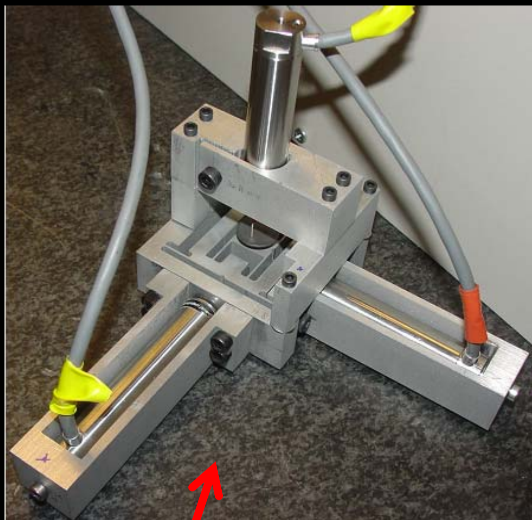
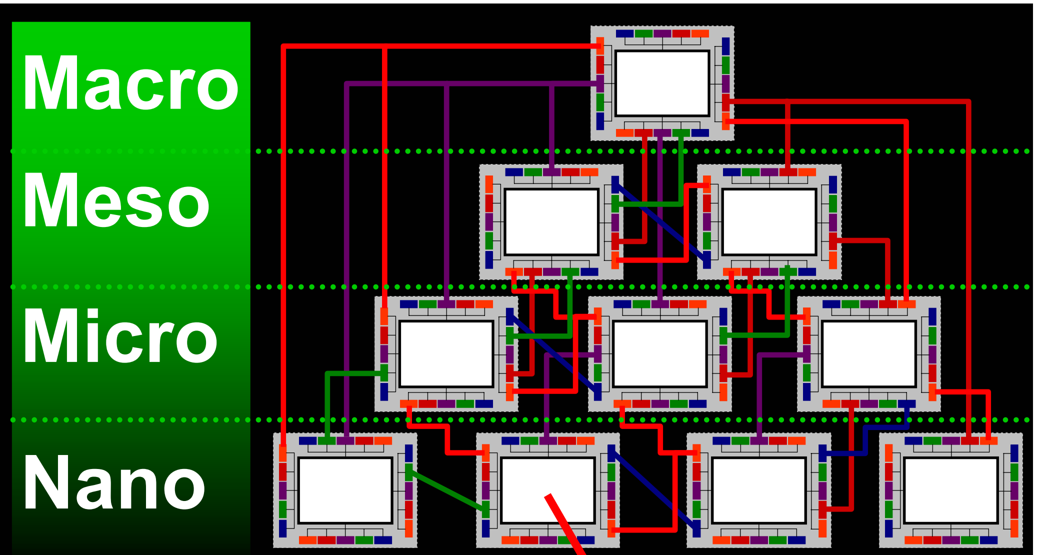
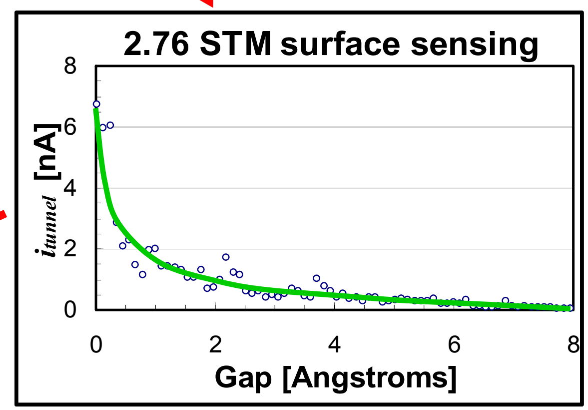
2.76 / 2.760 Lecture 10: Error modeling





$$\vec{e}_{system} = \vec{O}_{error} - \vec{O}_{nominal}$$

$$\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} \leftarrow \begin{vmatrix} f_{11}\left(SR\frac{Macro}{Macro}\right) & f_{12}\left(SR\frac{Meso}{Macro}\right) & f_{13}\left(SR\frac{Micro}{Macro}\right) & f_{14}\left(SR\frac{Nano}{Macro}\right) \\ f_{21}\left(SR\frac{Macro}{Meso}\right) & f_{22}\left(SR\frac{Meso}{Meso}\right) & f_{23}\left(SR\frac{Micro}{Meso}\right) & f_{24}\left(SR\frac{Nano}{Meso}\right) \\ f_{31}\left(SR\frac{Macro}{Micro}\right) & f_{32}\left(SR\frac{Meso}{Micro}\right) & f_{33}\left(SR\frac{Micro}{Micro}\right) & f_{34}\left(SR\frac{Nano}{Micro}\right) \\ f_{41}\left(SR\frac{Macro}{Nano}\right) & f_{42}\left(SR\frac{Meso}{Nano}\right) & f_{43}\left(SR\frac{Micro}{Nano}\right) & f_{44}\left(SR\frac{Nano}{Nano}\right) \end{vmatrix} \cdot \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix}$$



Announcements

Grades….

- I am behind, apologies…

Literature critique

- Posted: First come, First serve
- WAIT UNTIL AFTER CLASS!!!!!

Questions

1. How did the STM reading go?

2. What do you perceive as being the most helpful way we can (outside of lecture) help your group get started on the design?
   - ❑ Meetings
   - ❑ Recitation
   - ❑ Etc…

3. What would you like to hear about from Thursday's lecture?

Where are we at, where to go now…

Sept.

Oct.

Nov.

Dec.

Design
- Perception
- Approach

Model
- Components
- Interfaces
- System
- Examples

Project
- Model
- Design
- Integration
- Validation
- Characterize

PSets
- 3 p. max!
- Schedule
- Risk
- Mitigation

Purpose of today

Connect qualitative analysis/view with component-level view

Error models link components and system behavior

Error budgets set limits on system and component errors

Types of errors in systems

Principle of determinism

Systems transform inputs into output

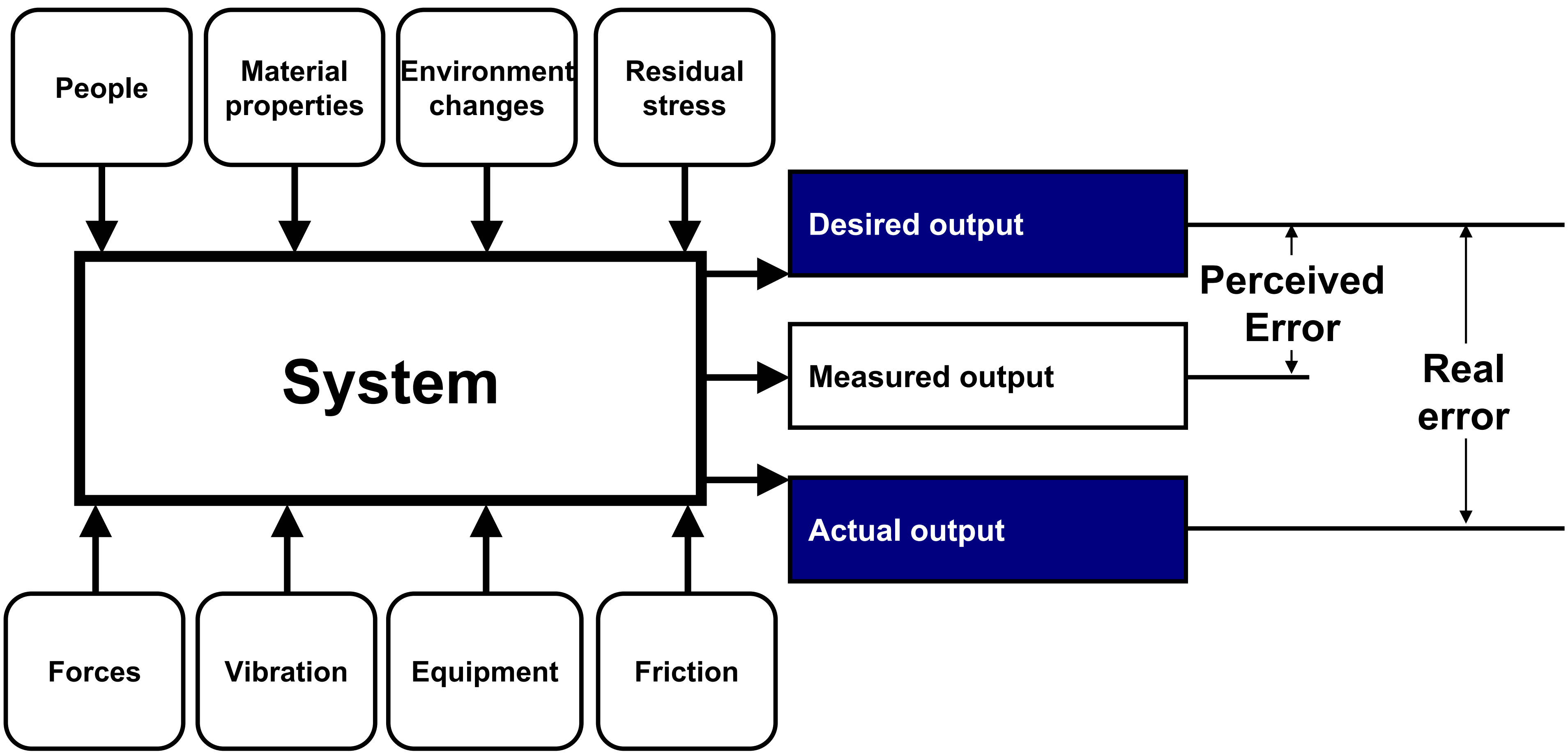
Desire a one to one relationship between inputs/outputs

Deterministic relationship = one relationship

Closed form modeling is then possible!

System design principles

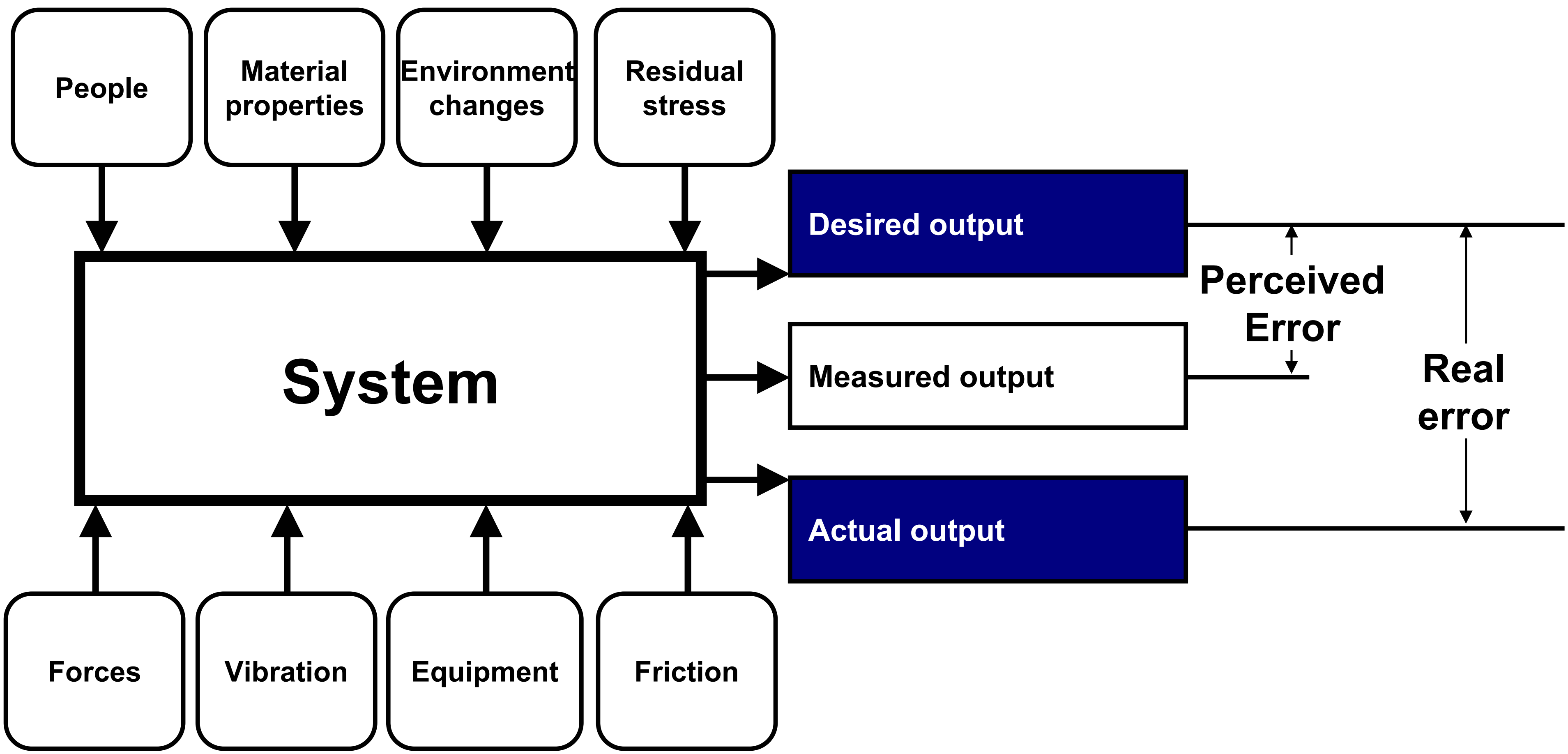
Inputs
System (Deterministic!!!)
Outputs



The strategy for dealing with errors

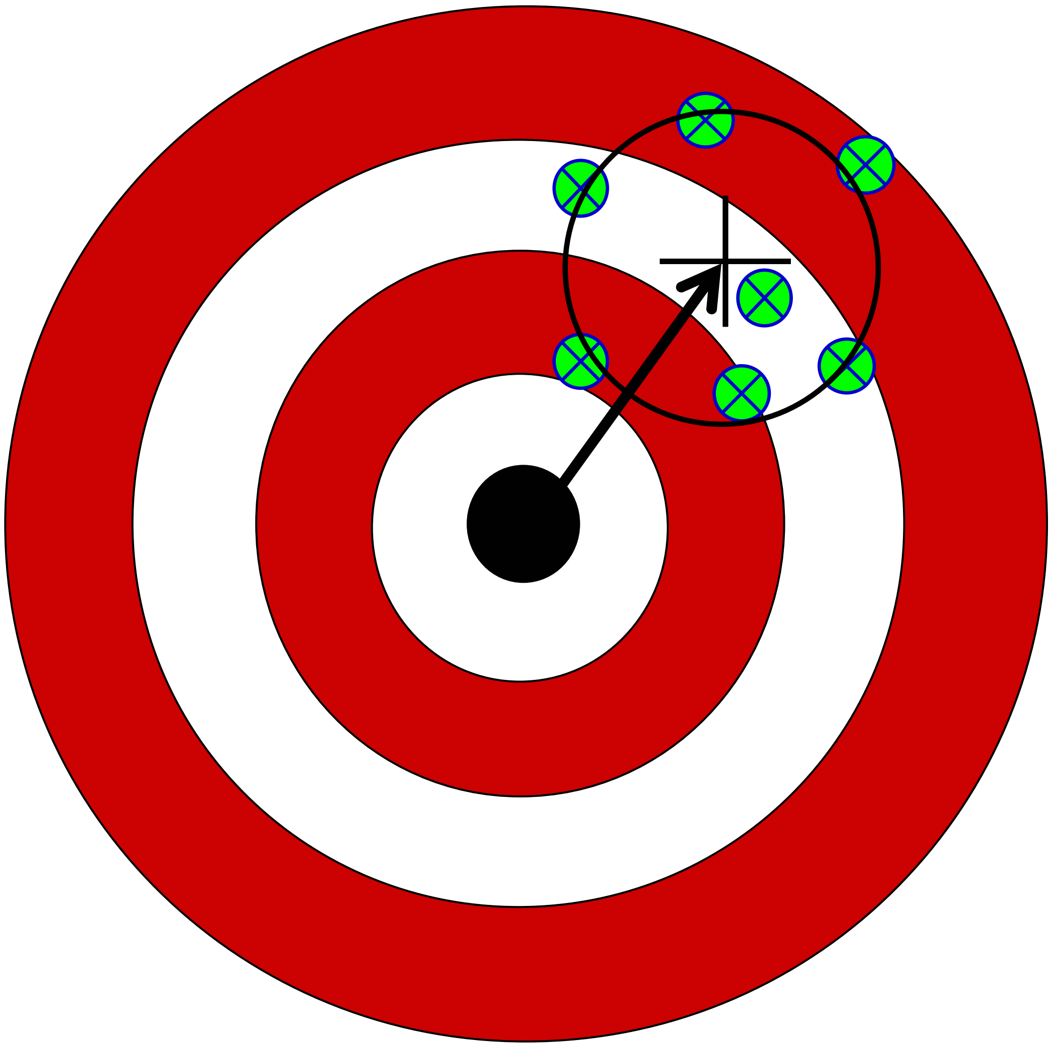
What is the nature of cause/effect?

- ❑ Deterministic: Is there 1 output for a set of given inputs
- ❑ Sensitivity: $\frac{d \text{ output}}{d \text{ error input}} = ?$



Principle of accuracy

How well you achieve the goal

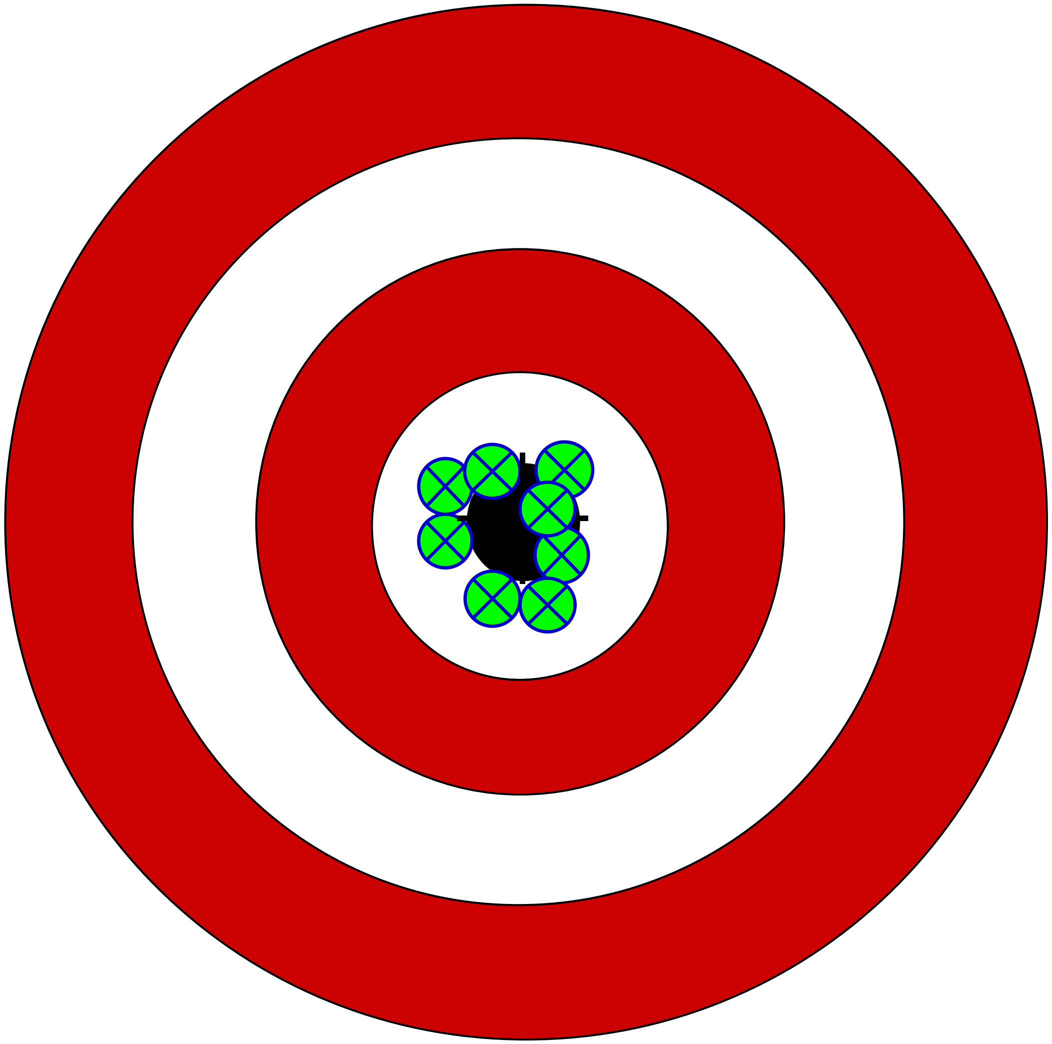


Principle of repeatability

- How well you can perform same function a multiplicity of times

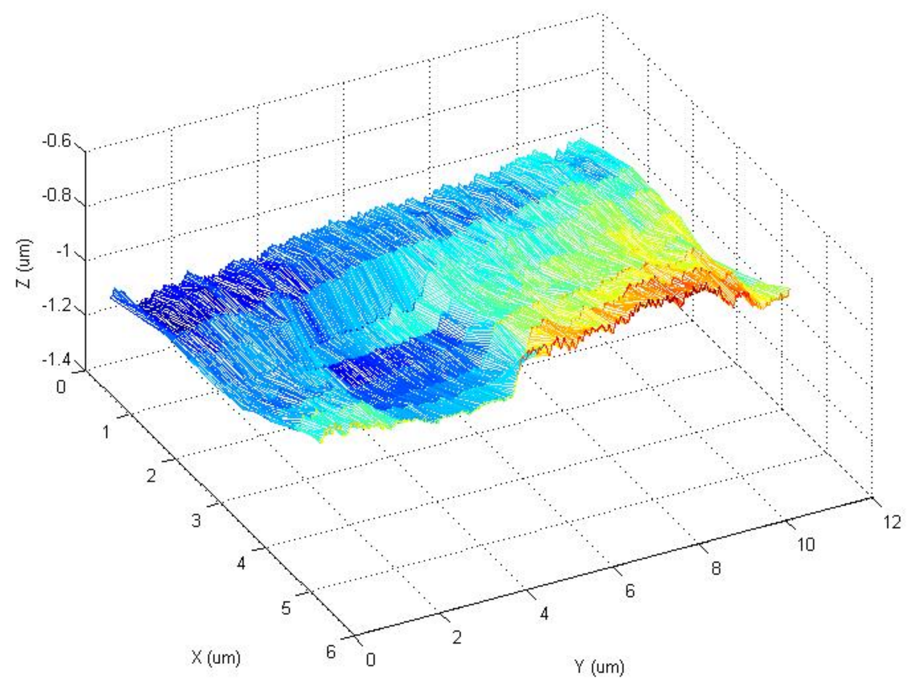
Accuracy and repeatability

Ideal situation, accuracy and repeatability



What are most concerned with in the STM?

Example



$$\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} \leftarrow \begin{vmatrix} f_{11}\left(SR_{\frac{Macro}{Macro}}\right) & f_{12}\left(SR_{\frac{Meso}{Macro}}\right) & f_{13}\left(SR_{\frac{Micro}{Macro}}\right) & f_{14}\left(SR_{\frac{Nano}{Macro}}\right) \\ f_{21}\left(SR_{\frac{Macro}{Meso}}\right) & f_{22}\left(SR_{\frac{Meso}{Meso}}\right) & f_{23}\left(SR_{\frac{Micro}{Meso}}\right) & f_{24}\left(SR_{\frac{Nano}{Meso}}\right) \\ f_{31}\left(SR_{\frac{Macro}{Micro}}\right) & f_{32}\left(SR_{\frac{Meso}{Micro}}\right) & f_{33}\left(SR_{\frac{Micro}{Micro}}\right) & f_{34}\left(SR_{\frac{Nano}{Micro}}\right) \\ f_{41}\left(SR_{\frac{Macro}{Nano}}\right) & f_{42}\left(SR_{\frac{Meso}{Nano}}\right) & f_{43}\left(SR_{\frac{Micro}{Nano}}\right) & f_{44}\left(SR_{\frac{Nano}{Nano}}\right) \end{vmatrix} \cdot \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix}$$

Nature of errors in systems

Excercise

In-class example

5 people measure

Tabulate measurements, one for each end

Will call on you in a minute

Nature and type of errors

Systematic errors

- Repeatable errors which are inherent to the system
- These errors are always present

Random errors

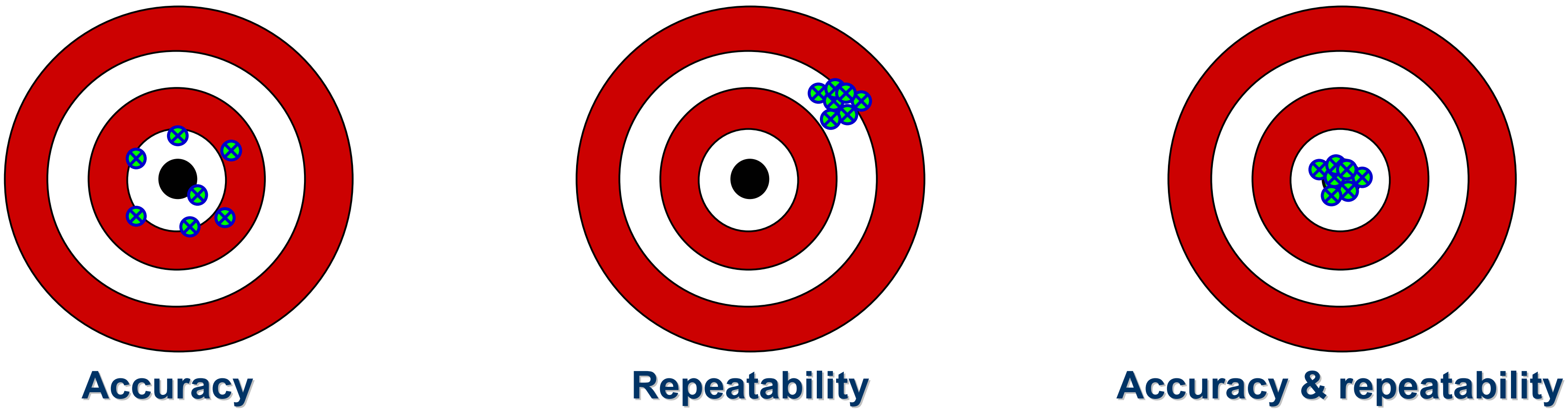
- Errors in a given system which are perceived to have a statistical nature to them

Error sources

- Thermal
- Compliance
- Manufacturing
- Etc..

Goal of system design

Apply physics to design systems to achieve?



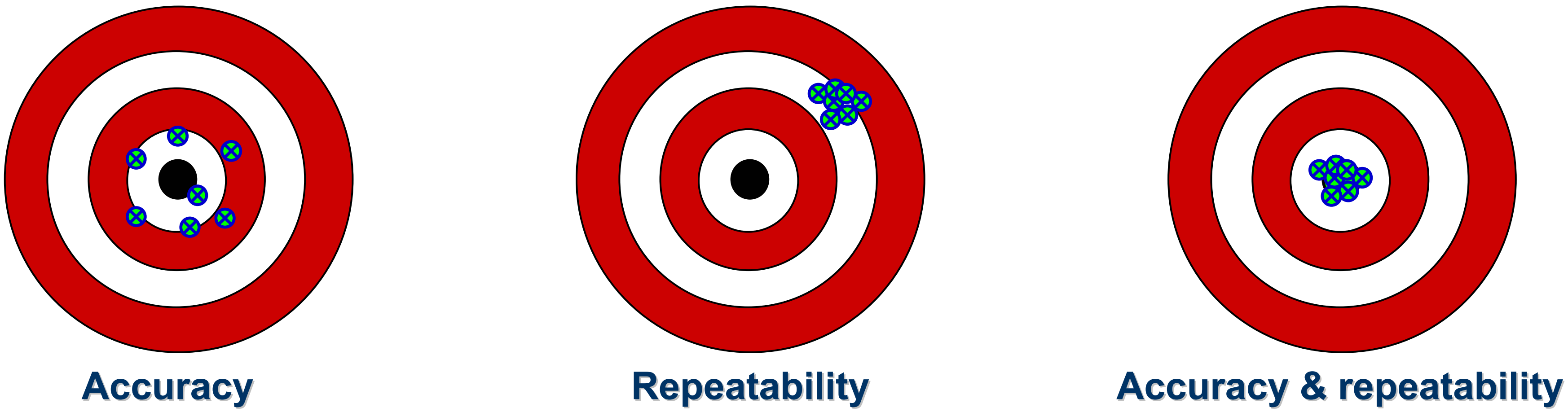
Error management is key. Types of errors

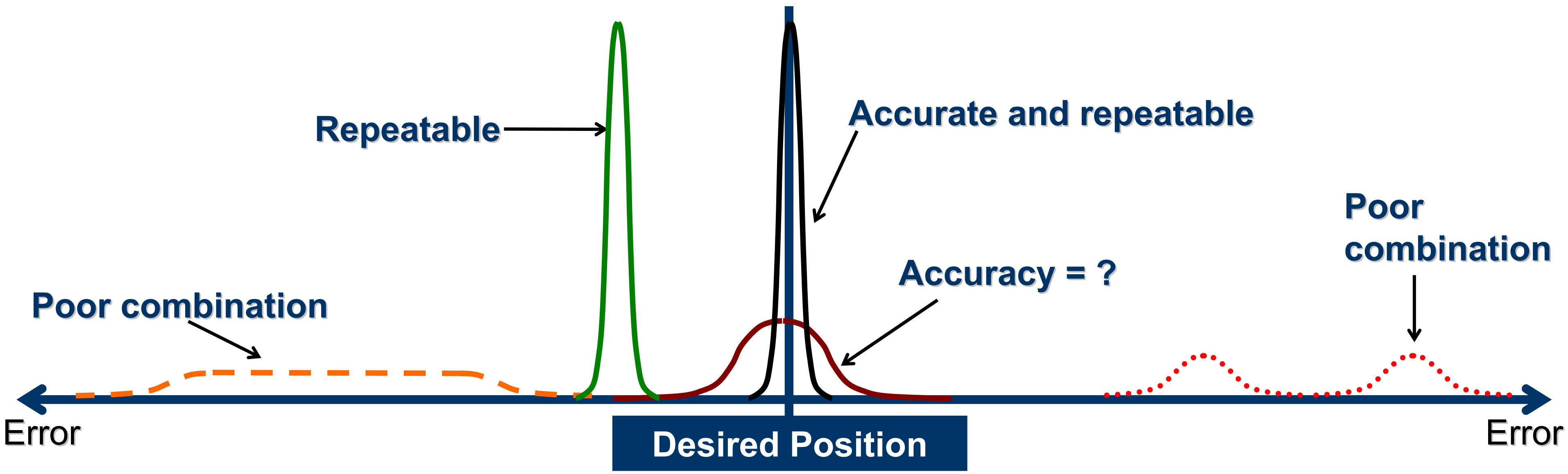
- ❑Random: Non-repeatable errors, no good way to model
- ❑Systematic: Repeatable, can be mapped/calibrated/corrected

Accuracy only as good as repeatability

Experimentation is not a bad thing, often necessary (e.g. random errors)!!!

Random nature of errors

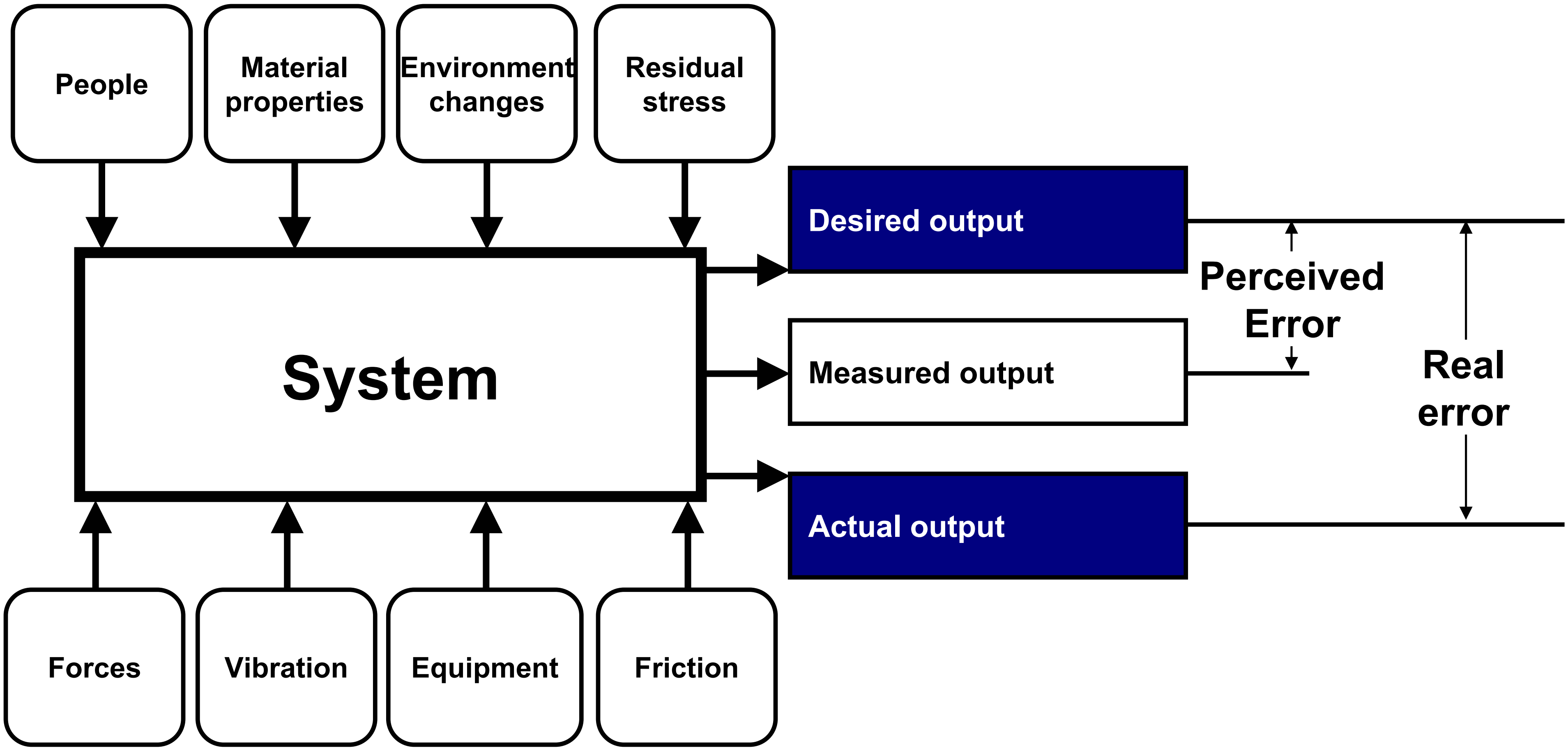




Common sources of errors

Common error sources

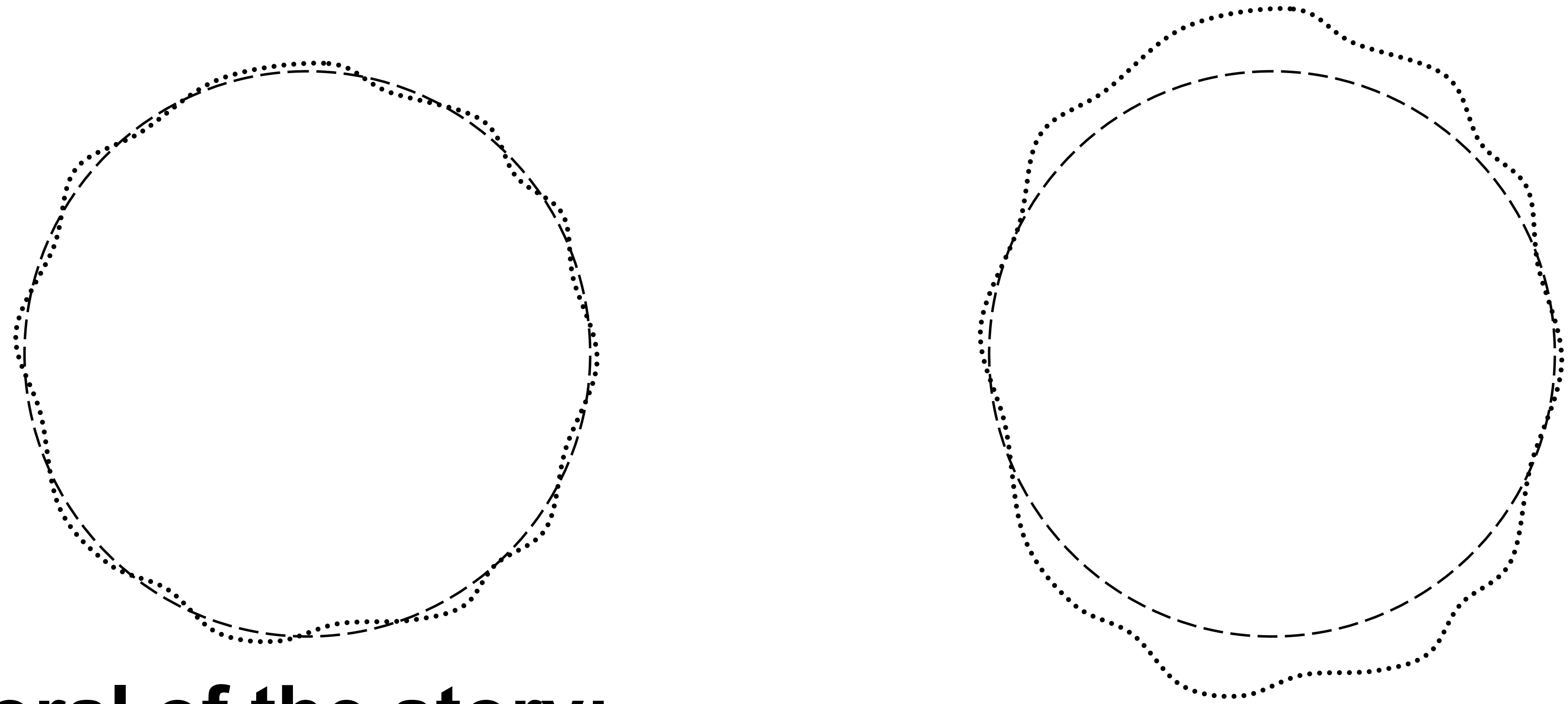
- Contacts Thermal Load Wear
- Vibration Measurements Constraint Stress
- Gravity Friction Others…..



Unexpected error sources

If you can't measure it, you can't make it

Measure the "right thing"… Example:



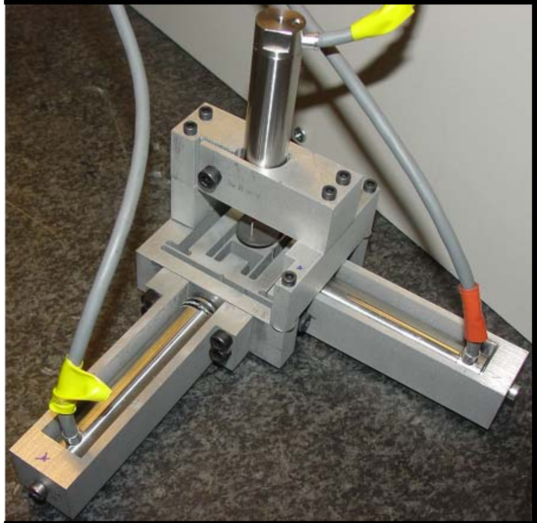
Moral of the story:

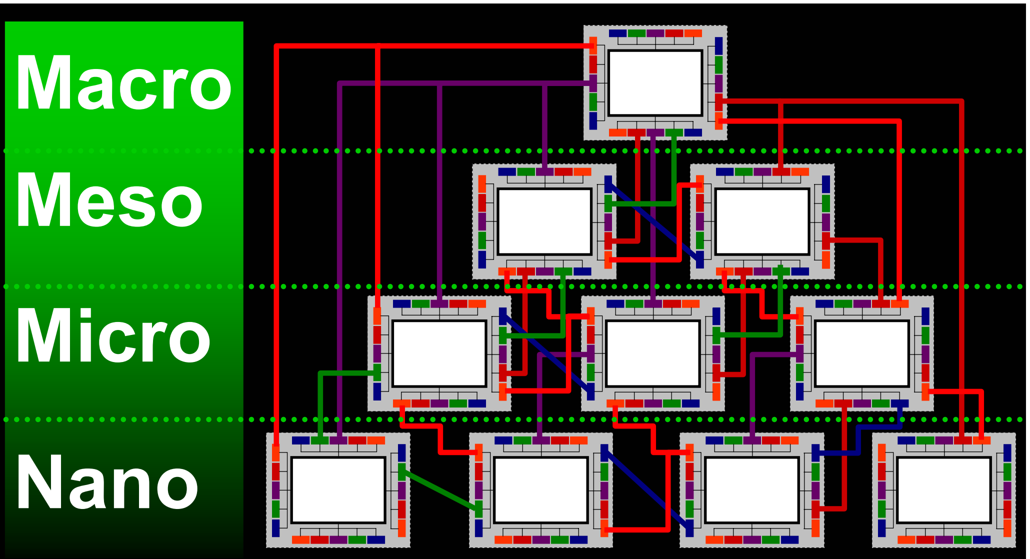
- People are lousy components of measurement systems
- Measurement gurus…

Error budgets

Error budgets 101

1. What are they needed?
2. Why are they needed?
3. How do you do predict errors?
4. How does this apply to project?





System modeling fundamentals

$$O_{MuSS} = G(SR) \cdot I_{MuSS}$$

$$\begin{vmatrix} O_{Macro} \\ O_{Meso} \\ O_{Micro} \\ O_{Nano} \end{vmatrix} = \begin{vmatrix} f_{11}\left(SR_{\frac{Macro}{Macro}}\right) & f_{12}\left(SR_{\frac{Meso}{Macro}}\right) & f_{13}\left(SR_{\frac{Micro}{Macro}}\right) & f_{14}\left(SR_{\frac{Nano}{Macro}}\right) \\ f_{21}\left(SR_{\frac{Macro}{Meso}}\right) & f_{22}\left(SR_{\frac{Meso}{Meso}}\right) & f_{23}\left(SR_{\frac{Micro}{Meso}}\right) & f_{24}\left(SR_{\frac{Nano}{Meso}}\right) \\ f_{31}\left(SR_{\frac{Macro}{Micro}}\right) & f_{32}\left(SR_{\frac{Meso}{Micro}}\right) & f_{33}\left(SR_{\frac{Micro}{Micro}}\right) & f_{34}\left(SR_{\frac{Nano}{Micro}}\right) \\ f_{41}\left(SR_{\frac{Macro}{Nano}}\right) & f_{42}\left(SR_{\frac{Meso}{Nano}}\right) & f_{43}\left(SR_{\frac{Micro}{Nano}}\right) & f_{44}\left(SR_{\frac{Nano}{Nano}}\right) \end{vmatrix} \cdot \begin{vmatrix} I_{Macro} \\ I_{Meso} \\ I_{Micro} \\ I_{Nano} \end{vmatrix}$$

Input-output mapping

$$O_{MuSS} = G(SR) \cdot I_{MuSS}$$

$$\downarrow$$

$$O_{ideal} = G(SR)_{ideal} \cdot I_{ideal}$$

$$\downarrow$$

$$O_{error} = G(SR)_{random+systematic} \cdot I_{random+systematic}$$

$$\downarrow$$

$$e_{MuSS} = O_{error} - O_{ideal}$$

How do we do this?

Example

Identify Coordinate systems (CSs)

- ❑ Look for symmetric centers and center of stiffness locations

Connect the CSs to form stick figures (SFs)

Assign length/orientation variables to SFs

Formulate component error equations between CSs

Formulate system error equation

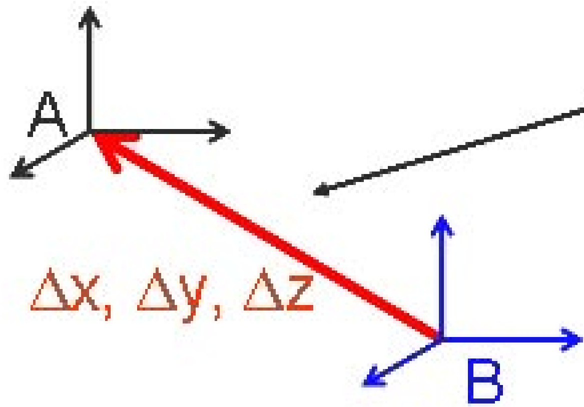
- ❑ Requires Homogeneous Transformation Matrices (HTM)

Obtain error vectors (ideally in parametric form)

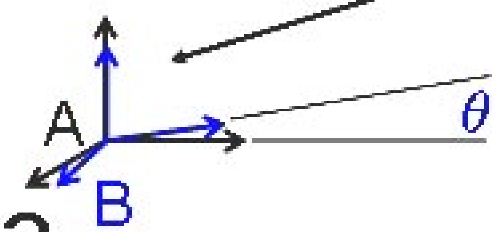
Homogeneous Transformation Matrices

What are they?

- 4X4 matrices which relate coordinate systems



$$ {}^{A}H_{B}(Translation) = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



$$ {}^{A}H_{B}(Rotation) = \begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

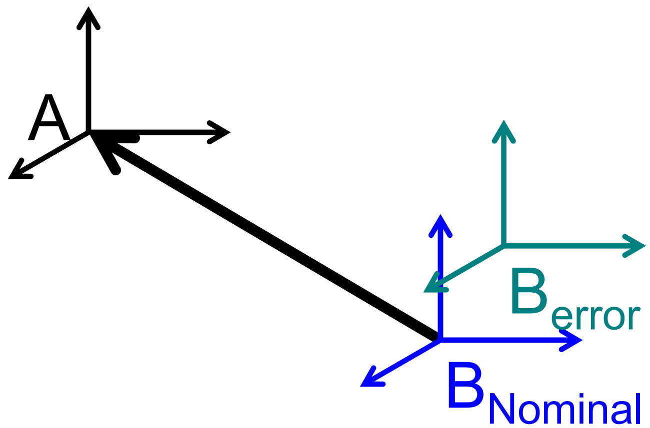
What do they do?

- Transfer locations/orientations in coordinate systems
- Enable parametric modeling of errors

$$ \begin{bmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{bmatrix} = H \cdot \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} $$

Homogeneous Transformation Matrices

What does this buy us? Example: Translation

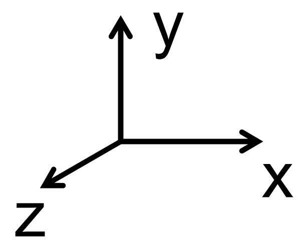


$$\begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix}_{\text{Nominal}} = H_{\text{Nominal}} \cdot \begin{vmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{vmatrix} \quad \& \quad \begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix}_{\text{Error}} = H_{\text{Error}} \cdot \begin{vmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{vmatrix}$$

$$e_{\text{Error}} = \begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix}_{\text{Error}} - \begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix}_{\text{Nominal}} = H_{\text{Error}} \cdot \begin{vmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{vmatrix} - H_{\text{Nominal}} \cdot \begin{vmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{vmatrix}$$

Homogeneous Transformation Matrices

Rotations:



$$^{A}H_{B}(\theta_x)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos(\theta_x) & -\sin(\theta_x) & 0\\ 0 & \sin(\theta_x) & \cos(\theta_x) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$^{A}H_{B}(\theta_y)=\begin{bmatrix}\cos(\theta_y) & 0 & \sin(\theta_y) & 0\\ 0 & 1 & 0 & 0\\ -\sin(\theta_y) & 0 & \cos(\theta_y) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

For small θ, cos(θ)~0 & sin(θ)~θ

$$^{A}H_{B}(\theta_z)=\begin{bmatrix}\cos(\theta_z) & -\sin(\theta_z) & 0 & 0\\ \sin(\theta_z) & \cos(\theta_z) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Order of multiplication is important for large θ

Example: Mach. vs. concept model

Big issues first

❑ Contacts	Thermal	Load	Wear
❑ Vibration	Measurements	Constraint	Stress
❑ Gravity	Friction		

Diagram removed for copyright reasons. Detailed schematic of machine tool, which forms the basis for the conceptual model (right).

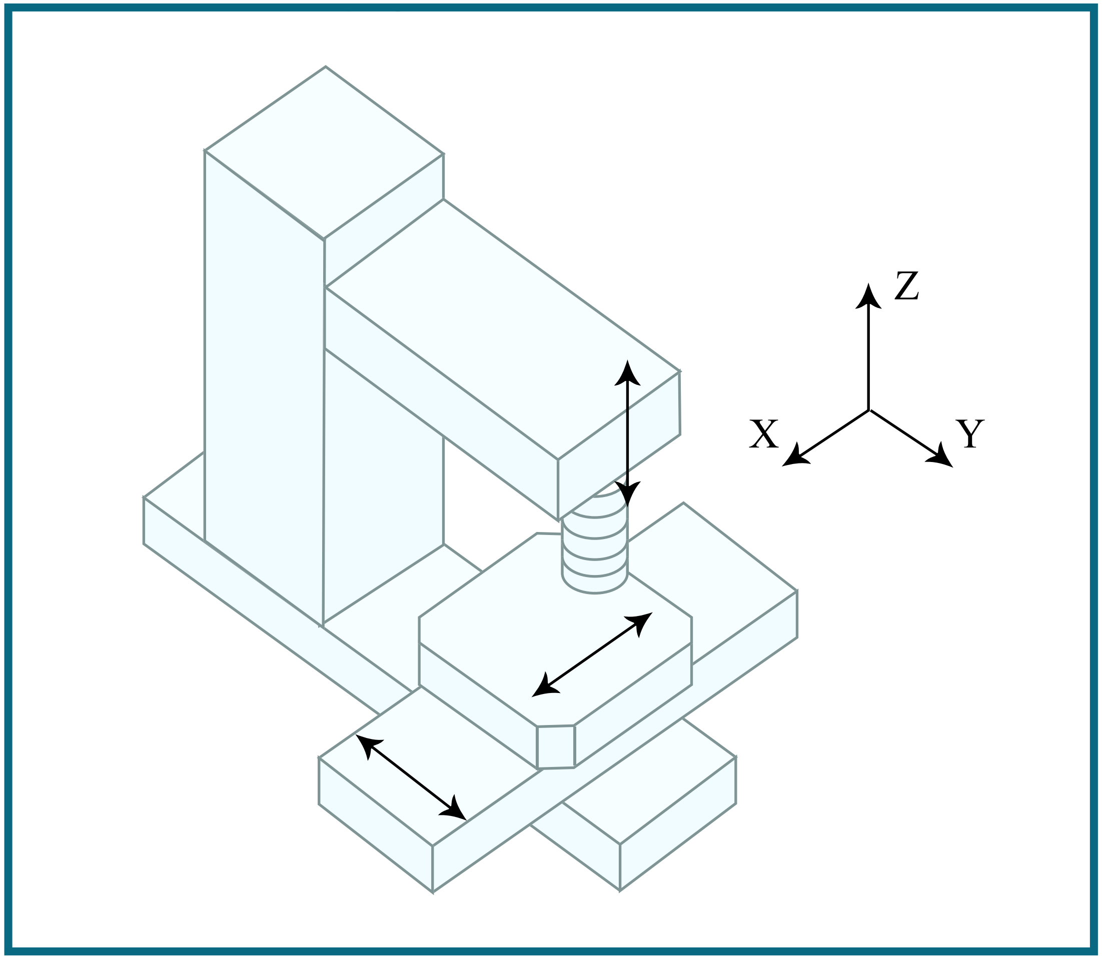


Figure by MIT OCW.

Example: Mach. vs. concept model

Stick figure model

CS A is at stationary part of bottom part

Diagram removed for copyright reasons. Detailed schematic of machine tool, which forms the basis for the conceptual model (right).

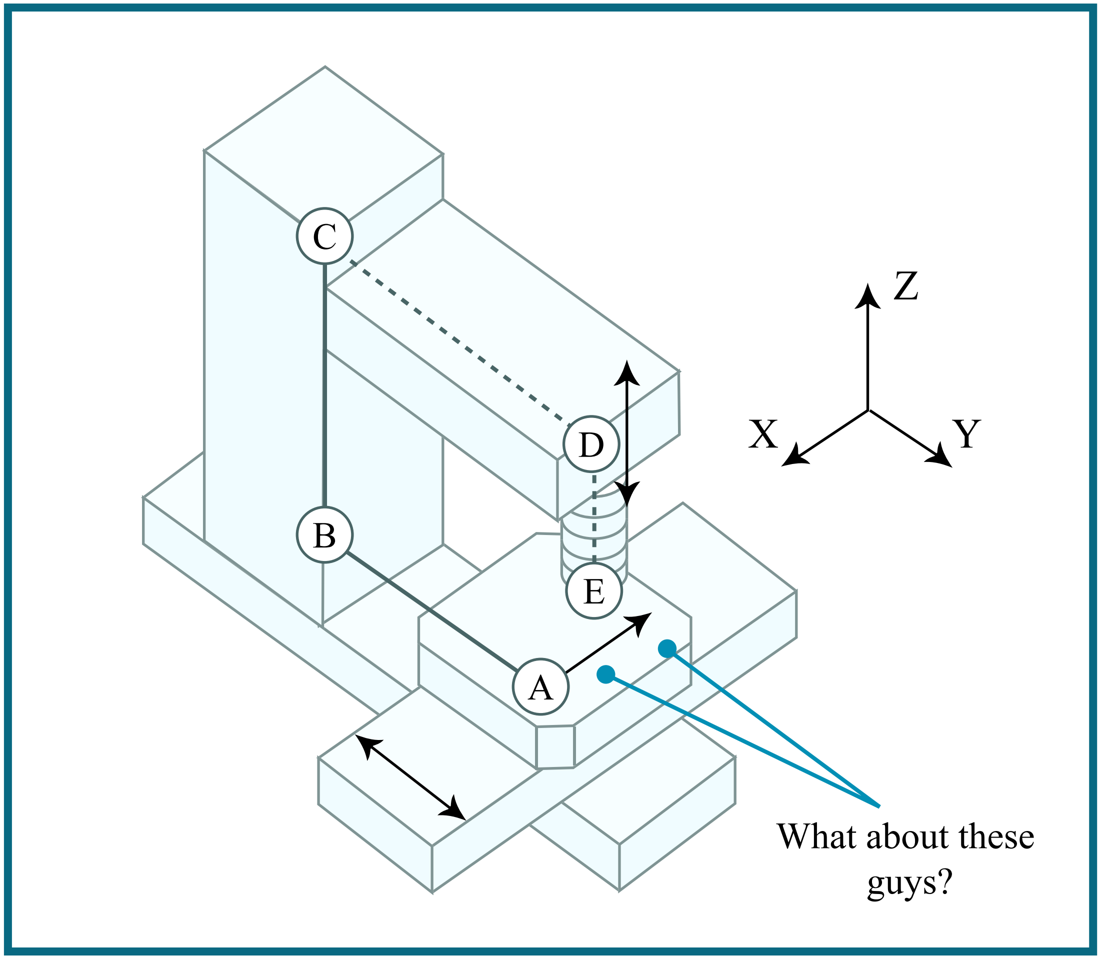
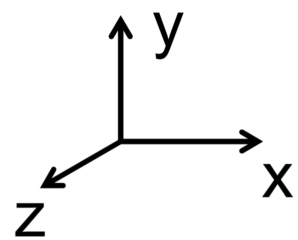


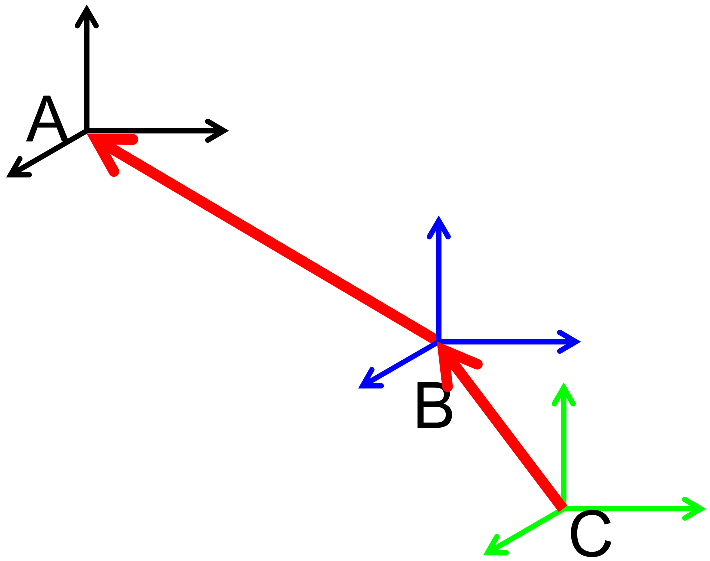
Figure by MIT OCW.

General form w/o rotations

Multiple transformations



$$\begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix} = {}^A H_B \cdot \begin{vmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{vmatrix} \qquad \begin{vmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{vmatrix} = {}^B H_C \cdot \begin{vmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{vmatrix}$$



$$\begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix} = {}^A H_B \cdot {}^B H_C \begin{vmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{vmatrix}$$

Example cont.

Analytic modeling of linear errors

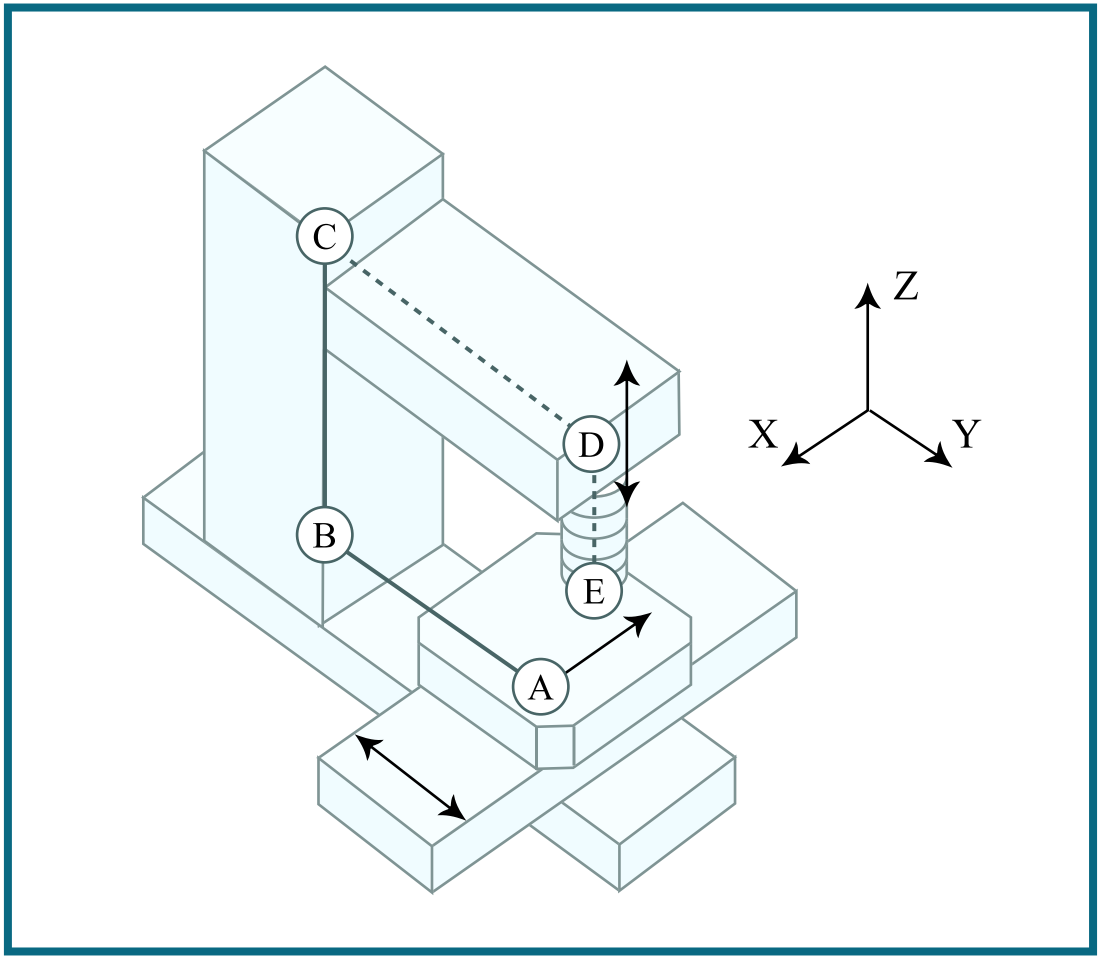


Figure by MIT OCW.

$$\begin{vmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{vmatrix} = {}^{A}H_B \cdot {}^{B}H_C \cdot {}^{C}H_D \cdot {}^{D}H_E \begin{vmatrix} X_E \\ Y_E \\ Z_E \\ 1 \end{vmatrix}$$

Statistical treatment of error models

Once you have a good parametric error model you can:

- Put into a spreadsheet/Matlab/MathCAD
- Run "what if" scenarios
- Run sensitivity analyses
- Run optimization
- Run statistical analyses (e.g. Monte Carlo analysis)