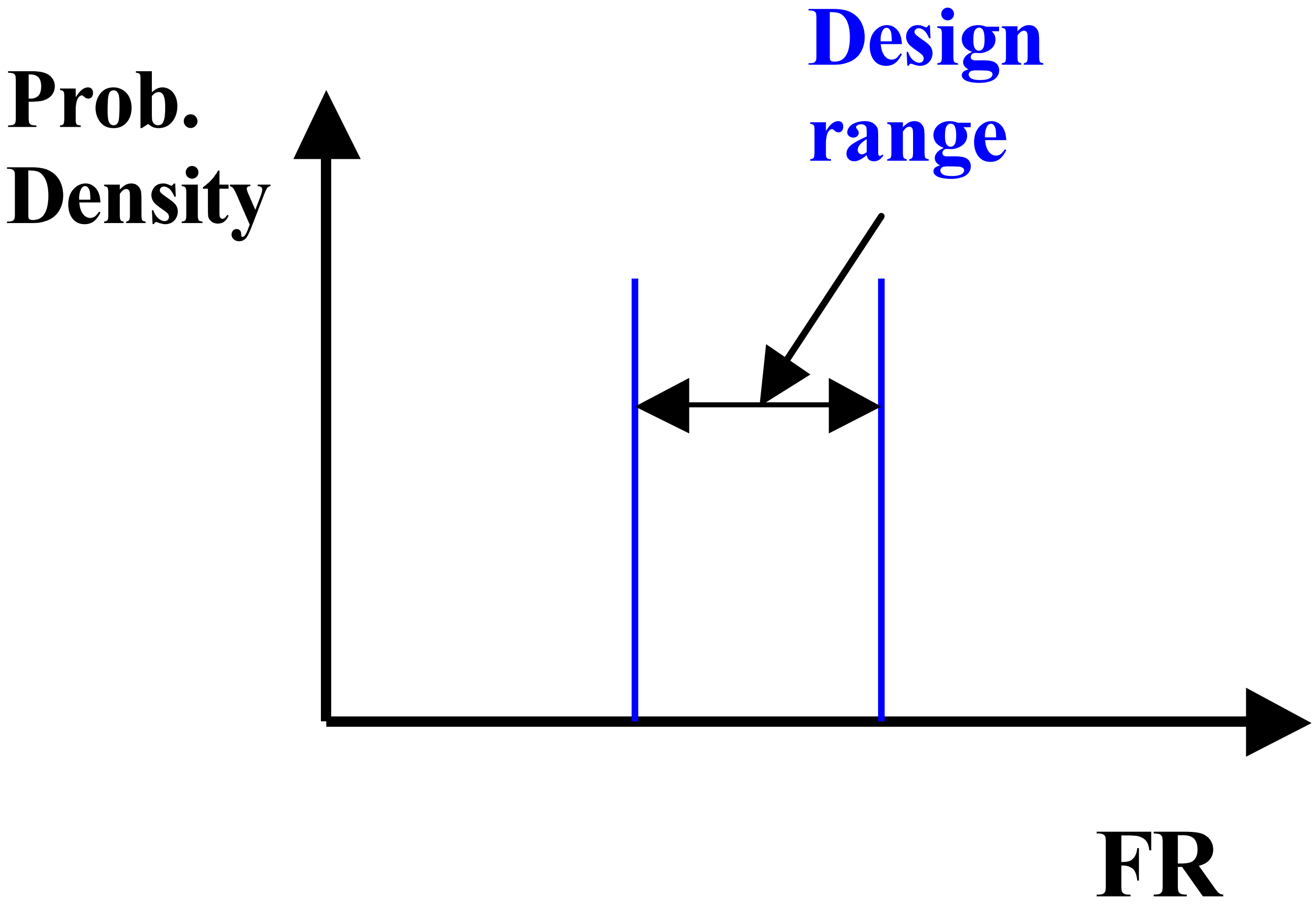
System Design & Development

V-Model

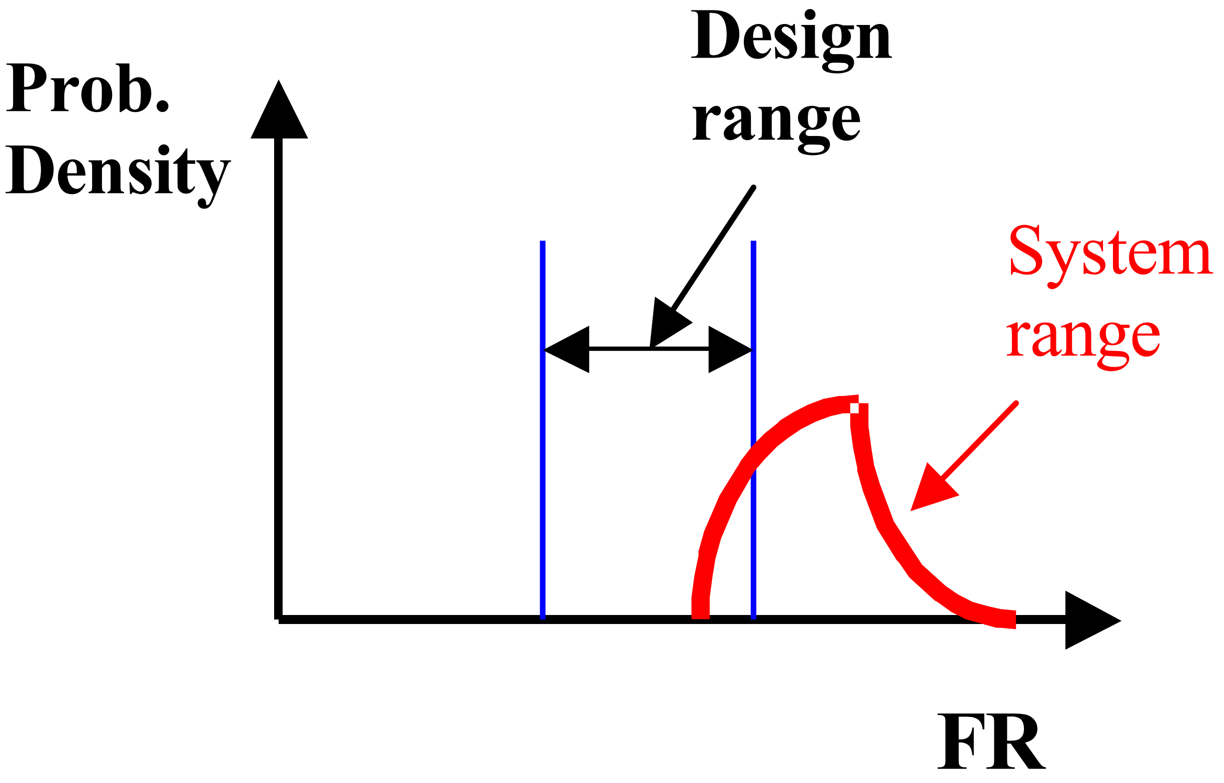


Mapping DPs into physical entities

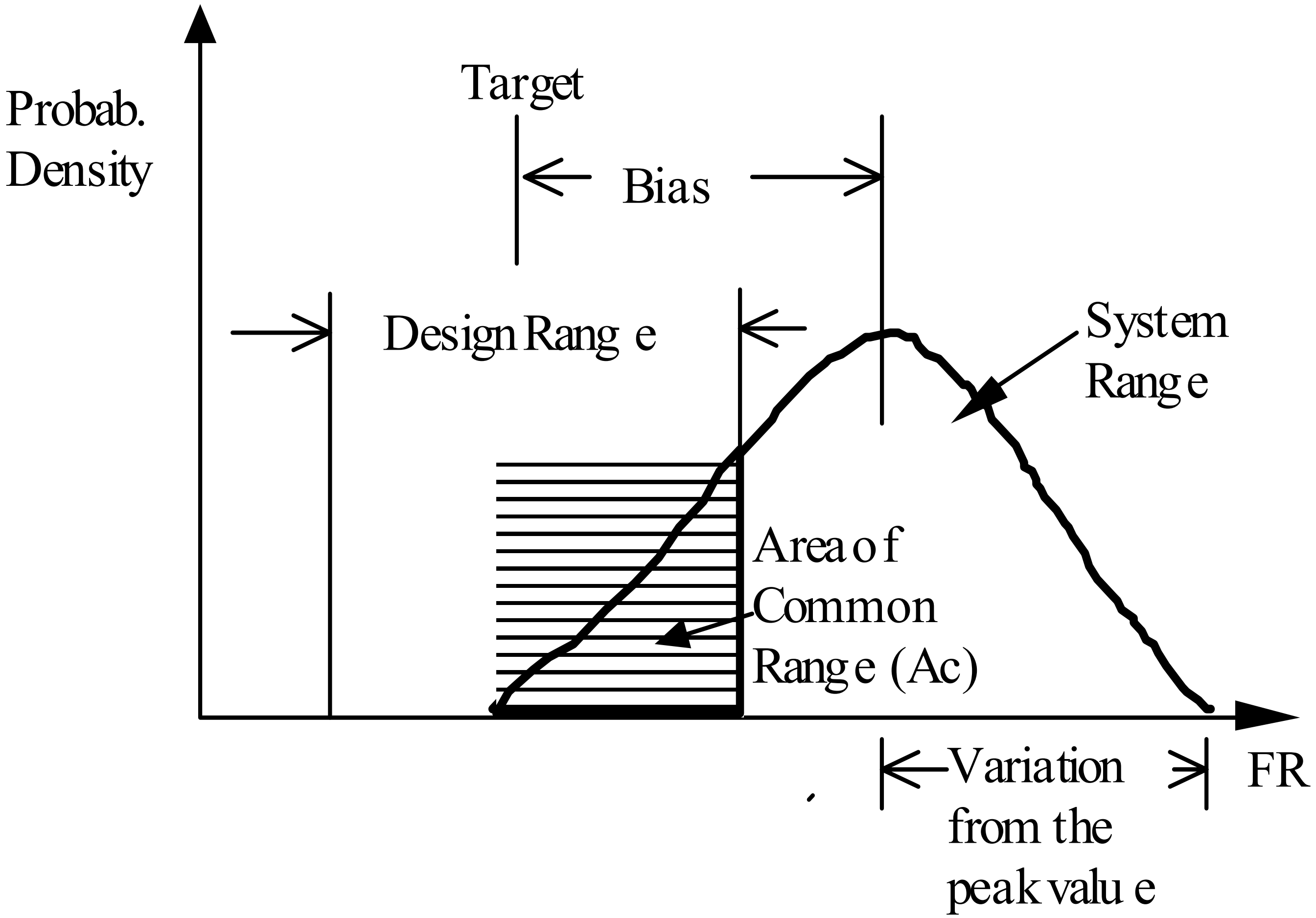
FR must be satisfied within the design range.



To satisfy the FR, we have to map FRs in the physical domain and identify DPs.



Design Range, System Range, and Common Range

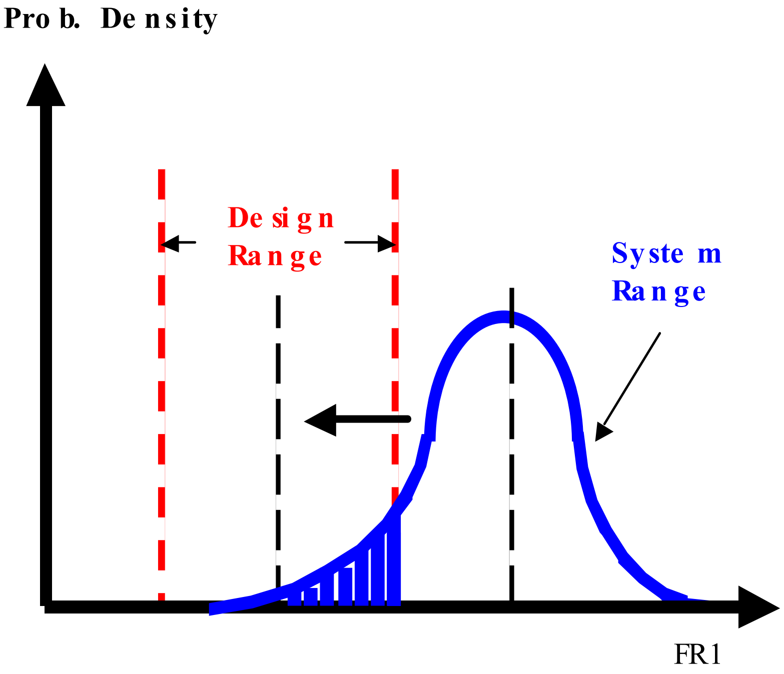


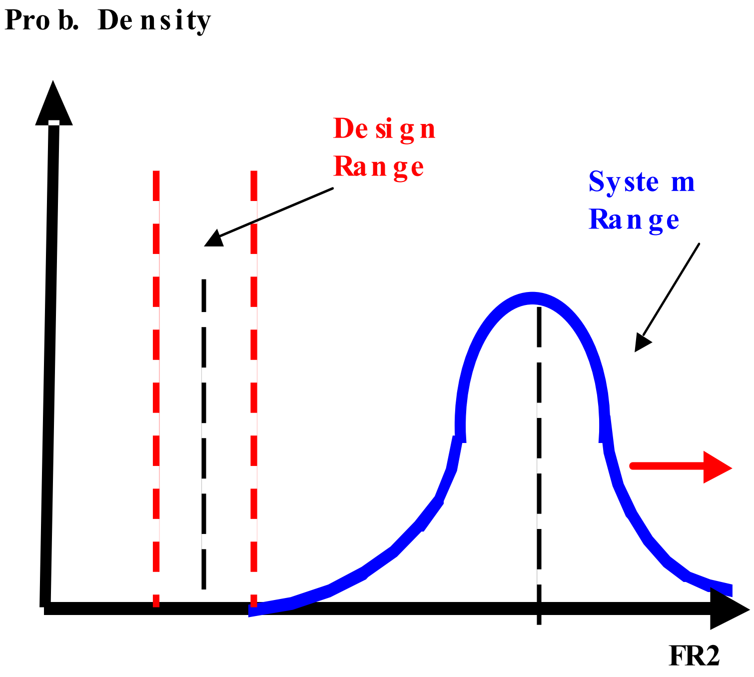
What happens when there are many FRs?

Most engineered systems must satisfy many FRs at each level of the system hierarchy.

The relationship between the FRs determines how difficult it will be to satisfy the FRs within the desired certainty and thus complexity.

If FRs are not independent from each other, the following situation may exist.





Coupling decreases the design range and thus robustness!!

Uncoupled

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix}$$

$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

$$\Delta DP2 = \frac{\Delta FR2}{A22}$$

$$\Delta DP3 = \frac{\Delta FR3}{A33}$$

Decoupled

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix}$$

$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

$$\Delta DP2 = \frac{\Delta FR2 - \left| A21 \cdot \Delta DP1 \right|}{A22}$$

$$\Delta DP3 = \frac{\Delta FR3 - \left| A31 \cdot \Delta DP1 \right| - \left| A32 \cdot \Delta DP2 \right|}{A33}$$

What is wrong with conventional connectors?

It violates the Independence Axiom, which states that

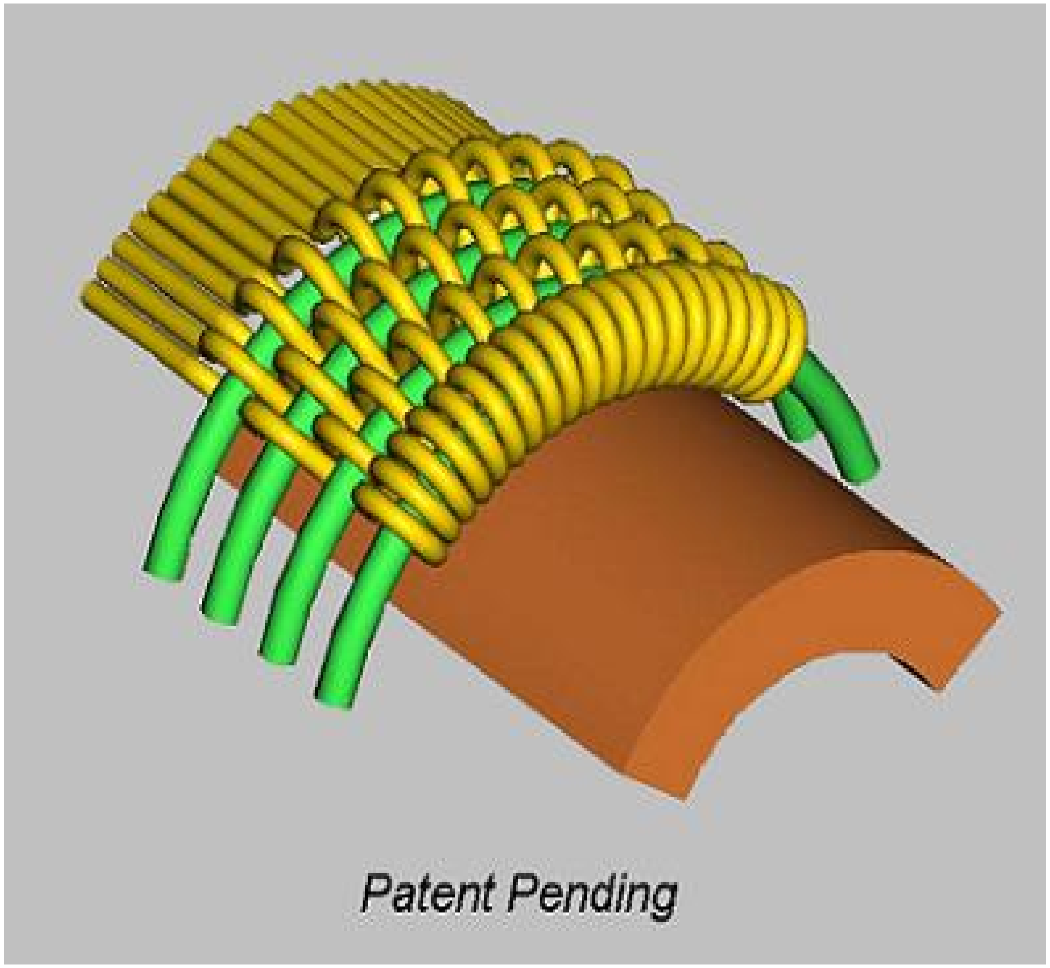
“Maintain the independence of Functional Requirements (FRs)”.

It is a coupled design.

What is the solution?

Tribotek connector: A woven connector

Tribotek Electrical Connectors
(Courtesy of Tribotek, Inc. Used with permission.)



Performance of “Woven” Power Connectors

Power density => 200% of conventional connectors

Insertion force => less than 5% of conventional connectors

Electric contact resistance = 5 m ohms

Manufacturing cost

Capital Investment

TMA Projection System

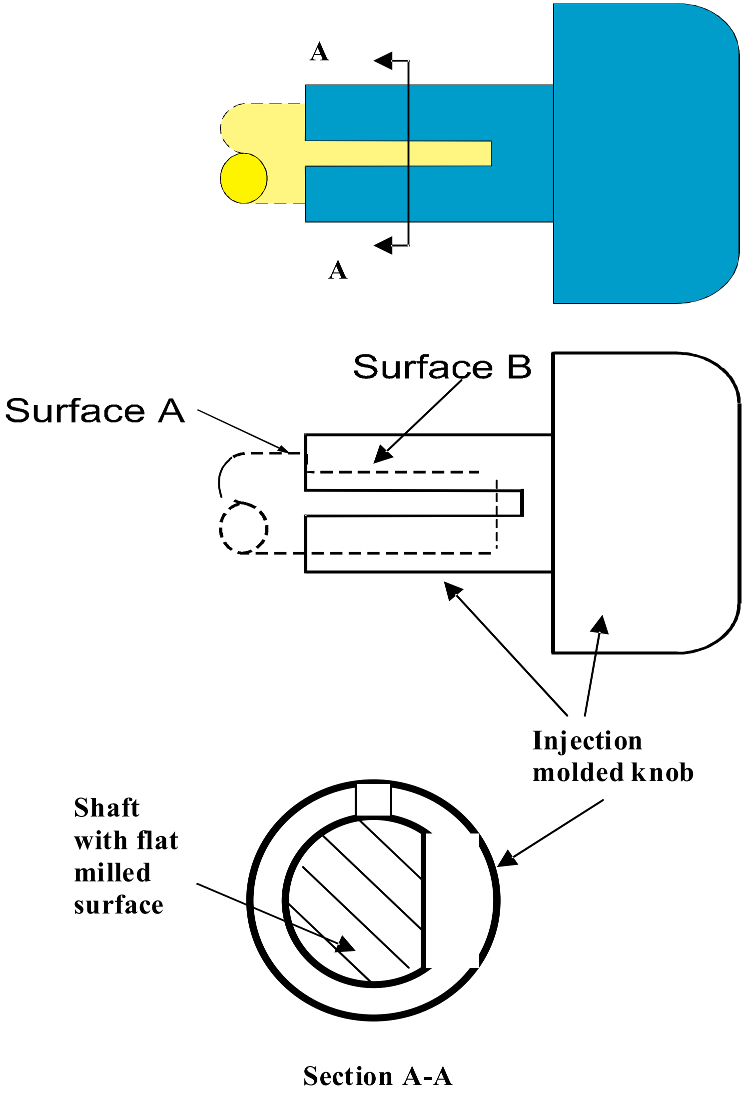
Photos removed for copyright reasons.

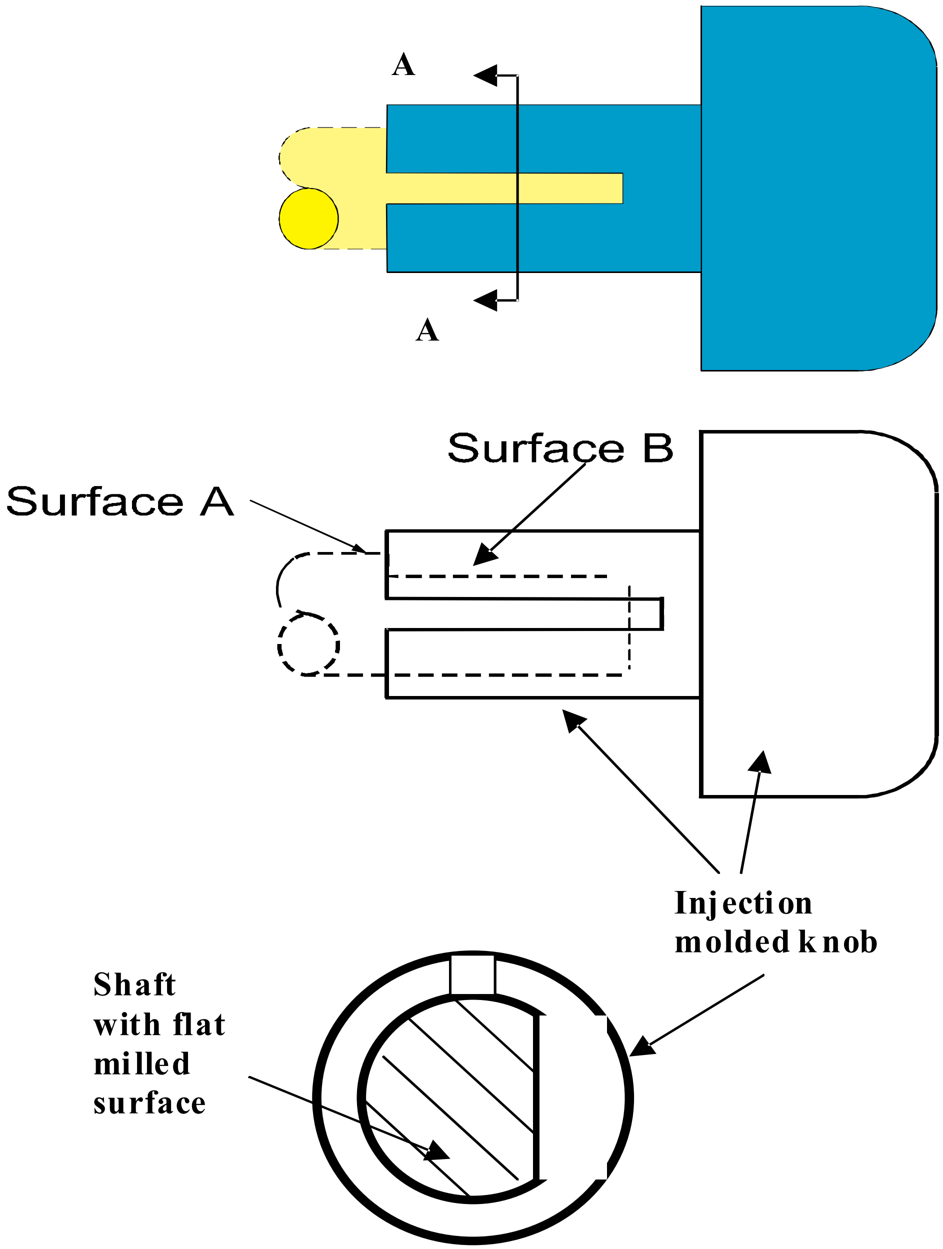
What are the FRs of a face seal that must isolate the lubricated section from the abrasives of the external environment?

There are many FRs.

They must be defined in a solution neutral environment.

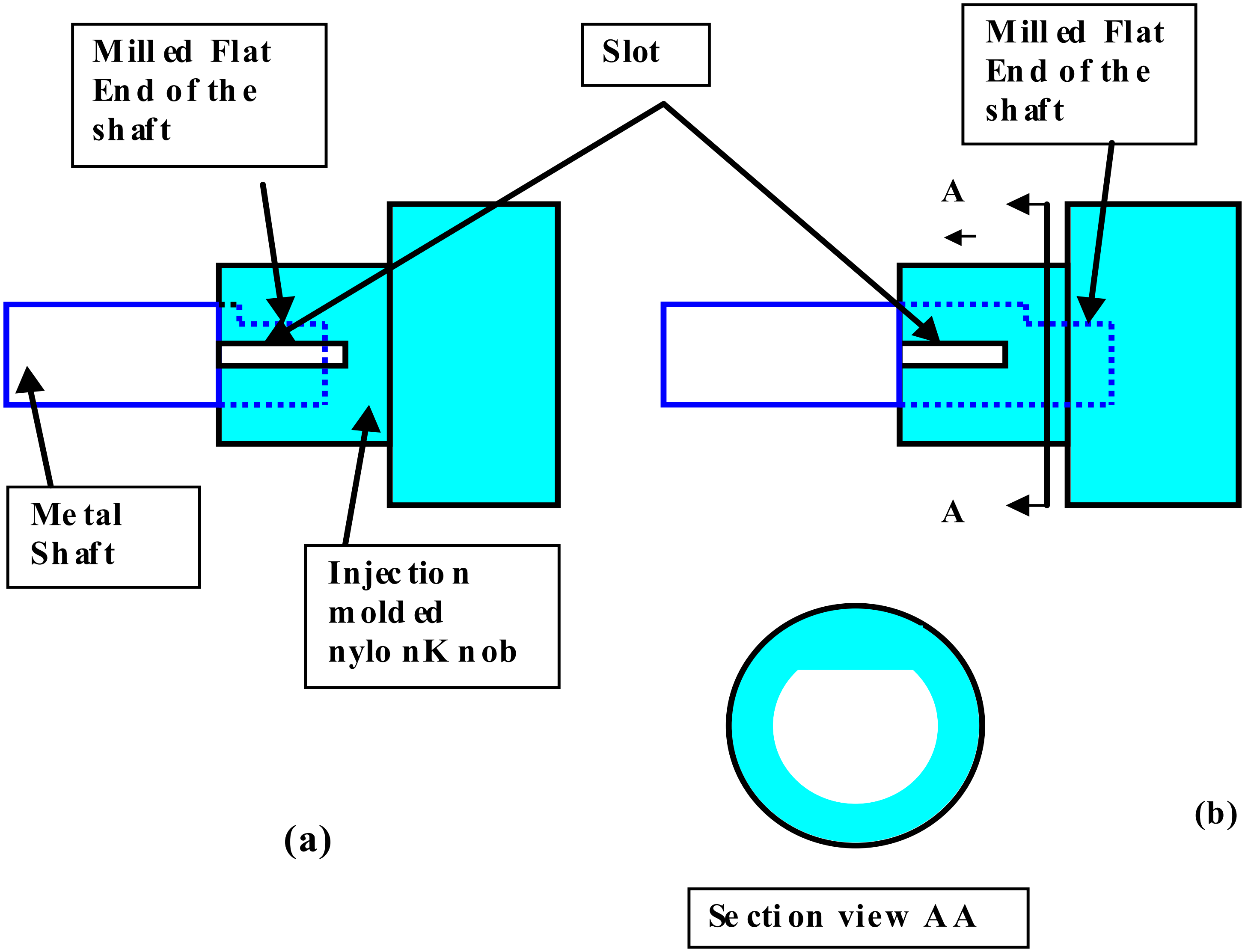
Is this knob a good design or a poor design?




A
A
Surface B
Surface A
Injection molded knob
Shaft with flat milled surface

Section A-A

Which is a better design?



Milled Flat End of the shaft

Slot

Milled Flat End of the shaft

A

A

Metal Shaft

Injection molded nylon Knob

(a)

(b)

Section view AA

History

Goal

To establish the *science base* for areas such as design and manufacturing

How do you establish science base in design?

Axiomatic approach

Algorithmic approach

References

N. P. Suh, *Axiomatic Design: Advances and Applications*. New York: Oxford University Press, 2001

N. P. Suh, *The Principles of Design*. New York: Oxford University Press, 1990

Axiomatic Design

Axiomatic Design applies to all designs:

- Hardware
- Software
- Materials
- Manufacturing
- Organizations

Axiomatic Design

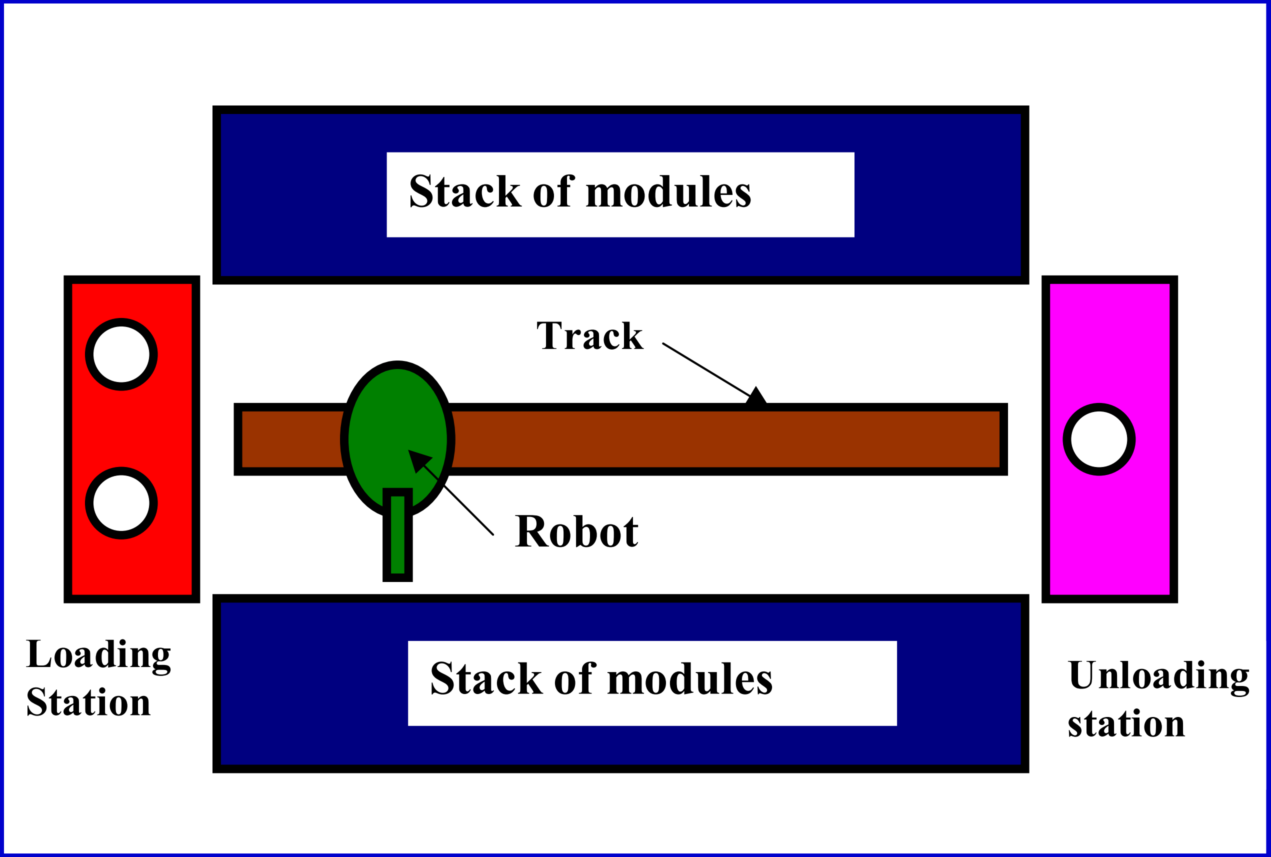
Axiomatic Design helps the design decision making process.

- Correct decisions
- Shorten lead time
- Improves the quality of products
- Deal with complex systems
- Simplify service and maintenance
- Enhances creativity

Axiomatic Design

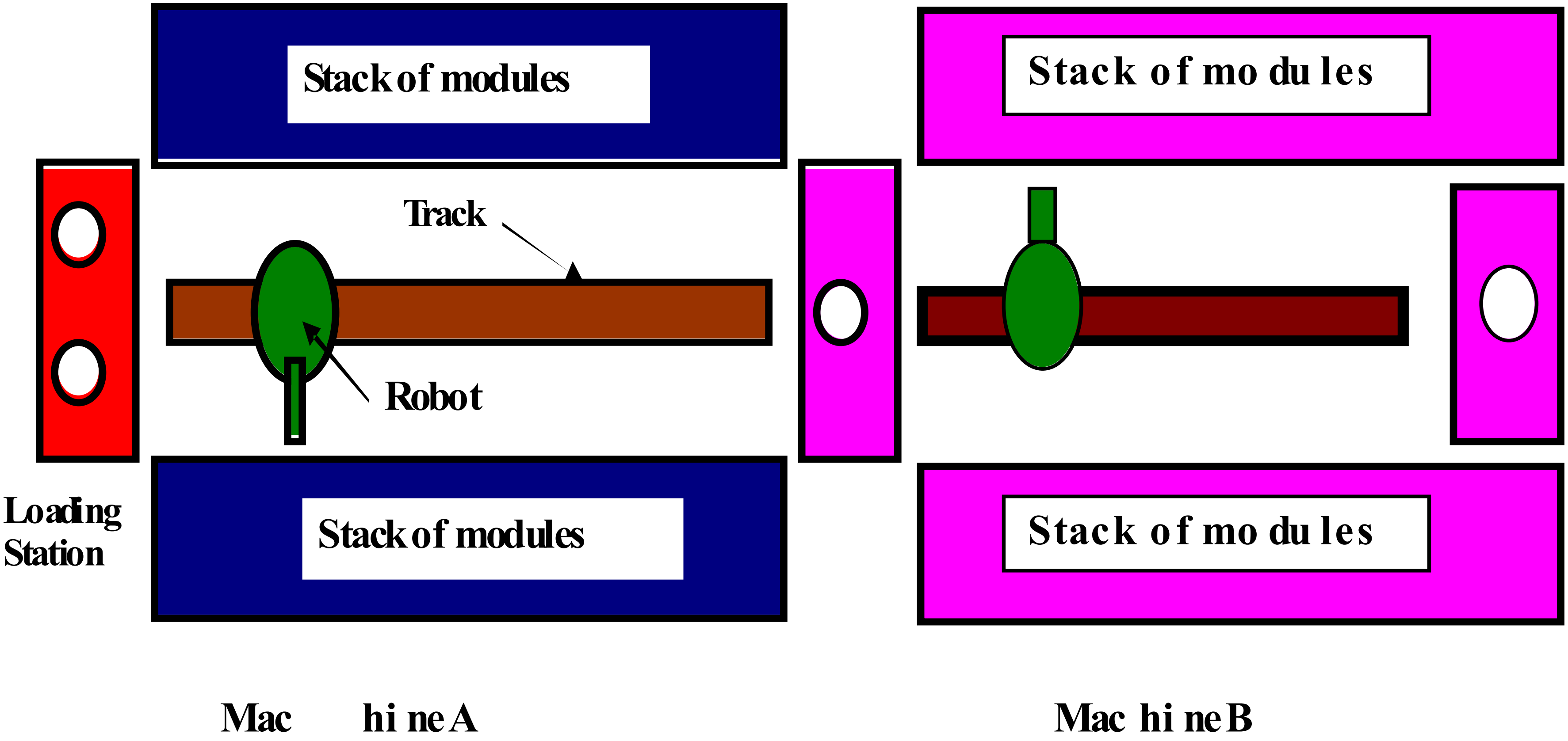
- Axioms
- Corollaries
- Theorems
- Applications -- hardware, software, manufacturing, materials, etc.
- System design
- Complexity

Introduction



Xerography machine design– See Example 9.2 in [Suh 2001].

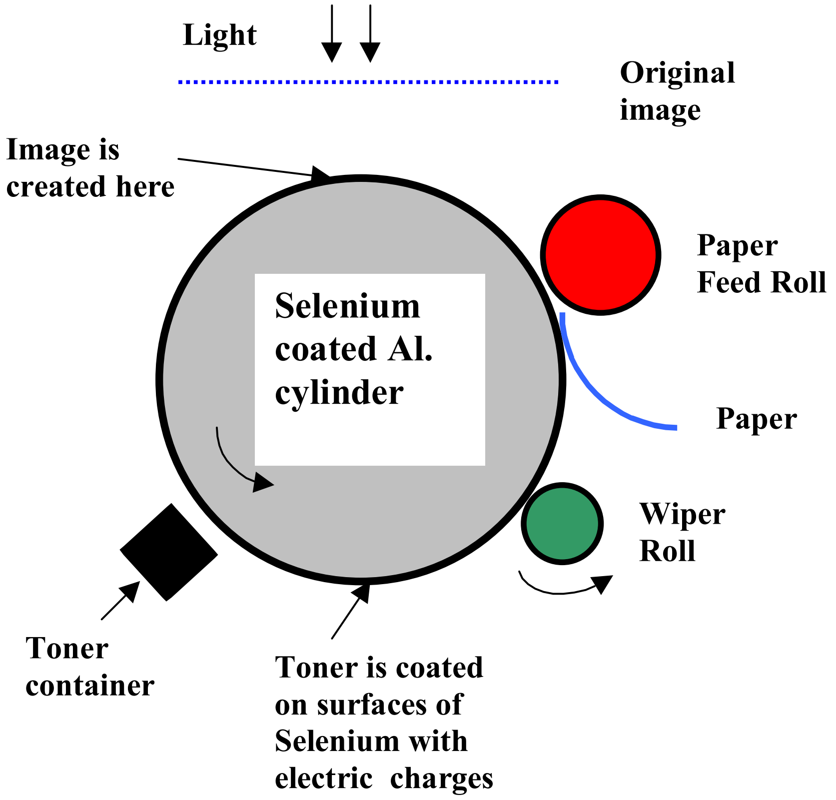
System integration



A cluster of two machines that are physically coupled to manufacture a part.

Introduction (cont'd)

Example1 Xerography-based Printing Machine



Schematic drawing of the xerography based printing machine.

Who are the Designers?
How do we design? What is design?

Is the mayor of Boston a designer?

Design Process

1. *Know* their *"customers' needs".*
2. *Define the problem* they must solve to satisfy the needs.
3. *Conceptualize the solution through synthesis*, which involves the task of satisfying several different functional requirements using a set of inputs such as product design parameters within given constraints.
4. Perform *analysis* to optimize the proposed solution.
5. *Check the resulting design solution* to see if it meets the original customer needs.

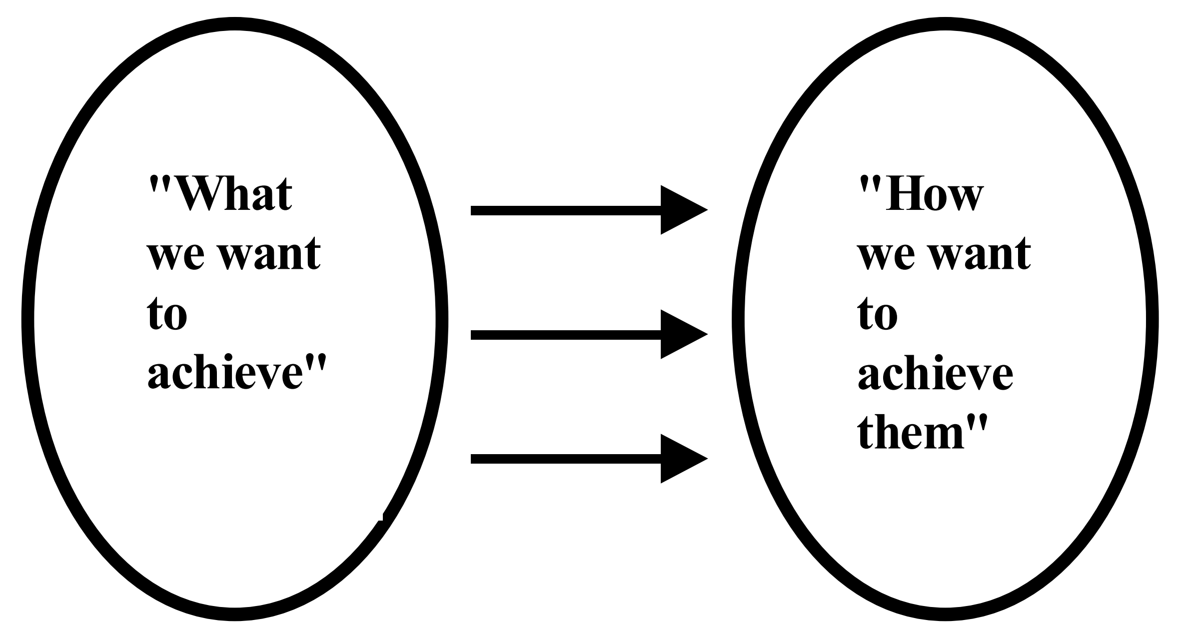
Definition of Design

Design is an interplay between

what we want to achieve and

how we want to achieve them.

Definition of Design



"What
we want
to
achieve"

"How
we want
to
achieve
them"

Example: Refrigerator Door Design

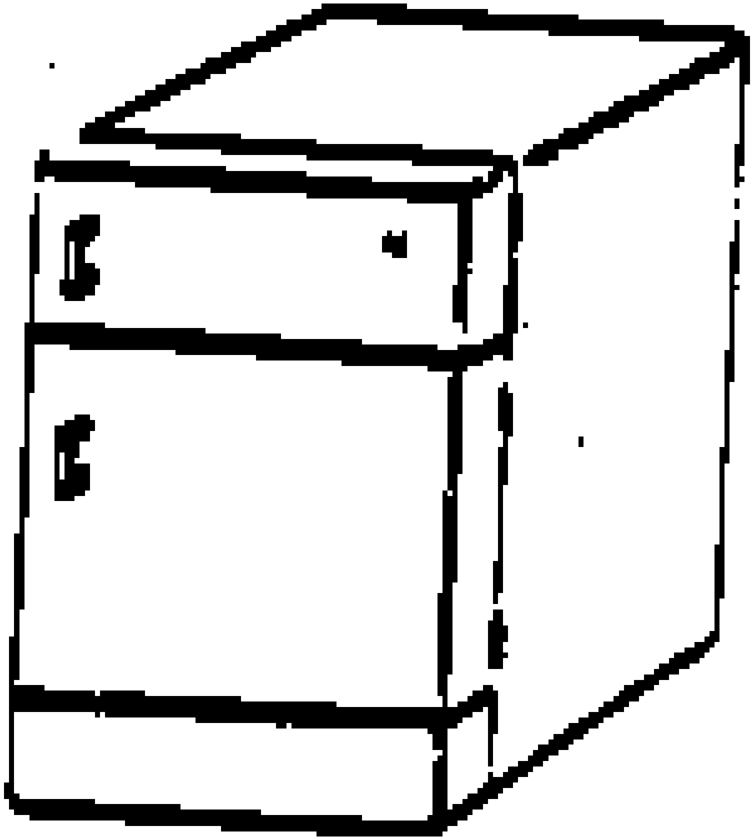


Figure ex.1.1.a   Vertically hung refrigerator door.

Ultimate Goal of Axiomatic Design

The ultimate goal of Axiomatic Design is to establish a science base for design and to improve design activities by providing the designer with a theoretical foundation based on logical and rational thought processes and tools.

Creativity and Axiomatic Design

Axiomatic design enhances creativity by eliminating bad ideas early and thus, helping to channel the effort of designers .

Historical Perspective on Axiomatic Design

Axioms are truths that cannot be derived but for which there are no counter-examples or exceptions.

Many fields of science and technology owe their advances to the development and existence of axioms.

(1) Euclid's geometry

(2) The first and second laws of thermodynamics are axioms

(3) Newtonian mechanics

Constraints

What are constraints?

Constraints provide the bounds on the acceptable design solutions and differ from the FRs in that they do not have to be independent.

There are two kinds of constraints:

input constraints
system constraints.

Example: Shaping of Hydraulic Tubes

To design a machine and a process that can achieve the task, the functional requirements can be formally stated as:

FR1= bend a titanium tube to prescribed curvatures

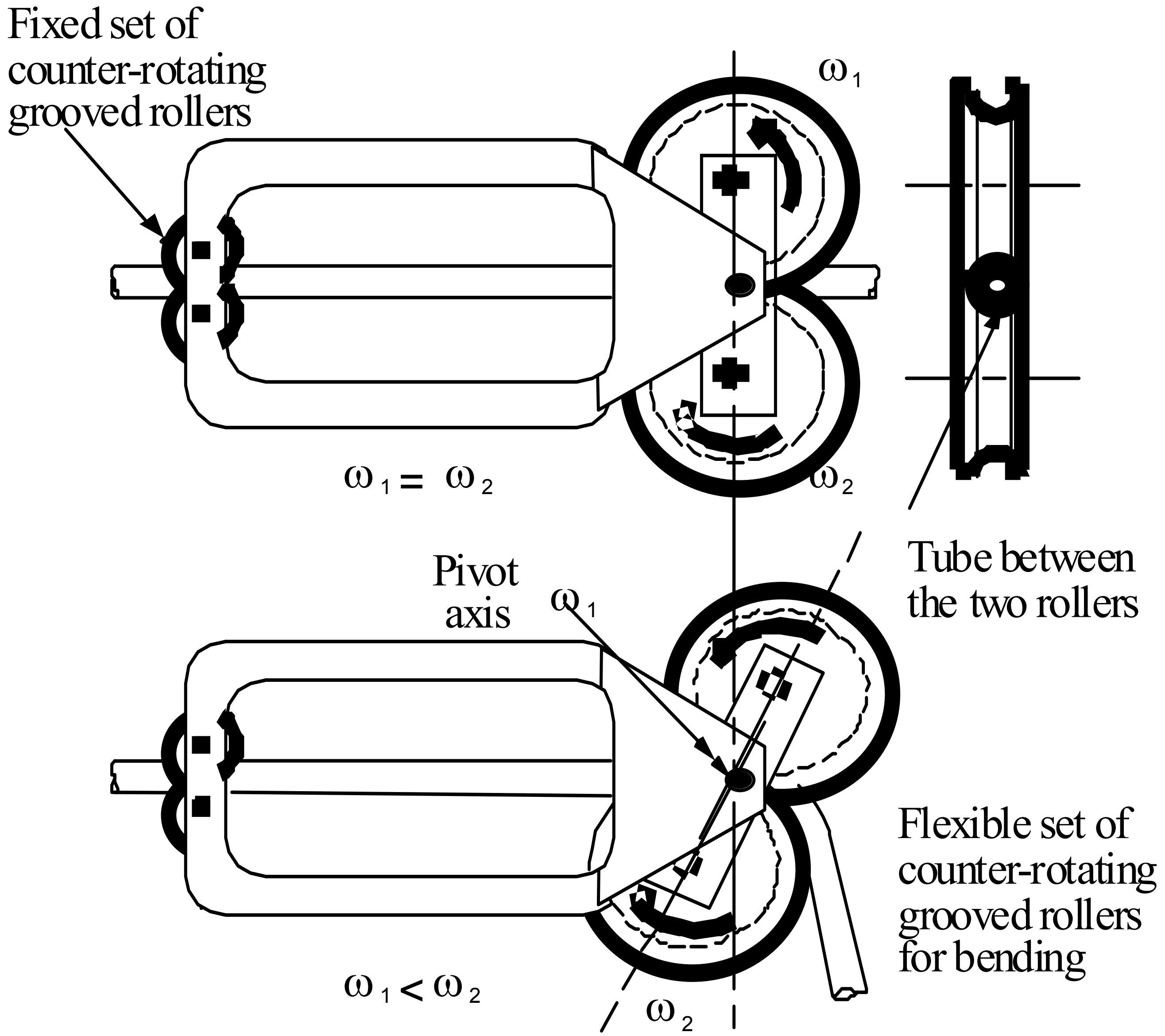
FR2= maintain the circular cross-section of the bent tube

Tube Bending Machine Design (cont’s)

Given that we have two FRs,

how many DPs do we need?

Example: Shaping of Hydraulic Tubes

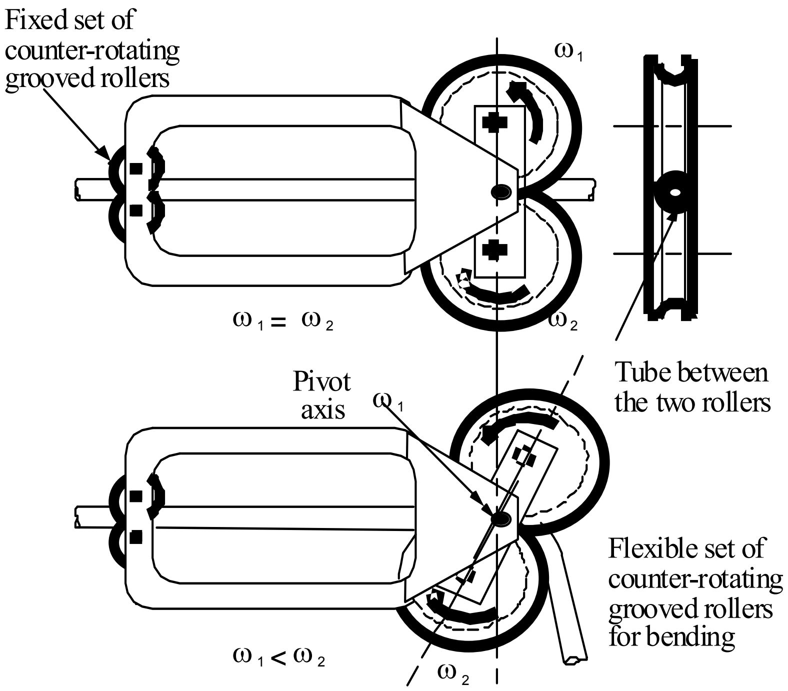


See Example 1.6 in [Suh 2001].

Example: Shaping of Hydraulic Tubes

DP1= Differential rotation of the bending rollers to bend the tube

DP2= The profile of the grooves on the periphery of the bending rollers



Tube bending apparatus

Example: Van Seat Assembly (Adopted from Oh, 1997)

See Example 2.6 in [Suh 2001].

Schematic drawing of a van seat that can be removed and installed easily using a pin/latch mechanism

Example: Van Seat Assembly

Solution

The FR of the seat engagement linkage is that the distance between the front leg and the rear latch when the seat engages the pins must be equal to the distance between the pins, which is 340 mm. The linkages [see Figures E2.6.b and c in Suh 2001] determine the FR = F. The following table shows the nominal lengths of the linkages.

Example: Van Seat Assembly

Traditional SPC Approach to Reliability and Quality

The traditional way of solving this kind of problem has been to do the following:

(a) ***Analyze the linkage to determine the sensitivity of the error.***

Table a Length of linkages and sensitivity analysis

Links	*Nominal Length (mm)*	*Sensitivity (mm/mm)*
L12	370.00	3.29
L14	41.43	3.74
L23	134.00	6.32
L24	334.86	1.48
L27	35.75	6.55
L37	162.00	5.94
L45	51.55	11.72
L46	33.50	10.17
L56	83.00	12.06
L67	334.70	3.71

New Manufacturing Paradigm – Robust Design

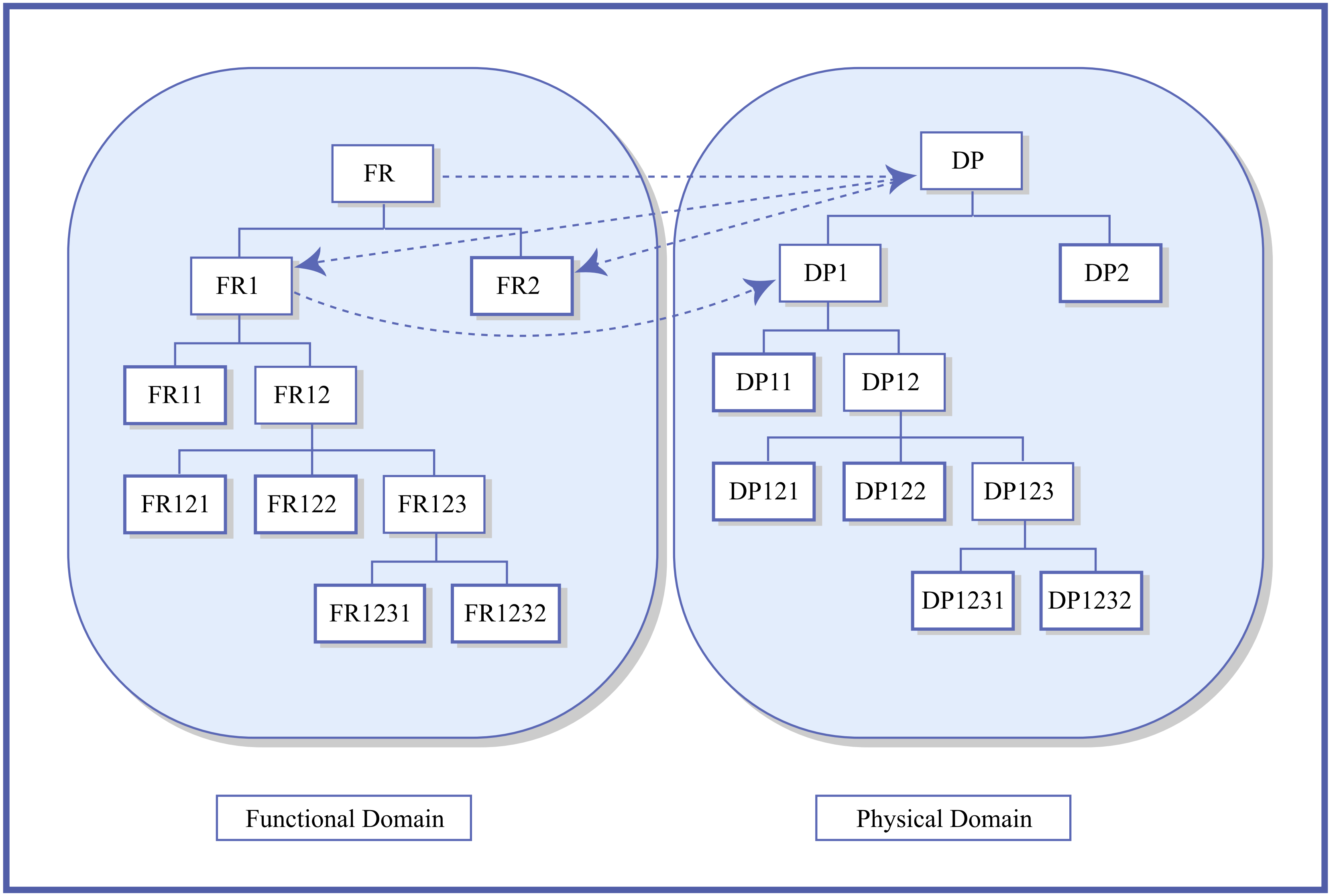
This design has one FR, i.e., F, the front to rear leg span . This is a function of 10 DPs, i.e., 10 linkages. This may be expressed mathematically as

$$F = f\left(DP^1, DP^2, \ldots DP^{10}\right) \qquad (e)$$

$$\delta F = \frac{\partial f}{\partial DP^x}\,\delta DP^x + \sum_{i=1,\, except\, i=x}^{10} \frac{\partial f}{\partial DP^i}\,\delta DP^i \qquad (f)$$

What we want to do is to make $\delta F=0$

Decomposition, Zigzagging and Hierarchy



Zigzagging to decompose in the functional and the physical domains and create the FR- and DP hierarchies

Figure by MIT OCW. After Figure 1.3 in [Suh 2001].

Identical Design and Equivalent Design

Equivalent Design:

When two different designs satisfy the same set of the highest-level FRs but have different hierarchical architecture, the designs are defined to be equivalent designs.

Identical Design:

When two different designs satisfy the same set of FRs and have the identical design architecture, the designs are defined to be identical designs.

Example: Refrigerator Design

FR1 = Freeze food for long-term preservation

FR2 = Maintain food at cold temperature for short-term preservation

To satisfy these two FRs, a refrigerator with two compartments is designed. Two DPs for this refrigerator may be stated as:

DP1 = The freezer section

DP2 = The chiller (i.e., refrigerator) section.

Example: Refrigerator Design

FR1 = Freeze food for long-term preservation

FR2 = Maintain food at cold temperature for short-term preservation

DP1 = The freezer section

DP2 = The chiller (i.e., refrigerator) section.

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Example: Refrigerator Design

Having chosen the DP1, we can now decompose FR1 as:

FR11 = Control temperature of the freezer section in the range of -18 C +/- 2 C

FR12 = Maintain the uniform temperature throughout the freezer section at the preset temperature

FR13 = Control humidity of the freezer section to relative humidity of 50%

Example: Refrigerator Design

FR11 = Control temperature of the freezer section in the range of -18 C +/- 2 C
FR12 = Maintain the uniform temperature throughout the freezer section at the preset temperature
FR13 = Control humidity of the freezer section to relative humidity of 50%

DP11 = Sensor/compressor system that turn on and off the compressor when the air temperature is higher and lower than the set temperature in the freezer section, respectively.

DP12 = Air circulation system that blows air into the freezer section and circulate it uniformly throughout the freezer section at all times

DP13 = Condenser that condenses the moisture in the returned air when its dew point is exceeded

Example: Refrigerator Design

Similarly, based on the choice of DP2 made, FR2 may be decomposed as:

FR21 = Control the temperature of the chilled section in the range of 2 to 3 C

FR22 = Maintain a uniform temperature throughout the chilled section within 1 C of a preset temperature

Example: Refrigerator Design

FR21 = Control the temperature of the chilled section in the range of 2 to 3 C

FR22 = Maintain a uniform temperature throughout the chilled section within 1 C of a preset temperature

DP11 = Sensor/compressor system that turn on and off the compressor when the air temperature is higher and lower than the set temperature in the chiller section, respectively.

DP12 = Air circulation system that blows air into the freezer section and circulate it uniformly throughout the freezer section at all times

Example: Refrigerator Design

Figures removed for copyright reasons.
See Example 1.7 in [Suh 2001].

Example: Refrigerator Design

The design equation may be written as:

$$\begin{Bmatrix} FR12 \\ FR11 \\ FR13 \end{Bmatrix} = \begin{bmatrix} XOO \\ XXO \\ XOX \end{bmatrix} \begin{Bmatrix} DP12 \\ DP11 \\ DP13 \end{Bmatrix}$$

Equation (a) indicates that the design is a decoupled design.

	DP22	DP21
FR22	X	0
FR21	X	X

Full DM of Uncoupled Refrigerator Design

	DP1			DP2		
		DP12	DP11	DP13	DP22	DP21
FR1	FR21	X	0	0	0	0
	FR11	X	X	0	0	0
	FR13	X	0	X	0	0
FR2	FR22	0	0	0	X	0
	FR21	0	0	0	X	X

Full DM of Uncoupled Refrigerator Design

		DP1			DP2	
		DP12	DP11	DP13	DP22	DP21
FR1	FR12	X	0	0	0	0
	FR11	X	X	0	0	0
	FR13	X	0	X	0	0
FR2	FR22	X	0	0	0	0
	FR21	0	0	0	X	0/X

Analysis

When do we perform analysis during the design process?

Requirements for Concurrent Engineering

	[A]	**[B]**	**[C] = [A] {B]**
1. Both diagonal	[\ ]	[\ ]	[\]
2. Diag x Full	[\ ]	[X]	[X]
3. Diag x triang.	[\ ]	[LT]	[LT]
4. Tria. x Triang	[LT]	[LT]	[LT]
5. Tria. x Triang	[LT]	[UT]	[X]
6. Full x Full	[X]	[X]	[X]

Table 1.3 The characteristic of concurrent engineering matrix [C].

Ideal Design, Redundant Design, and Coupled Design - A Matter of Relative Numbers of DPs and FRs

Depending on the relative numbers of DPs and FRs the design can be classified as coupled, redundant and ideal designs.

Case 1: Number of DPs < Number of FRs: Coupled Design

When the number of design parameters is less than the number of functional requirements, we always have a coupled design. This is stated as Theorem 1.

Case 2: Number of DPs > Number of FRs: Redundant Design

When there are more design parameters than the functional requirements, the design is called a redundant design. A redundant design may or may not violate the Independence Axiom.

Ideal Design, Redundant Design, and Coupled Design - A Matter of Relative Numbers of DPs and FRs

Consider the following two dimensional redundant design:

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & A13 & A14 & A15 \\ A21 & A22 & 0 & A24 & 0 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \\ DP4 \\ DP5 \end{Bmatrix}$$

(Theorem 3 in Appendix 1-A)

Case 3: Number of DPs = Number of FRs: Ideal Design

When the number of FRs and DPs are the same, the design is an ideal design, provided that the Independence Axiom is satisfied.

(Theorem 4 in Appendix 1-A)

The Second Axiom: The Information Axiom

Axiom 2: The Information Axiom

Minimize the information content.

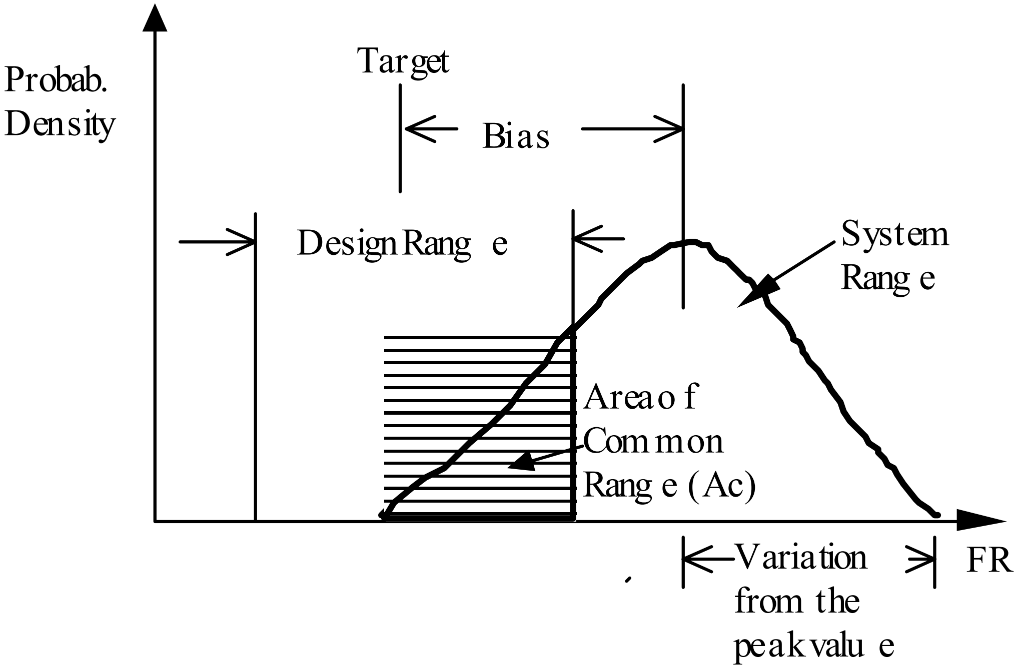
Information content I is defined in terms of the probability of satisfying a given FR.

$$I = \log_2 \frac{1}{P} = -\log_2 P$$

In the general case of n FRs for an uncoupled design, I may be expressed as

$$I = \sum_{i=1}^{n} \log_2 \frac{1}{P_i} = -\sum_{i=1}^{n} \log_2 P_i$$

Design Range, System Range, and Common Range



Design Range, System Range, and Common Range in a plot of the probability density function (pdf) of a functional requirement. The deviation from the mean is equal to the square root of the variance. The design range is assumed to have a uniform probability distribution in determining the common range.

Measure of Information Content in Real Systems

The probability of success can be computed by specifying the *Design Range (dr)* for the FR and by determining the *System Range (sr)* that the proposed design can provide to satisfy the FR.

$$I = \log_2 \frac{A_{sr}}{A_{cr}} \tag{1.9}$$

where A_{sr} denotes the area under the System Range and A_{cr} is the area of the Common Range.

Furthermore, since A_{sr} = 1.0 in most cases (since the total area of the probability distribution function is equal to the total probability, which is one) and there are n FRs to satisfy, the information content may be expressed as

$$I = \sum_{i=1}^{n} \log_2 \frac{1}{A_{cr}} \tag{1.10}$$

Example: Buying a House

FR1 = Commuting time for Prof. Wade must be in the range of 15 to 30 minutes.

FR2 = The quality of the high school must be good, i.e., more than 65 % of the high school graduates must go to reputable colleges.

FR3 = The quality of air must be good, i.e., the air quality must be good over 340 days a year.

FR4 = The price of the house must be reasonable, i.e., a four bed room house with 3,000 square feet of heated space must be less than $650,000.

Example: Buying a House

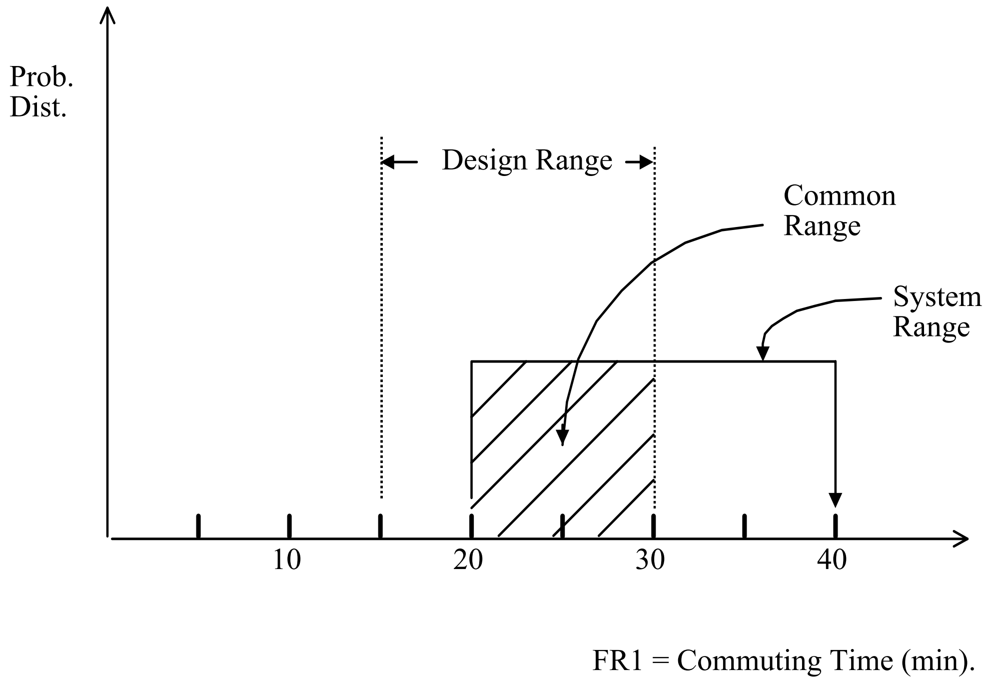
They looked around towns A, B, C and collected the following data:

Town	FR1=Comm. time [min]	FR2=Quality of school [%]	FR3=Quality of air [days]	FR4=Price [$]
A	20 to 40	50 to 70	300 to 320	450k to 550k
B	20 to 30	50 to 75	340 to 350	$450k to 650k
C	25 to 45	50 to 80	350 and up	$600k to 800k

Which is the town that meets the requirements of the Wade family the best? You may assume uniform probability distributions for all FRs.

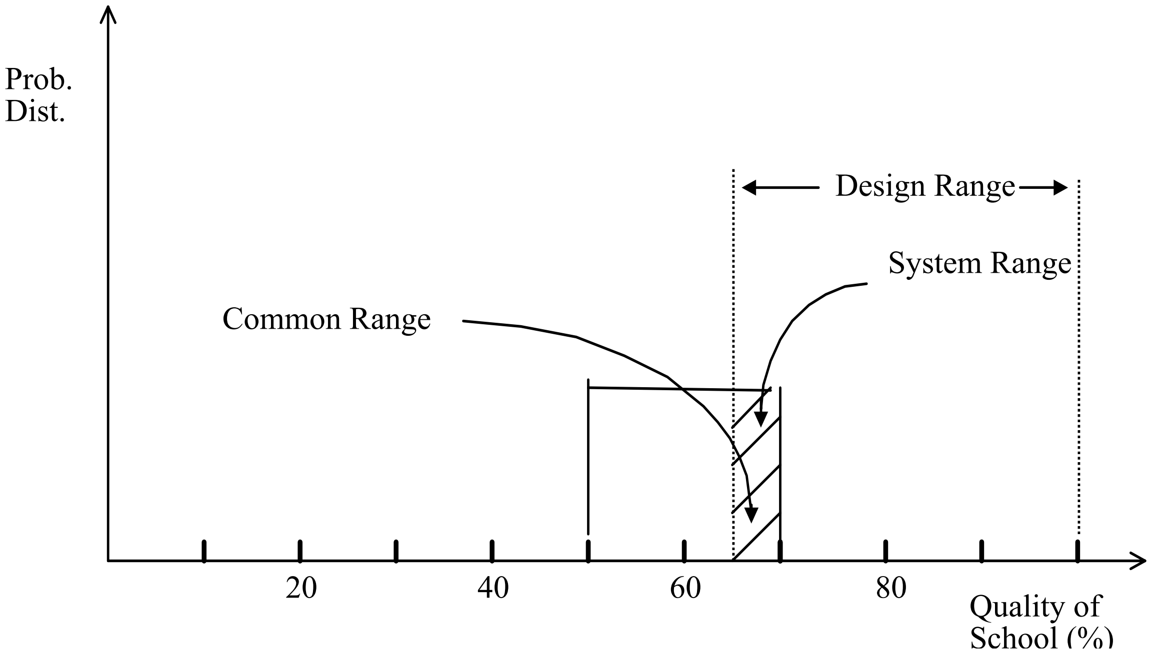
Example: Buying a House

Solution



Probability distribution of commuting time

Example: Buying a House



Probability distribution of the quality of schools

Example: Buying a House

The information content of Town A is infinite since it cannot satisfy FR3, i.e., the design range and the system range do not overlap at all. The information contents of Towns B and C are computed using Eq. (1.8) as follows:

Town	I_1 [bits]	I_2 [bits]	I_3 [bits]	I_4 [bits]	ΣI [bits]
A	1.0	2.0	Infinite	0	Infinite
B	0	1.32	0	0	1.32
C	2.0	1.0	0	2.0	5.0

1.8 Common Mistakes Made by Designers

i. Coupling Due to Insufficient Number of DPs (Theorem 1)

ii. Not Recognizing a Decoupled Design

iii. Having more DPs than the number of FRs

iv. Not creating a robust design -- not minimizing the information content through elimination of bias and reduction of variance

v. Concentrating on Symptoms rather than Cause -- Importance of Establishing and Concentrating on FR.