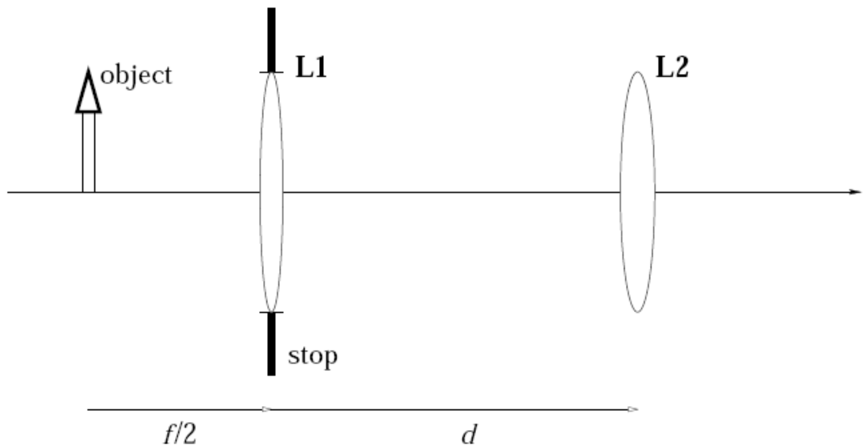
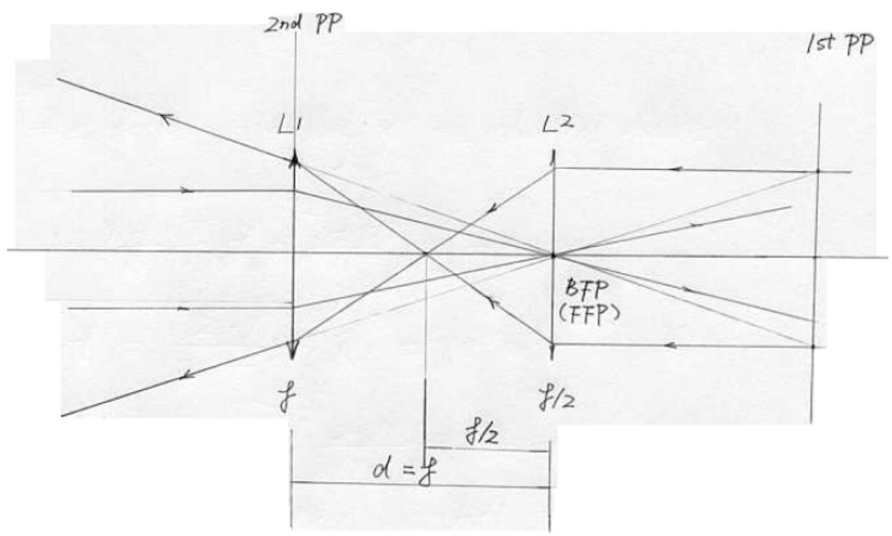
1. Consider the two-lens system shown below. Lens L1 has focal length f, and lens L2 has focal length $f/2$.





(a) Set the separation distance d such that the Effective Focal Length (EFL) of the combination equals f.

Solution: Using the matrix method,

$$\begin{bmatrix} 1 & -\frac{2}{f} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{2d}{f} & -\left(\frac{3}{f}-\frac{2d}{f}\right) \\ d & 1-\frac{d}{f} \end{bmatrix}$$

The power of this system is: $P = \dfrac{3}{f} - \dfrac{2d}{f}$

In order to make the EFL $= f$, we have: $\dfrac{3}{f} - \dfrac{2d}{f} = \dfrac{1}{f} \Rightarrow d = f$

(b) Locate the principal planes.

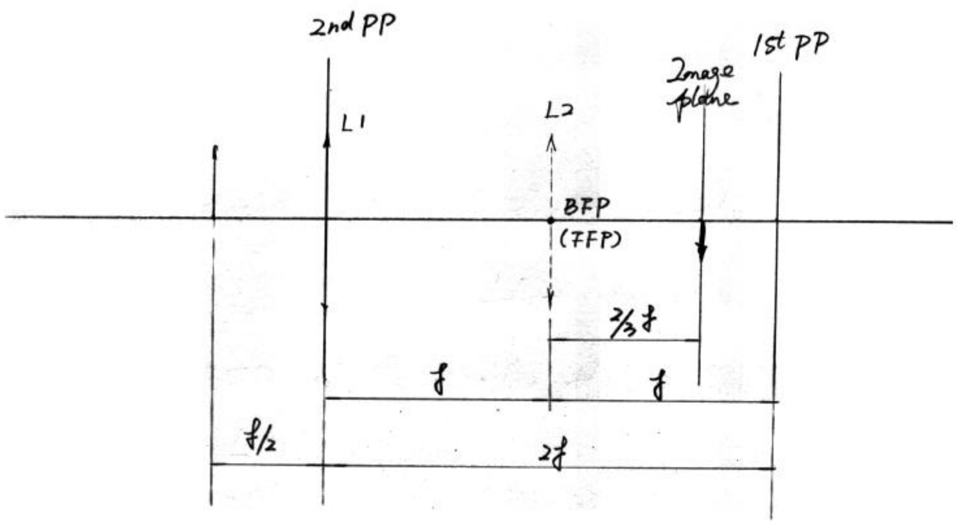
Solution: The BFP and 2nd principle plane are very easy to find. Because $d = f$, the parallel rays pass through L1 and focus at the center of L2. Thus L2 has no effect on these rays. The BFP is at the center of L2 and the 2nd PP is at L1.

The FFP and 1st PP can be obtained by thinking about parallel rays coming from right to left. After passing through L2, they focus at the distance of $f/2$ to L2. Thus, to lens L1, the object distance is:

$$S_o = d - \frac{f}{2} = f - \frac{f}{2} = \frac{f}{2}$$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \Rightarrow S_i = -f$$

The FFP is also at the center of L2. The 1st PP is located at a distance f to the right of lens L2. (Note: To find the principle plane, we must trace back to obtain the intersection from the same ray. So the 1st PP is not to the left, at L1.)



(c) Locate the image plane.

Solution: Let's redraw the system. The object distance is:

$$S_o = \frac{f}{2} + 2f = \frac{5}{2}f$$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \Rightarrow S_i = \frac{5}{3}f$$

The image plane is at the distance $\frac{5}{3}f$ to the 2nd PP. The distance between the image plane and L2 is: $\frac{5}{3}f - f = \frac{2}{3}f$

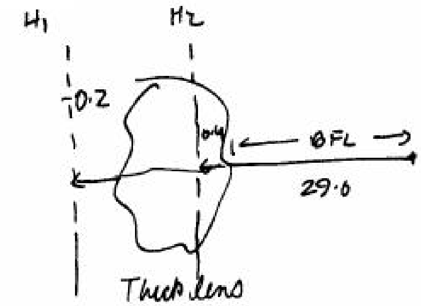
(d) What are the lateral and angular magnifications?

Solution:

The lateral magnification is: $M_T = -\dfrac{S_i}{S_o} = -\dfrac{2}{3}$

The angular magnification is: $M_A = \dfrac{1}{M_T} = -\dfrac{3}{2}$

2. It is found that sunlight is focused to a spot 29.6 cm from the back face of a thick lens, which has principal planes P_1 at +0.2 cm to the front face and P_2 at -0.4 cm to the back face. Determine the location of the image of a candle that is placed 49.8 cm in front of the lens.



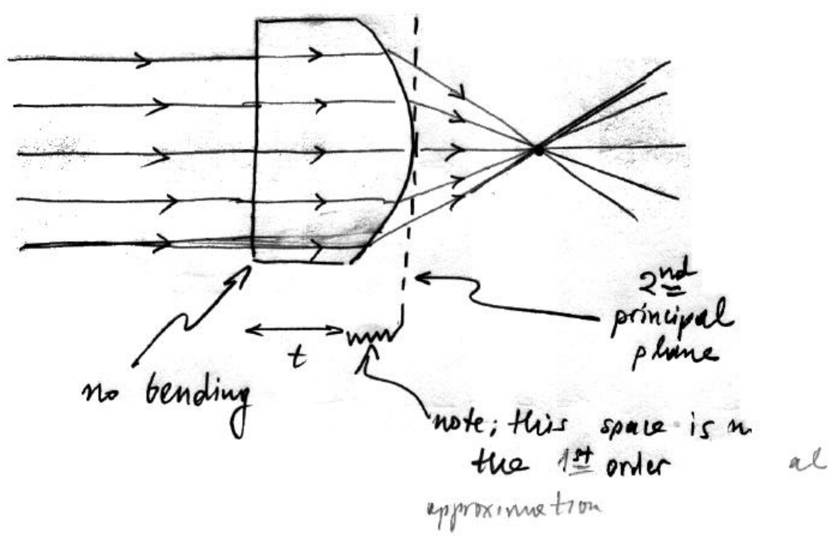
Solution: From the geometry in the figure, we can immediately see that BFL = 29.6 cm and EFL = 29.6 + 0.4 = 30 cm. Just use the principal planes and the imaging condition now.

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{\text{EFL}}$$

$$S_1 = 49.8 + 0.2 = 50$$

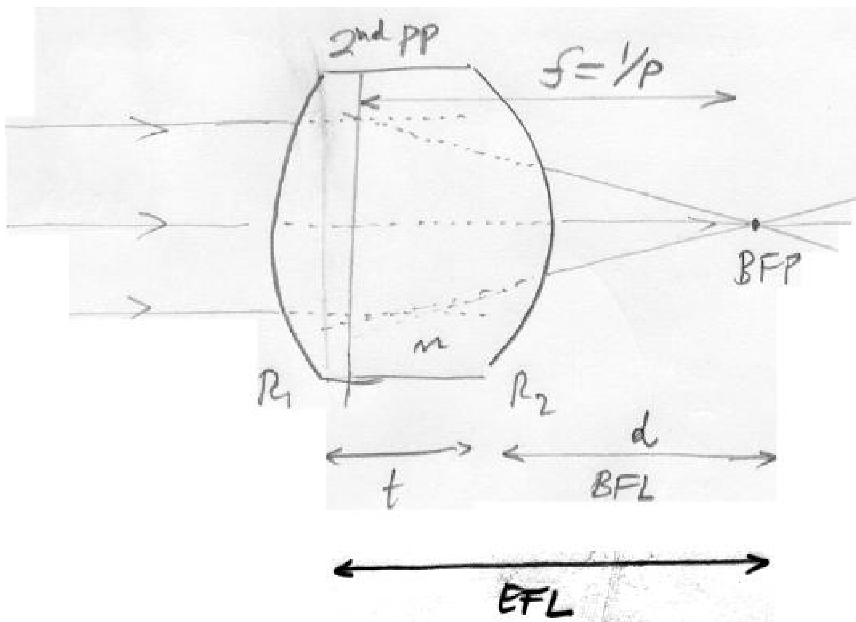
$$\therefore \frac{1}{50} + \frac{1}{S_2} = \frac{1}{30} \Rightarrow S_2 = 75 \text{ cm}$$

3. Show that one of the principle planes of a plano-convex or plano-concave lens is tangential to the curved surface.



Proof: Consider a parallel ray bundle entering through a planar surface. Obviously this surface has no power, so no changes in curvature can occur there. Focusing occurs due to the curved surface, hence this is the location of the principal plane. If the planar surface is on the left as above, then the curved surface is the 2nd PP.

Alternative proof using matrices:



$$P_1 = (n-1)\tfrac{1}{R_1}$$
$$P_2 = (1-n)\tfrac{1}{R_2}$$

If we locate the BFP, we can calculate the BFL, and vice versa.

$$\begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix}\begin{pmatrix} 1 & -P_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{t}{n} & 1 \end{pmatrix}\begin{pmatrix} 1 & -P_1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} =$$

$$\begin{pmatrix} 1-\frac{tP_2}{n} & -P \\ \frac{t}{n}+d\left(1-\frac{tP_2}{n}\right) & 1-\frac{tP_1}{n}-dP \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} -Py \\ \left(1-\frac{tP_1}{n}-dP\right)y \end{pmatrix},$$

$$P = P_1 + P_2 - \frac{tP_1P_2}{n}$$

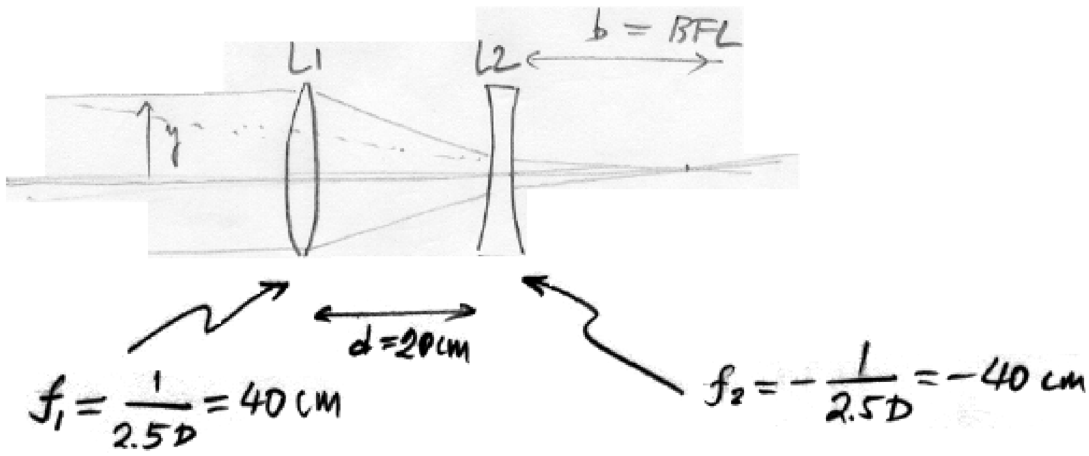
The focus condition is: $1-\dfrac{tP_1}{n}-dP=0 \Rightarrow d=\dfrac{1}{P}\left(1-\dfrac{tP_1}{n}\right)$

The 2nd PP is: $f-d=\dfrac{1}{P}-\dfrac{1}{P}\left(1-\dfrac{tP_1}{n}\right)=\dfrac{tP_1}{n}$ to the left of the curved (back) surface.

Now if $R_1=\infty$ (i.e. the front surface is flat) $\Rightarrow P_1=0 \Rightarrow$ 2nd PP coincides with the curved surface

4. A compound lens consists of a thin positive lens of power +2.5D followed by an interval of 20 cm followed by a thin negative lens of power -2.5D. Locate the principal planes and determine the EFL, BFL, and FFL.

Solution:



First find the BFL:

$$\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}\begin{pmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix}\begin{pmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} =$$

$$\begin{pmatrix} 1-\frac{d}{f_2} & -\frac{1}{f} \\ d+b\left(1-\frac{d}{f_2}\right) & 1-\frac{d}{f_1}-\frac{b}{f} \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} -\frac{y}{f} \\ \left(1-\frac{d}{f_1}-\frac{b}{f}\right)y \end{pmatrix}$$

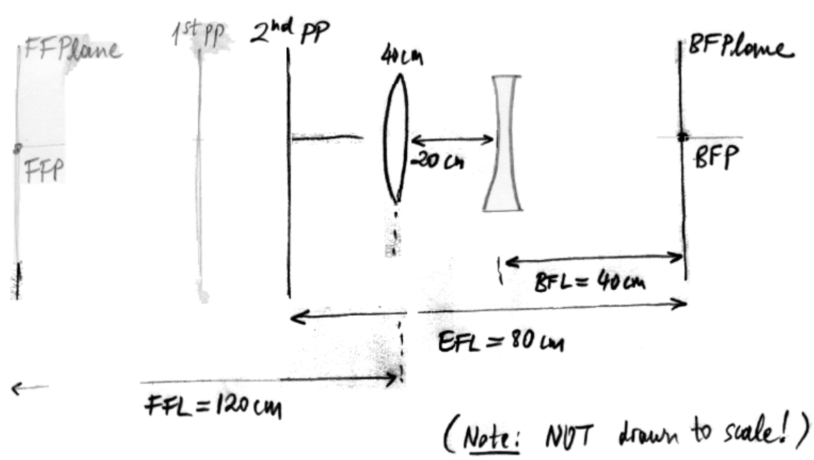
$$\frac{1}{f} = \frac{1}{\text{EFL}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{40} - \frac{1}{40} - \frac{20}{40\times(-40)} \Rightarrow f = 80 \text{ cm (EFL)}$$

The focusing condition is:

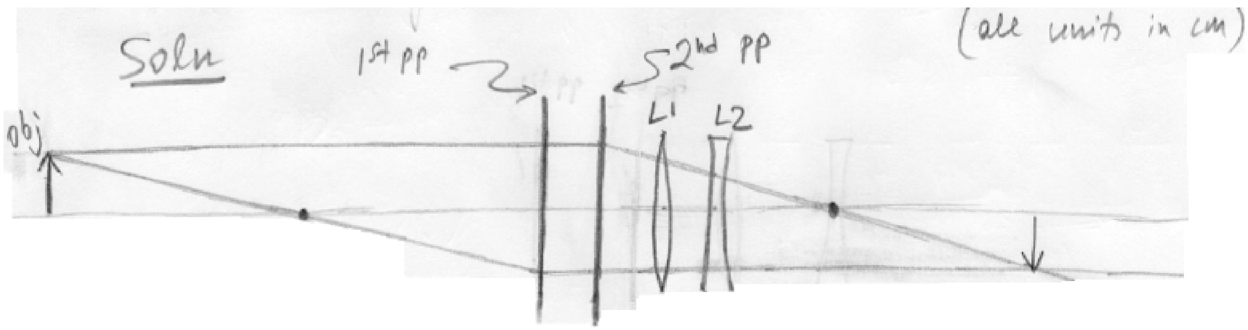
$$1 - \frac{d}{f_1} - \frac{b}{f} = 0 \Rightarrow b = f\left(1-\frac{d}{f_1}\right) = 80\times\left(1-\frac{20}{40}\right) \Rightarrow 40 \text{ cm (BFL)}$$

The 2nd PP is located at $|b-f| = 40$ cm to the <u>left</u> of lens L2.

Do yourself the FFL and 1st PP. You should find:



5. An object is placed 200 cm to the left of the first lens of problem 4. Where does the image form and what is the lateral magnification?



Solution: First, we see that $S_o = 160$ cm.

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \Rightarrow \frac{1}{160} + \frac{1}{S_i} = \frac{1}{80} \Rightarrow S_i = 160 \text{ cm}$$

The image forms 120 cm to the <u>right</u> of lens L2.

$-\dfrac{S_i}{S_o} = -1$, so the image is real and inverted.