1. Solution:

$$g_1(x,z) = a_1 \exp\left\{i2\pi\left(\frac{x}{\lambda}\sin\frac{\pi}{6} + \frac{z}{\lambda}\cos\frac{\pi}{6}\right)\right\}, \qquad a_1 = 1$$
$$g_2(x,z) = a_2 \exp\left\{i2\pi\left(-\frac{x}{\lambda}\sin\frac{\pi}{3} + \frac{z}{\lambda}\cos\frac{\pi}{3}\right)\right\}, \qquad a_2 = \frac{1}{2}$$

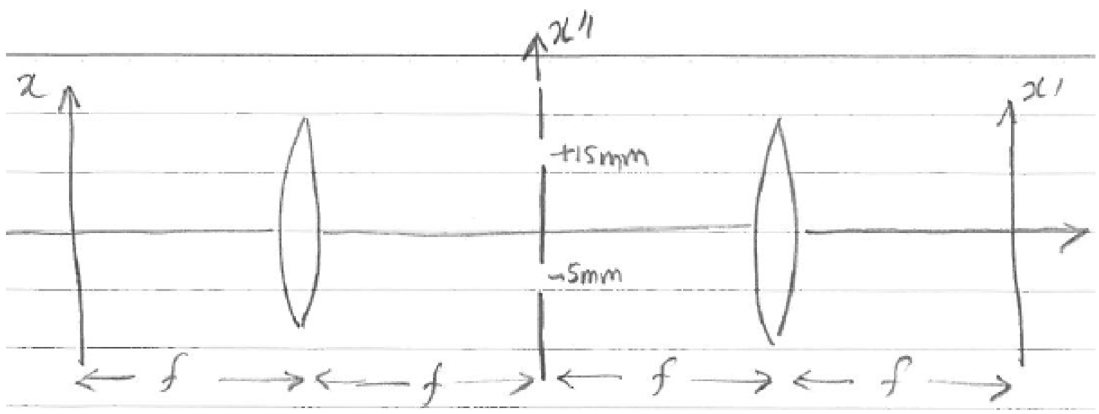
$$\text{At } z=0, \quad I(x) = |g_1(x,0)+g_2(x,0)|^2 = \left|a_1 e^{i2\pi\frac{x}{\lambda}\cdot\frac{1}{2}} + a_2 e^{-i2\pi\frac{x}{\lambda}\cdot\frac{\sqrt{3}}{2}}\right|^2$$
$$= (a_1^2 + a_2^2)\left[1 + \frac{2a_1a_2}{a_1^2+a_2^2}\cos\left(2\pi\frac{x}{\lambda}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right)\right]$$
$$= \left(1+\frac{1}{4}\right)\left[1 + \frac{2\times 1\times\frac{1}{2}}{1+\frac{1}{4}}\cos\left(2\pi\frac{x}{\lambda}\cdot\frac{1+\sqrt{3}}{2}\right)\right]$$
$$= \frac{5}{4}\left[1+\frac{4}{5}\cos\left(2\pi\frac{x}{\Lambda}\right)\right] \quad \text{where } \Lambda = \frac{2\lambda}{1+\sqrt{3}}$$

$$m = \frac{4}{5} \rightarrow \text{ contrast} \qquad \Lambda = \frac{2\lambda}{1+\sqrt{3}} \rightarrow \text{ period}$$
$$I(0) = \frac{5}{4}\times\left(1+\frac{4}{5}\right) = \frac{5}{4}\times\frac{9}{5} = \frac{9}{4} = 2\frac{1}{4}$$

After phase-shifting g_1 by $\pi/2$, we get

$$I(x) = \frac{5}{4}\left[1+\frac{4}{5}\cos\left(2\pi\frac{x}{\Lambda}+\frac{\pi}{2}\right)\right] \Rightarrow I(0) = \frac{5}{4} = 1\frac{1}{4}$$

2. Solution:



$$\lambda = 1.0\mu\text{m}, \quad f = 5\text{cm}, \quad \Lambda = 10\mu\text{m}$$

According to Lecture 19, slide 12, with duty cycle $\alpha = \frac{1}{2}$,

$$g_t(x) = \frac{1}{2} \sum_{q=-\infty}^{\infty} \operatorname{sinc}\left(\frac{q}{2}\right) e^{i2\pi q \frac{x}{\Lambda}}$$

$$G_t(u) = \frac{1}{2} \sum_{q=-\infty}^{\infty} \operatorname{sinc}\left(\frac{q}{2}\right) \delta\left(u - \frac{q}{\Lambda}\right)$$

$$\operatorname{sinc}\left(\frac{q}{2}\right) = \frac{\sin\left(\frac{\pi q}{2}\right)}{\frac{\pi q}{2}} = \begin{cases} \frac{2(-1)^{\frac{q-1}{2}}}{\pi q} & \text{if } q = \text{ odd} \\ 1 & \text{if } q = 0 \\ 0 & \text{if } q \neq 0, \text{ even} \end{cases}$$

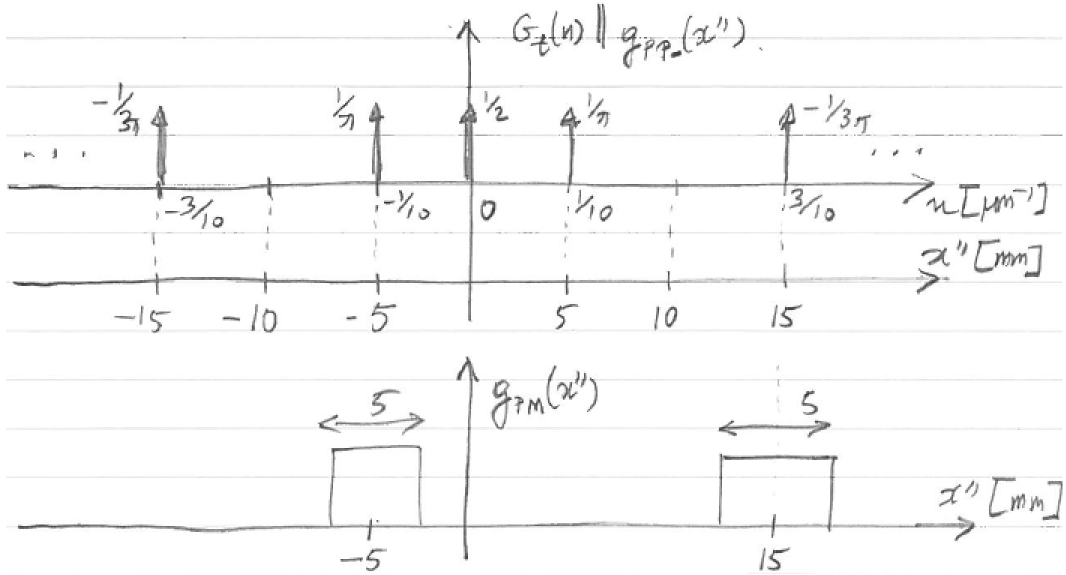
$$G_t(u) = \cdots \underbrace{- \frac{1}{3\pi}\delta\left(u + \frac{3}{10\mu\text{m}}\right)}_{\text{-3rd order}} + \underbrace{\frac{1}{\pi}\delta\left(u + \frac{1}{10\mu\text{m}}\right)}_{\text{-1st order}}$$
$$+ \underbrace{\frac{1}{2}\delta(u)}_{\substack{\text{0th}\\ \text{order}\\ \text{(DC)}}} + \underbrace{\frac{1}{\pi}\delta\left(u - \frac{1}{10\mu\text{m}}\right)}_{\text{+1st order}} \underbrace{- \frac{1}{3\pi}\delta\left(u - \frac{3}{10\mu\text{m}}\right)}_{\text{+3rd order}} + \cdots$$

We see that the even orders are missing.

$$g_{pp-}(x'') \propto G_t\left(\frac{x''}{\lambda f}\right) = \cdots - \frac{1}{3\pi}\delta\left(x'' + \frac{3\lambda f}{\Lambda}\right) + \frac{1}{\pi}\delta\left(x'' + \frac{\lambda f}{\Lambda}\right) + \frac{1}{2}\delta(x'')$$
$$+ \frac{1}{\pi}\delta\left(x'' - \frac{\lambda f}{\Lambda}\right) - \frac{1}{3\pi}\delta\left(x'' - \frac{3\lambda f}{\Lambda}\right) + \cdots$$

Since $\dfrac{\lambda f}{\Lambda} = \dfrac{10^{-6} \times 5 \times 10^{-2}}{10 \times 10^{-6}} = 5\text{mm}$,

$$g_{pp-}(x'') \propto \cdots - \frac{1}{3\pi}\delta(x''+15\text{mm}) + \frac{1}{\pi}\delta(x''+5\text{mm}) + \frac{1}{2}\delta(x'') + \frac{1}{\pi}\delta(x''-5\text{mm}) - \frac{1}{3\pi}\delta(x''-15\text{mm}) + \cdots$$



$$g_{pp+}(x'') = g_{pp-}(x'')g_{pm}(x'') = \frac{1}{\pi}\delta(x''+5\text{mm}) - \frac{1}{3\pi}\delta(x''-15\text{mm})$$

$$g_{\text{out}}(x') \propto G_{pp+}\left(\frac{x'}{\lambda f}\right) = \frac{1}{\pi}e^{-i2\pi\frac{x'}{10\mu\text{m}}} - \frac{1}{3\pi}e^{+i2\pi\frac{3x'}{10\mu\text{m}}}$$

$$I_{\text{out}}(x') = |g_{\text{out}}(x')|^2 = \left(\frac{1}{\pi}\right)^2 + \left(\frac{1}{3\pi}\right)^2 + 2\left(\frac{1}{\pi}\right)\left(-\frac{1}{3\pi}\right)\cos\left(2\pi\frac{2x'}{10\mu\text{m}}\right)$$

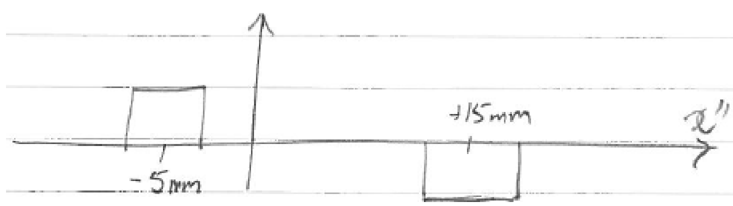
$$= \frac{10}{9\pi^2} - \frac{2}{3\pi^2}\cos\left(2\pi\frac{2x'}{10\mu\text{m}}\right)$$

Reminder:

$$|a_1e^{i\phi_1} + a_2e^{i\phi_2}|^2 = (a_1e^{i\phi_1} + a_2e^{i\phi_2})(a_1e^{-i\phi_1} + a_2e^{-i\phi_2})$$
$$= a_1^2 + a_2^2 + a_1a_2e^{i(\phi_1-\phi_2)} + a_1a_2e^{i(\phi_2-\phi_1)}$$
$$= a_1^2 + a_2^2 + 2a_1a_2\cos(\phi_2-\phi_1)$$

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{(\frac{10}{9\pi^2} + \frac{2}{3\pi^2}) - (\frac{10}{9\pi^2} - \frac{2}{3\pi^2})}{(\frac{10}{9\pi^2} + \frac{2}{3\pi^2}) + (\frac{10}{9\pi^2} - \frac{2}{3\pi^2})} = \frac{\cancel{2} \times \frac{2}{3\cancel{\pi^2}}}{\cancel{2} \times \frac{10}{9\cancel{\pi^2}}} = \frac{3}{5}$$

Now modify the pupil mask so that its complex transmissivity is:



This can be done by inserting a piece of glass of optical thickness equal to π in the upper hole. The physical thickness, if $n = 1.5$, is:

$$\frac{2\pi}{\lambda}(n-1)t = (2k+1)\pi \Rightarrow t = \frac{(2k+1)\lambda}{2(n-1)} = \frac{(2k+1)\times 1\mu\text{m}}{2\times(1.5-1)} = (2k+1)\mu\text{m}, \text{ where } k \text{ is any integer}$$

With this modification,

$$g_{\text{out}}(x') = \frac{1}{\pi}e^{-i2\pi\frac{x'}{10\mu\text{m}}} \underbrace{+}_{\text{result of } \pi-\text{phase shift in the +3rd order}} \frac{1}{3\pi}e^{+i2\pi\frac{3x'}{10\mu\text{m}}}$$

$$\Rightarrow I_{\text{out}} = \frac{10}{9\pi^2} + \frac{2}{3\pi^2}\cos\left(2\pi\frac{2x'}{10\mu\text{m}}\right)$$

Now the intensity is maximum at $x' = 0$.

2.71 / 2.710 Optics
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