Reminder: Final Exam (May 20, 1:30-4:30 in 37-212, closed book, 1 page equation sheet allowed)

Outline:

Geometrical Optics

- **Reflection, Refraction, Fermat's Principle,**
- **Prisms, Lenses, Mirrors, Stops**
- **Lens/Optical Systems**
- **Analytical Ray Tracing, Matrix Methods**

Wave Optics

- **From wavefront and eikonal equations**
- **Interference and Interferometry**
- **Fraunhofer diffraction and diffraction gratings**
- **Optical Imaging and Spatial Filtering**

A. Relationship between Geometrical and Wave Optics

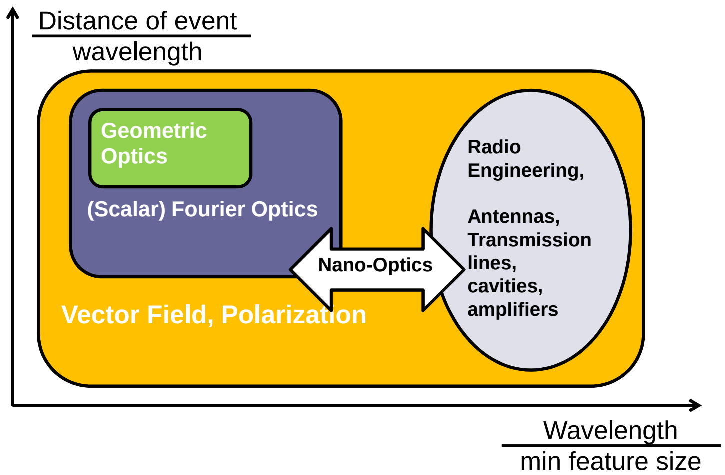
Based on the specific method of approximation, optics has been broadly divided into two categories, namely:

I. Geometrical Optics (ray optics) treated in the first half of the class;

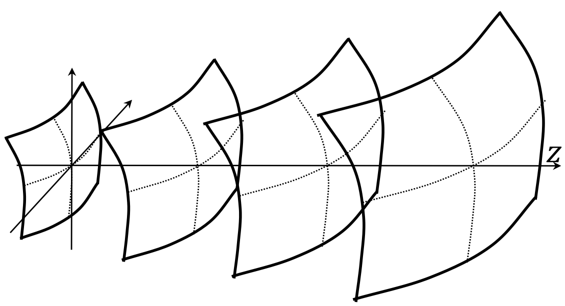
- Emphasis on finding the light path; it is especially useful for:
  - Designing optical instruments;
  - or tracing the path of propagation in inhomogeneous media.

II. Wave Optics (physical optics) treated in the second half of the class:

- Emphasis on analyzing interference and diffraction
- Gives more accurate determination of light distributions
- Basic concept: propagation of wavefront and intensity



Relationship between wavefronts and rays:



Wavefronts:

1) A geometrical surface at which the wave phase is constant

2) As time evolves, the wavefronts propagate at the speed of wave and expand outwards while preserving the wave's energy.

Properties of rays:

1) Rays are normals to the wavefront surfaces
2) trajectories of "particles of light"
3) Normal to the wavefront surfaces
4) Continuous and piece-wise differentiable
5) Ray trajectories are such as to minimize the "optical path"

How can we obtain Geometric optics picture such as ray tracing from wave equations? We can decompose the field $\mathbf{E}(\mathbf{r}, \omega)$ into two forms: a fast oscillating component $\exp(ik_0\Phi)$, $k_0 = \omega/c_0$ and a slowly varying envelope $\mathbf{E}_0(\mathbf{r})$.

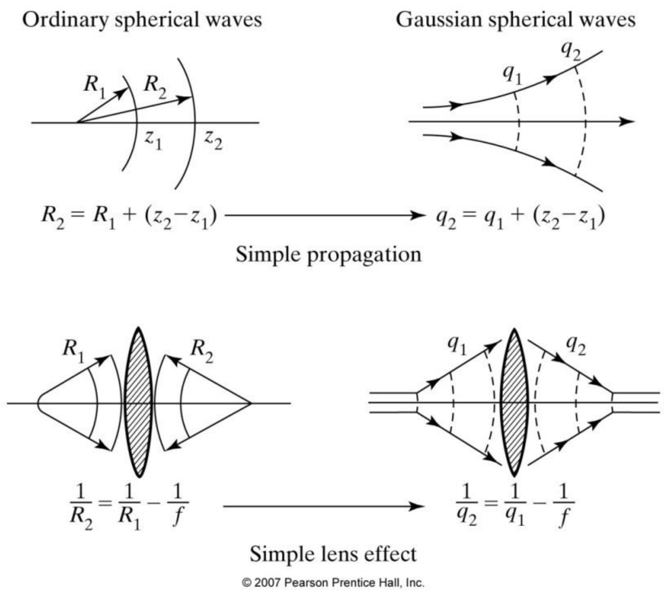
Eikonal Equation describes the variation of wavefront Φ:

$$\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2 = \varepsilon(x,y,z) = n^2(x,y,z)$$

Associated with the **transport of intensity** $E_0 = \sqrt{I}$:

$$\boxed{E_0\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}\right) + 2\left(\frac{\partial E_0}{\partial x}\frac{\partial\Phi}{\partial x} + \frac{\partial E_0}{\partial y}\frac{\partial\Phi}{\partial y} + \frac{\partial E_0}{\partial z}\frac{\partial\Phi}{\partial z}\right) = 0}$$

- ***A precursor to Gaussian Optics (not for the finals)**: use a complex radius of curvature ***q*** to mix and match of rays and wavefronts!



B. Origin of Interference (Coherence)

- The (generalized) wavefront of a wave train varies with **space and time**:

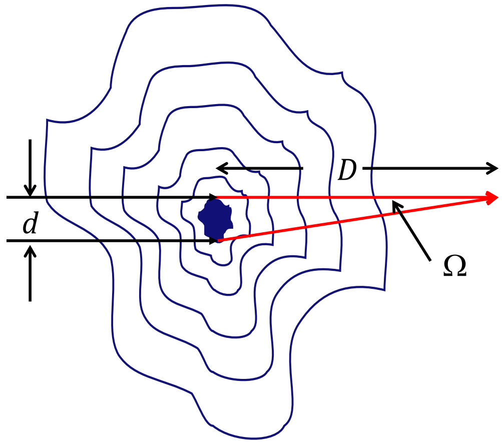
$$\delta(x,y,z,t) = \mathrm{k}_0\Phi = k_x x + k_y y + k_z z - \omega t + \varphi$$

The fringes of intensity suggest the similarity of the measured field over a given time or spatial period: this property can be used to measure **correlation** or *degree of coherence*.

Temporal coherence:

Correlation in phase of the radiation field measured at different time intervals

Spatial coherence:
Correlation in phase of the radiation field measured at different spatially distinct points. In general, wave-fronts smooth out as they propagate away from the source.



*The **van Cittert-Zernike Theorem** states that the spatial coherence area A_c is given by:

$$A_c = \frac{D^2\lambda^2}{\pi d^2} = \frac{\lambda^2}{\pi\Omega}$$

where d is the diameter of the light source and D is the distance away, and $\Omega = d^2/D^2$ is the solid angle extended by the source.

- Two or more wavefronts of different sources (with same polarization) may cross each other:

$$E_x = E_{1x} + E_{2x} = E_{1x}(0)\exp[i\delta_1] + E_{2x}(0)\exp[i\delta_2]$$

$$I = c\varepsilon[E_{1x}^2(0) + E_{2x}^2(0)] + 2c\varepsilon E_{1x}(0)E_{2x}(0)\langle\cos(\delta_1 - \delta_2)\rangle$$

e.g.
$\delta_1 - \delta_2 = (k_1 sin\theta_1 - k_2 sin\theta_2)x + (k_1 cos\theta_1 - k_2 cos\theta_2)z + (\varphi_1 - \varphi_2)$

Image of wavefront splitting inferometry removed due to copyright restrictions.

C. Fourier Transform and some famous functions:

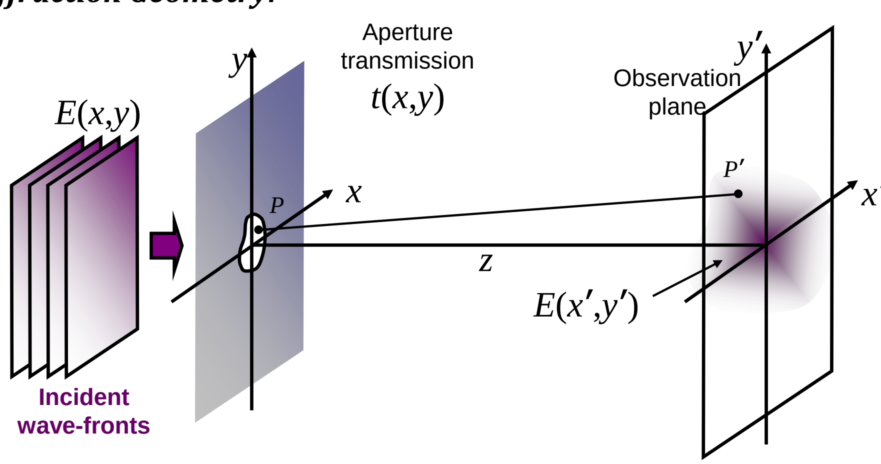
$$f(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(k_x,k_y)\exp(ik_x x)\exp(ik_y y)\,dk_x dk_y$$

$$F(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp(-ik_x x)\exp(-ik_y y)\,dxdy$$

Accordingly, the functions $f(x,y)$and $F(k_x,k_y)$are referred to as spatial-Fourier Transform pairs.

Functions	*Fourier Transform Pairs*
$rect\left(\frac{x}{a}\right)$	$\lvert a\rvert sinc\left(\frac{ak_x}{2\pi}\right)$
$sinc\left(\frac{x}{a}\right)$	$\lvert a\rvert rect\left(\frac{ak_x}{2\pi}\right)$
$\Lambda\left(\frac{x}{a}\right)$	$\lvert a\rvert^2 sinc^2\left(\frac{ak_x}{2\pi}\right)$
$\mathrm{comb}\left(\frac{x}{a}\right)$	$\lvert a\rvert comb\left(\frac{ak_x}{2\pi}\right)$
Gaussian $exp\left(-\frac{x^2}{a^2}\right)$	$exp\left(-\frac{a^2}{4\pi}{k_x}^2\right)$
Step function $H(x)$	$\frac{1}{ik_x}+\frac{1}{2}(\delta(k_x))$
$circ\left(\frac{\sqrt{x^2+y^2}}{a}\right)$	$\lvert a\rvert^2\frac{2\pi J_1\left(a\sqrt{{k_x}^2+{k_y}^2}\right)}{a\sqrt{{k_x}^2+{k_y}^2}}$

D. General *Diffraction Geometry:*



$$E(x',y') = \iint h(x'-x, y'-y, z)t(x,y)E(x,y)dxdy$$

a. *Fraunhofer Diffraction:*

$$\frac{k(x^2+y^2)}{2z} \ll 1 \text{ (\textit{difficult to achieve!})}$$

$$E(x',y') \approx \frac{1}{z}\iint \exp(-ik(\theta_{x'}x + \theta_{y'}y))t(x,y)E(x,y)dxdy$$

b. *Fresnel Diffraction*

$$When\ \frac{k\left(x'^2+y'^2\right)}{2z} \sim 1,\ \ \frac{k(x^2+y^2)}{2z} \sim 1,$$

we are in the domain of Fresnel Diffraction. The quadratic phase terms cannot be neglected in the **Fresnel propagator**:

$$\mathrm{h(x,y,x',y',z)} = \frac{\exp(ikz)}{z} exp(ik\frac{(x'-x)^2+(y'-y)^2}{2z})$$

You may recognize it is a Gaussian function with respect to x and y, and the wavefront is diverging.

$$\text{Using } x' = k_x \frac{z}{k}, y' = k_y \frac{z}{k}$$

The corresponding transfer function is:

$$H(k_x,k_y) = \frac{\exp(ikz)}{z}\iint exp\left(ik\frac{x^2+y^2}{2z}\right)\exp\left(-ik_x x - ik_y y\right) dxdy$$

$$H(k_x,k_y) = \frac{\exp(ikz)}{k} exp\left(-iz\frac{{k_x}^2+{k_y}^2}{2k}\right)$$

The Fourier transform of a Gaussian function is still a Gaussian function. The above property is often used in analyzing the depth of focus (DOF).

c. *Diffraction using a lens*

$$t(x,y) = exp\left[-\frac{ik(x^2+y^2)}{2f}\right]$$

cancels quadratic phase terms.

$$E_{out}(x',y') \approx \iint \quad E_{in}(x,y)\exp\left\{\frac{-ik[x'x+y'y]}{f}\right\}dxdy$$

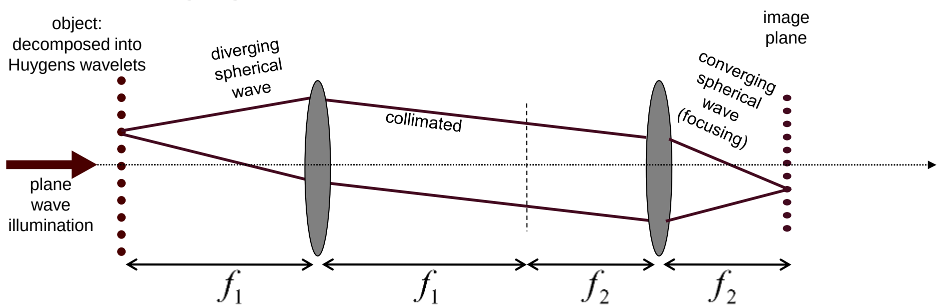
$x' = k_x \frac{f}{k}, y' = k_y \frac{f}{k}$ matches the Fourier transform.

d. Diffraction using a grating

$$t(x) = \sum_{m=0}^{\infty} t_m cos\left(\frac{2\pi m x}{\Lambda}\right)$$

Maximum of diffraction order occur if $\Lambda \sin(\theta_m) = m\lambda$ (at normal incidence)

E. Abbe's theory of Imaging

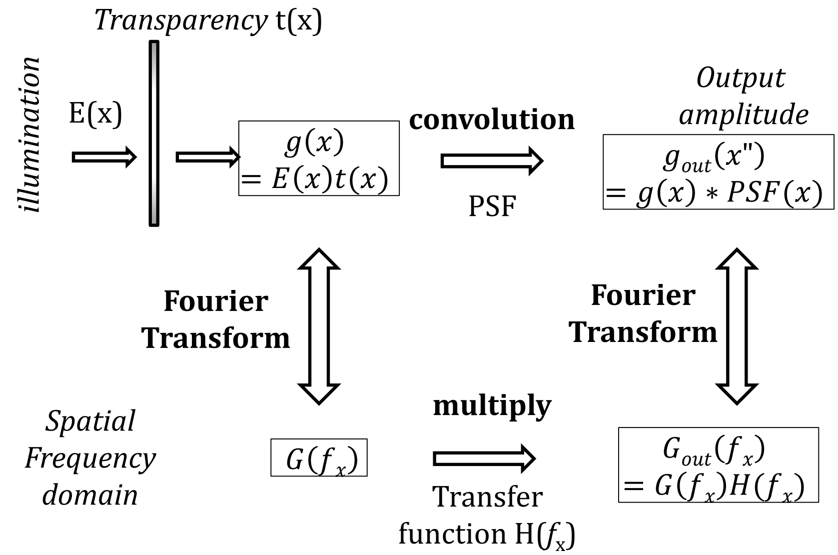


The transmission $AS(x', y')$ of the aperture stop, will contribute to the image formation through spatial filtering at Fourier planes:

$$AS(x', y') = AS(k_x \frac{f_1}{k},\ k_y \frac{f_1}{k})$$ (**Amplitude Transfer Function**(ATF))

a. **Coherent imaging** (superposition of E-field)
$\mathcal{F}\left[AS(k_x \frac{f_1}{k},\ k_y \frac{f_1}{k})\right]$ is called **Point Spread Function**(PSF) (since it is the spread of an ideal point source $\delta(x, y)$ at the image).

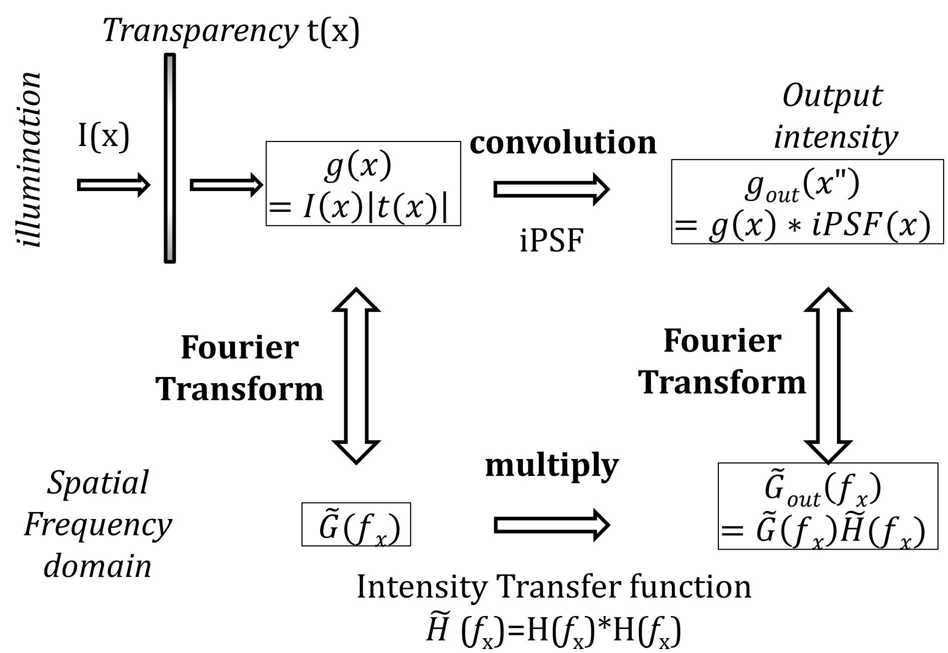
- The following diagram elaborate the procedure of coherent imaging as a linear, shift invariant system.



Transfer function H(f_x) (=AS(x')) is also called the pupil function.

b. Incoherent imaging
Under (spatially) **incoherent** illumination, the <u>image intensity</u> is a convolution of object <u>intensity</u> with <u>intensity of</u> point spread function (iPSF=|PSF|²). Correspondingly, the (complex) Optical Transfer Function (OTF) is the Fourier transform of iPSF.

- The following diagram elaborate the procedure of incoherent imaging as a linear, shift invariant system.



F. Aberration (not included in final exam, supplement Pedrotti Chapter 20, and for your study only)

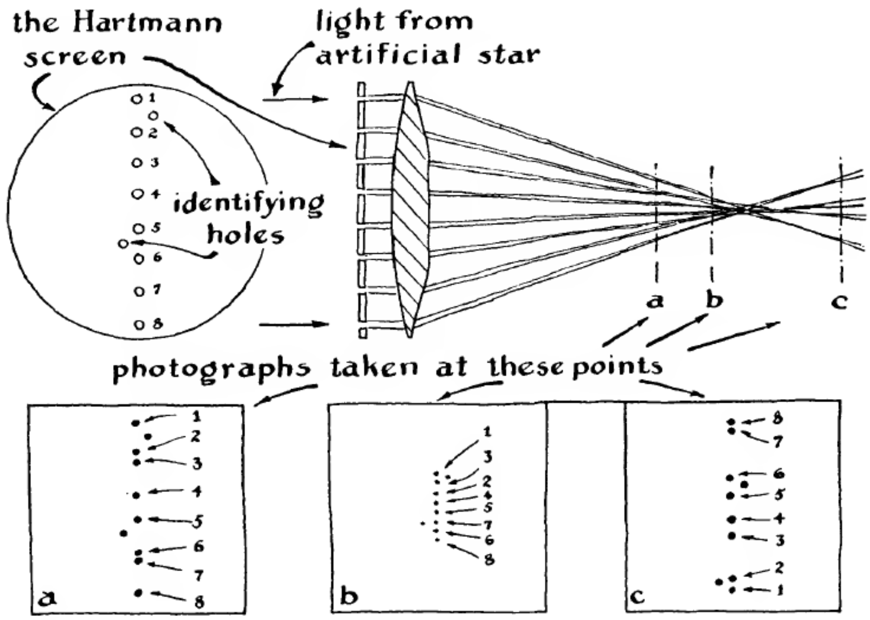
Very few optical systems give images that are free from all defects. Fortunately, certain defects can be tolerated in a particular application, if the produced images have the specified quality. Thus, experimentalists can solve optical problems in the real world, bearing the imperfections in mind. Optical designers control the important aberration effects by the position of stops (to minimize field curvature), by the "bending" of components (to minimize aperture defect), or by the choice of glasses, thicknesses, and spacing for components (to manage achromatism, *etc.).* Recently, designers have been beginning to use aspheric surfaces more extensively.

Most aberrations cannot be modeled with ray matrices. Designers beat them with lenses of multiple elements, that is, several lenses in a row. Some zoom lenses can have as many as a dozen or more elements.

- *Image Testing:*

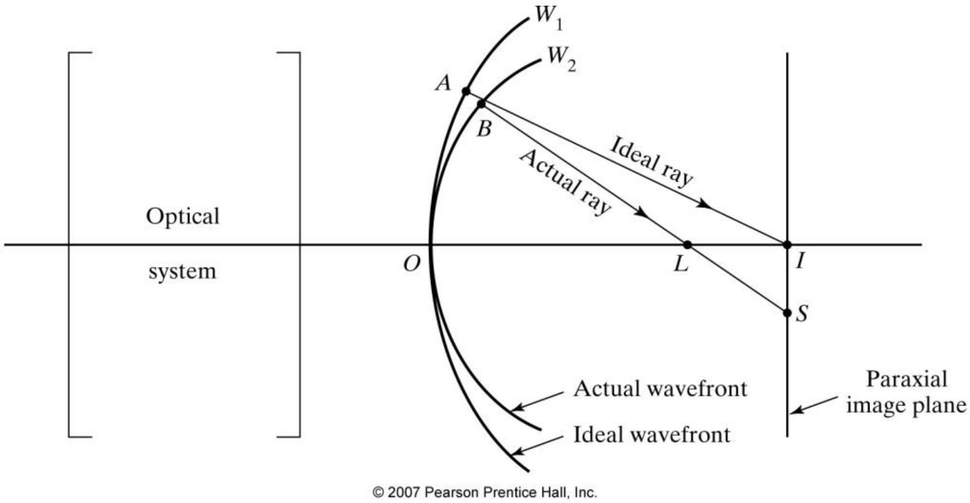
Ray tracing, to guide optical design, has been practiced since it was introduced in the time of Newton. An optical lens system may be evaluated before fabrication and assembly by computational ray tracing. When the lens is constructed, it may be further evaluated by experimental testing.

Modern ray tracing is done with high-speed computing softwares so that, for each point in the object plane, several hundred rays may be calculated, and the points where they penetrate the image plane may be plotted to predict the imaging characteristics. The surface density of these points in the image plane are used to predict the performance of the optical system with respect to the illumination condition. Often the experimental test shows that the dot diagram predictions are pointing to the worst case scenario- that is, they predict a poorer quality of the image of a point source than experiment limit. But despite of this lack of full agreement for ray tracing, these computational evaluations are widely used in practice.



The above schematic illustrates the **Hartmann test**. This experimental method for testing an optical system has much in common with theoretical ray tracing. In the Hartmann test a screen obscures all the rays collected by the lens, except a set of narrowly defined beams. These sampling beams are uniformly distributed over the aperture, as illustrated. The trajectory of each of these beams may be traced by means of a series of (digital) photographs made at regular intervals along the optical axis in front of, and behind, their best union, as shown.

The **knife-edge test** is also a kind of a ray tracing in reverse. It is evident that by such test we may determine where rays from each part of the aperture penetrate the focal plane. This determination follows from the position of the knife edge and the observed location of its shadow.



Pedrotti Figure 20-1. Illustration of ray and wave aberrations.

- **Seidel's Third Order Theory:**

  The ray optics theory for longitudinal and lateral image positions was based on the approximate polynomial series representation of the sinusoidal functions:

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + O(\theta^5)$$

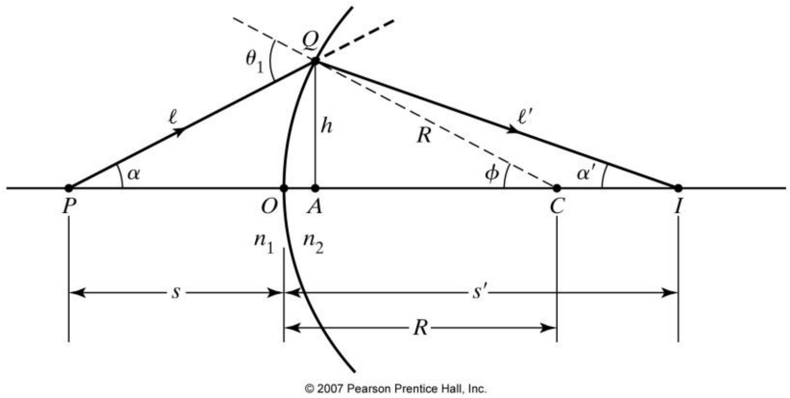
$$\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - O(\theta^6)$$

  The paraxial ray theory using $\sin\theta = \theta$ and $\cos(\theta) = 1$ is first-order approximation. The values of θ, representing angles of incidence or refraction, etc, are constrained by the paraxial approximation to have such modest magnitudes that all higher order terms of θ in the expansions are negligible. In contrast, a *third-order* optical theory for longitudinal and lateral image positions which retains the third order terms in $\sin\theta$, is first developed around 1857 by Philip Ludwig von Seidel.

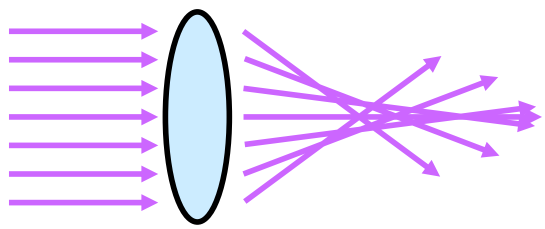
- **Spherical aberration** ($\propto \mathbf{r^4}$)

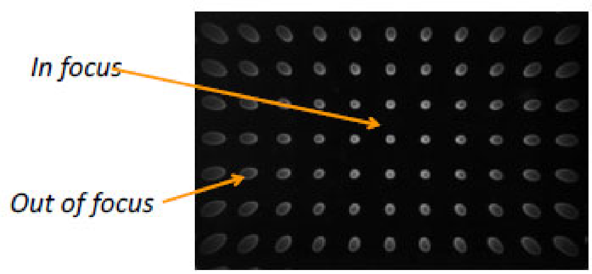
  The difference in optical path length between paraxial rays and marginal rays, proportional to the square of the lens diameter.

Example:



Pedrotti Figure 20-3: Refraction of ray at a spherical surface.





$$a(Q) = (n_1 l + n_2 l') - (n_1 s + n_2 s')$$

$$l^2 = \mathrm{R}^2 + (\mathrm{s}+\mathrm{R})^2 - 2R(\mathrm{s}+\mathrm{R})\cos\phi$$

$$l'^2 = \mathrm{R}^2 + (\mathrm{s}'-\mathrm{R})^2 + 2R(\mathrm{s}'-\mathrm{R})\cos\phi$$

$$\boxed{a(Q) = -\frac{h^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]}$$

- **Coma** ($\propto \mathbf{h'r^3}\boldsymbol{cos\theta}$)
  A trailing "comet-like" blur directed away from the optic axis.
  - image becomes increasingly blurred toward the edges;
  - can be partially corrected by tilting the lens.

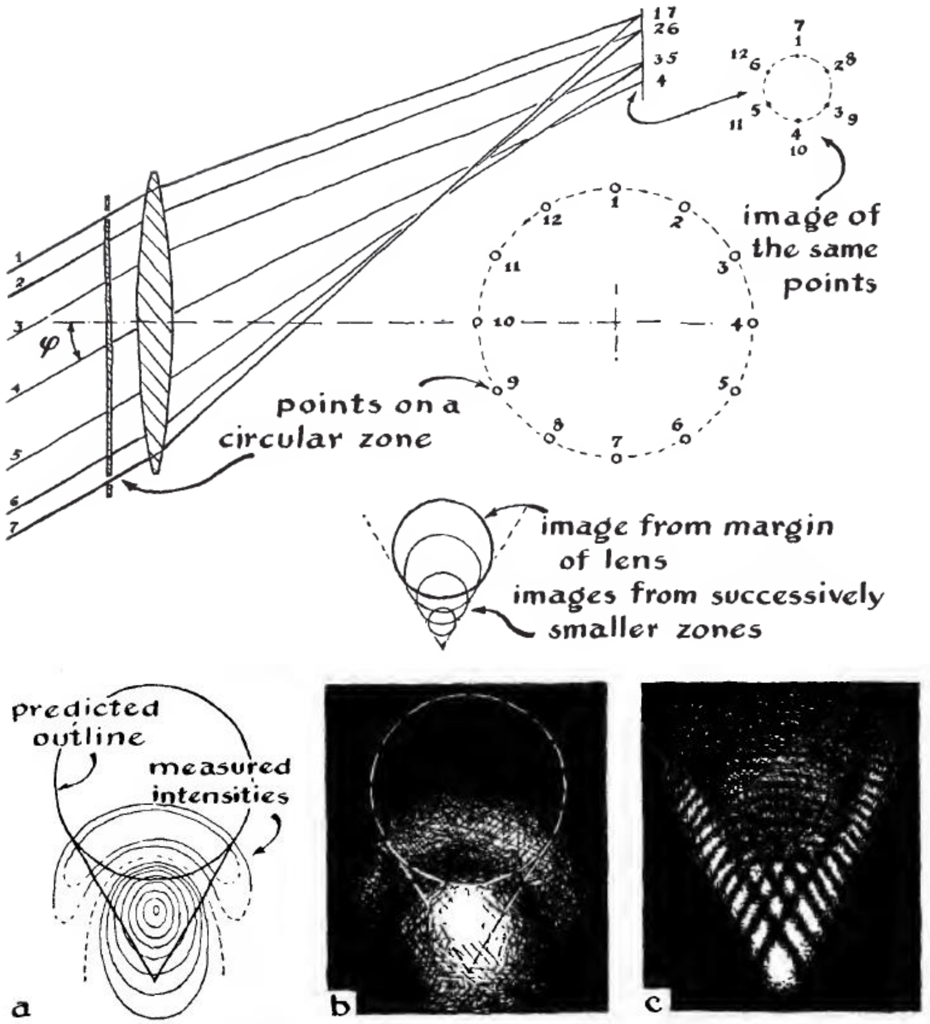


Figure: Coma predicted by ray tracing and as photographed (b) (redrawn from Martin, Technical Optics) and (c) coma manifesting diffraction details (redrawn from Kingslake, Photographic Lenses)

Astigmatism: ($\propto \mathbf{h'^2 r^2}\boldsymbol{cos^2\theta}$)
The image of an off-axis point forms focal lines at the sagittal and tangential foci and in between an elliptical shape.

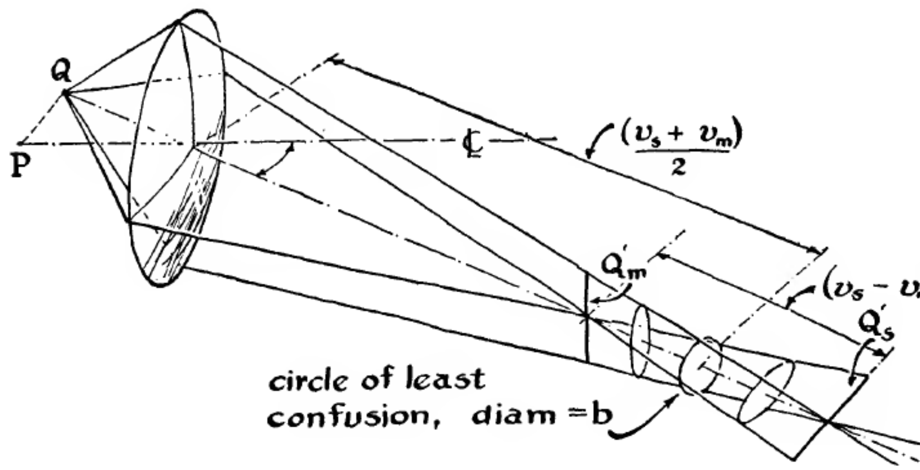


Figure: Astigmatism due to a point source Q located off the optical axis.

Curvature of Field: ($\propto \mathbf{h'^2 r^2}$)
Curvature of field causes a planar object to project a curved (non-planar) image. Rays at a large angle see the lens as having an effectively smaller diameter and an effectively smaller focal length, forming the image of the off axis points closer to the lens. This causes problems when a flat imaging device is used e.g., a photographic plate or CCD image sensor. For more details please visit Nikon Microscopy University: http://www.microscopyu.com/tutorials/java/aberrations/curvatureoffield/index.html

Distortion: ($\propto$ $\mathbf{h'^3 r\cos\theta}$)

A radial distortion that occur from the geometry of the lens (such as thick double convex lens).

Figure: The effect of aperture stop to control the distortion of an image.