2.71 Optics   **Spring '09**
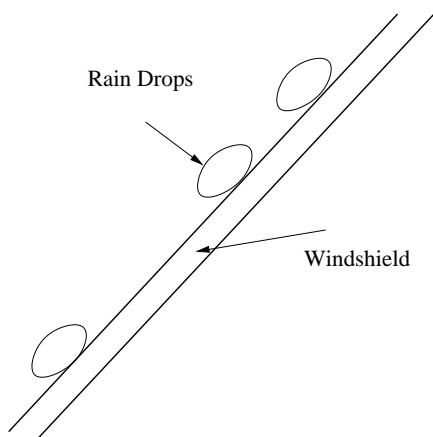Problem Set #2   Posted Feb. 17, 2009 — Due <u>Wednesday, Feb. 25, 2009</u>

1. **Wiper speed control** Design an optical system which can detect the amount of water present on a car's windshield (see the schematic above) to adjust the wiper speed. Use indices of refraction $n_{\mathrm{glass}} = 1.5, n_{\mathrm{water}} = 1.33, n_{\mathrm{air}} = 1.0$.

2. **Telescope with infinite conjugates** A parallel ray bundle of width a_1 and propagation angle α_1 is incident from the left on a two-lens system composed of two positive lenses **L1** (focal length $f_1 > 0$) and **L2** (focal length $f_2 > 0$) as shown below.
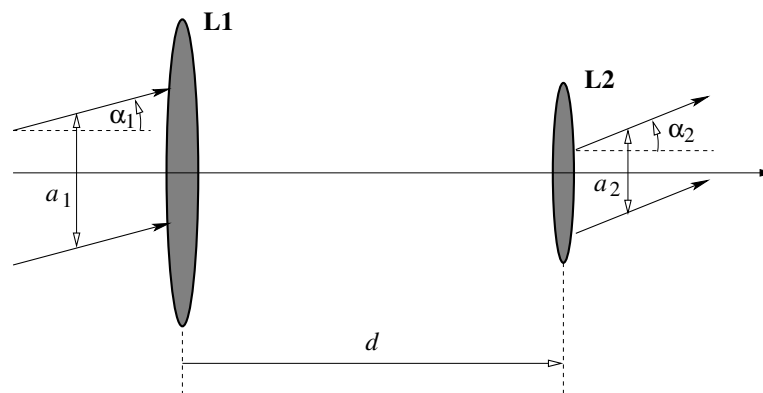

   a) Show that if the separation d between the two lenses equals the sum $f_1 + f_2$ of focal lengths, then a parallel ray bundle emerges from the system.

   b) What is the optical power of this composite optical system? What is the effective focal length?

c) What are the propagation angle α_2 and width a_2 of this outgoing ray bundle?

d) Is an image of the off–axis object at infinity formed anywhere within the given optical system? If so, is the image real or virtual and what is its elevation with respect to the optical axis? Sketch ray traces through the optical system to indicate image formation.

**e)** Repeat the last two questions (c&d) by assuming that L1 is a negative lens ($f_1 < 0$.)

(Note We will see later that an optical system with such specifications is called *afocal* and is also known as a *telescope*.)

3. **Telescope with finite conjugates** Consider an optical system identical to that of Problem **2** with positive lenses L1, L2 but with a point object instead, located at distance f_1 to the left of L1 and elevation x_1 with respect to the optical axis.
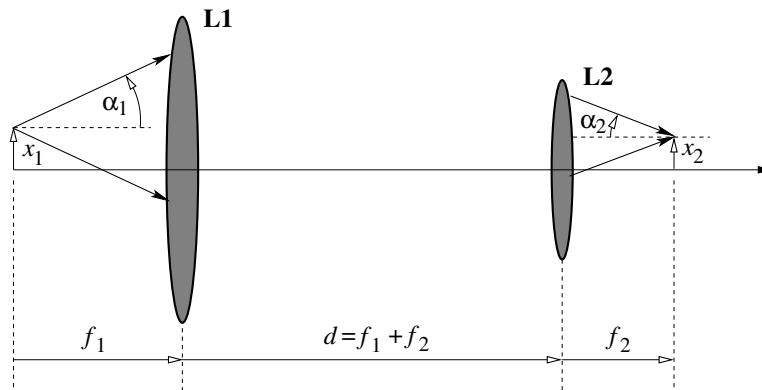
a) Show that an image is formed at distance f_2 to the right of L2.

b) Derive the elevation x_2 of the image and, hence, the lateral magnification M_T.

c) Consider a ray departing from the point object at angle α_1 with respect to the horizontal. Derive the arrival angle α_2 of the same ray at the point image and, hence, the angular magnification M_A. Sketch ray traces through the optical system to support your conclusions in questions (b&c).

d) Repeat the last two questions (b&c) by assuming that L2 is a negative lens ($f_2 < 0$.)

4. **Immersion lens** In class we derived the focal length and imaging condition of a glass lens surrounded by air. In practice, lenses are often "immersed" in liquids. Particularly popular are "immersion microscopy," where the object is immersed and "immersion lithography," where the image is immersed. We will learn later

that the most common purpose of immersion is to increase the numerical aperture of the optical system and, hence, achieve better resolution. For now, we are interested in the effects of immersion on focal length, imaging condition, and magnification. Consider a thin lens of refractive index n_0, with front and back surface curvatures R_1 and R_2, respectively, immersed in liquids of indices n_1, n_2 in the front and back, respectively, as shown in the schematic below.
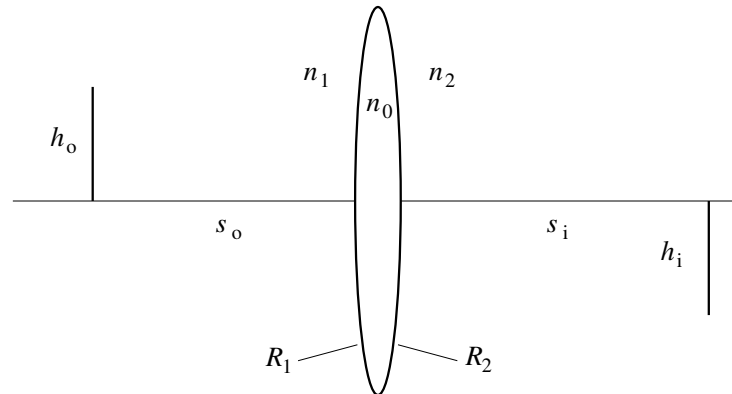
a) What is the focal length f_i of the immersed lens? Make a table that indicates whether the immersed focal length is longer or shorter than the focal length f_a of the same lens in air, depending on the values of n_1, n_2 (possibilities: equal to 1, and larger than 1) and R_1, R_2 (possibilities: negative, zero, positive.)

b) Derive the imaging condition for an object located s_o to the left of the lens, with the image presumed to be s_i to the right of the lens, as shown.

c) Derive the lateral and angular magnifications.

d) Consider the ray that originates at the object's tip (elevation h_o) and aims at the optical center of the lens, *i.e.* the intersection of the lens' left surface with the optical axis. What is the angle of propagation of the same ray after it emerges on the right-hand side of the lens? Provide a simple intuitive explanation for your result.

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