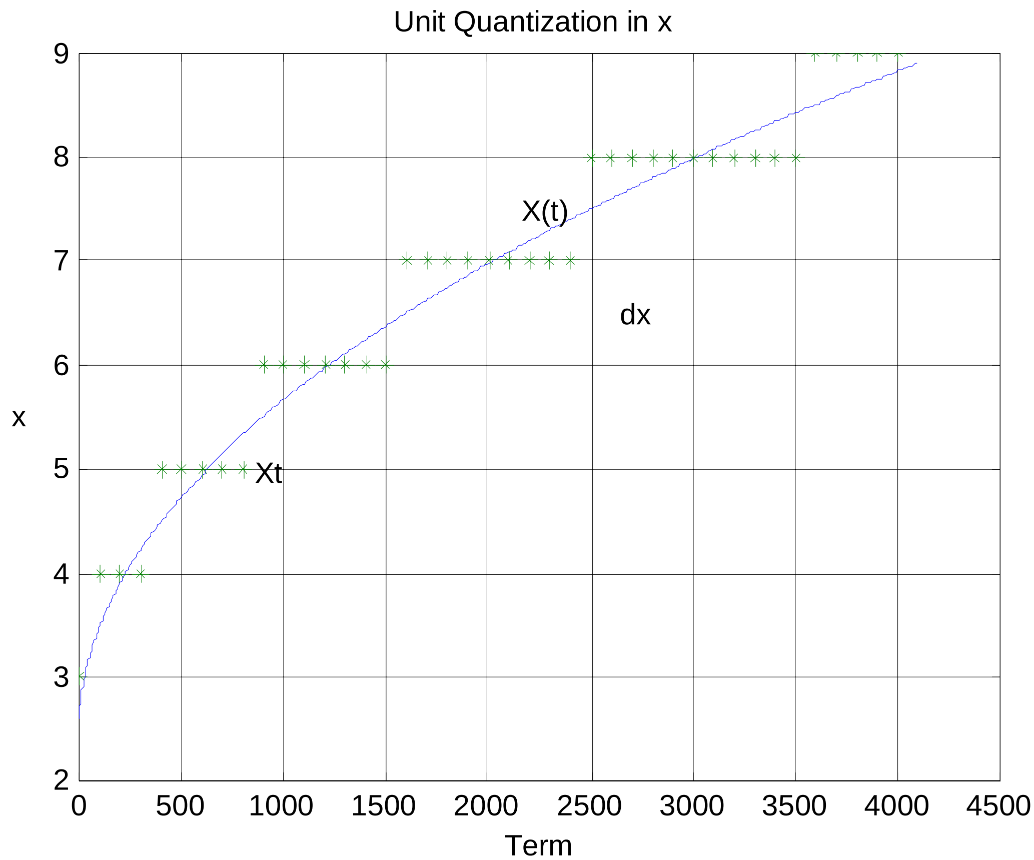
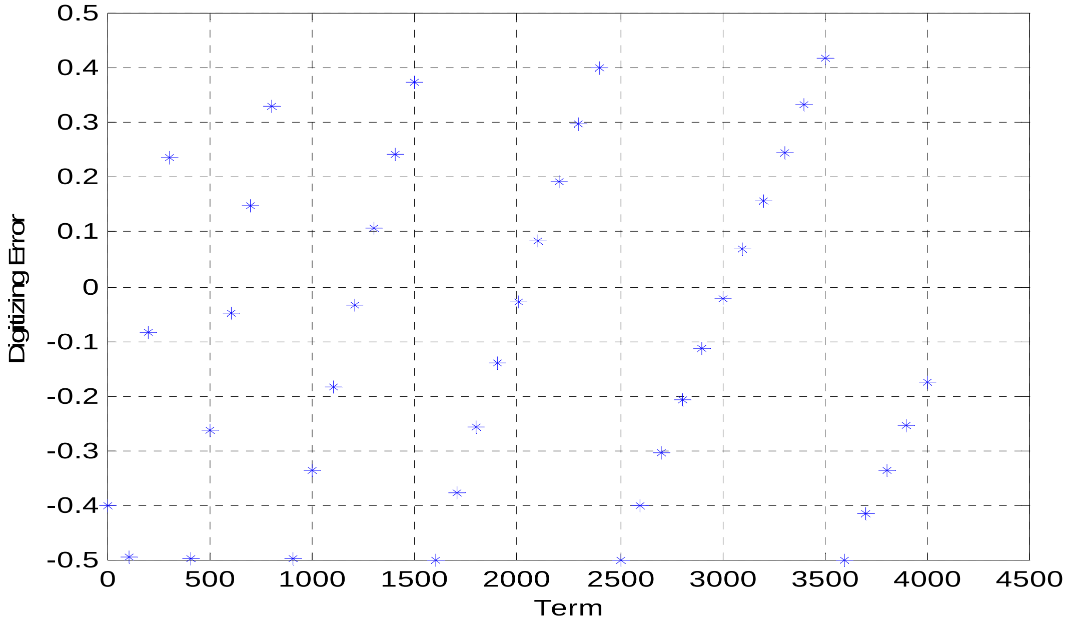
Sampling the environment properly

We are after a statistical description of the environment. We have introduced some new theoretical ideas above, now we apply these concepts to sampling the environment. With the rules above, we ought to be able to design a properly sampled experiment. Since our connection with the world is through a sensor of some sort, we have to also consider the problem of sensor calibration and response, and data system effects.

Digitizing Effects: Real recording systems not only sample at discrete intervals in time (our sampling process discussed above) but also with finite resolution in the quantity itself. To examine this, let our continuous geophysical process be represented by x(t), and the discretely sampled series by x_t where the sample interval is δt and the length of series is m data points. Now instead of an infinitely fine resolution or accuracy in determining the value of x_t, let us suppose that our recording equipment will resolve the value of the signal with a digitizing interval or least count interval of δx.



There is some error, ε_t, associated with each sample point, and graphing this error



If we assume that ε is uniformly distributed (any value between -½ and ½ is equally likely), then it can be shown that the mean of the error, for large m,

$$\mu = 1/m \sum_{t=1}^{m} \varepsilon_t = 0 \qquad \text{Eq 77}$$

and the variance of this error is,

$$\sigma^2 = 1/m \sum_{t=1}^{m} \varepsilon_t^2 = \delta x^2/12 \qquad \text{Eq 78}$$

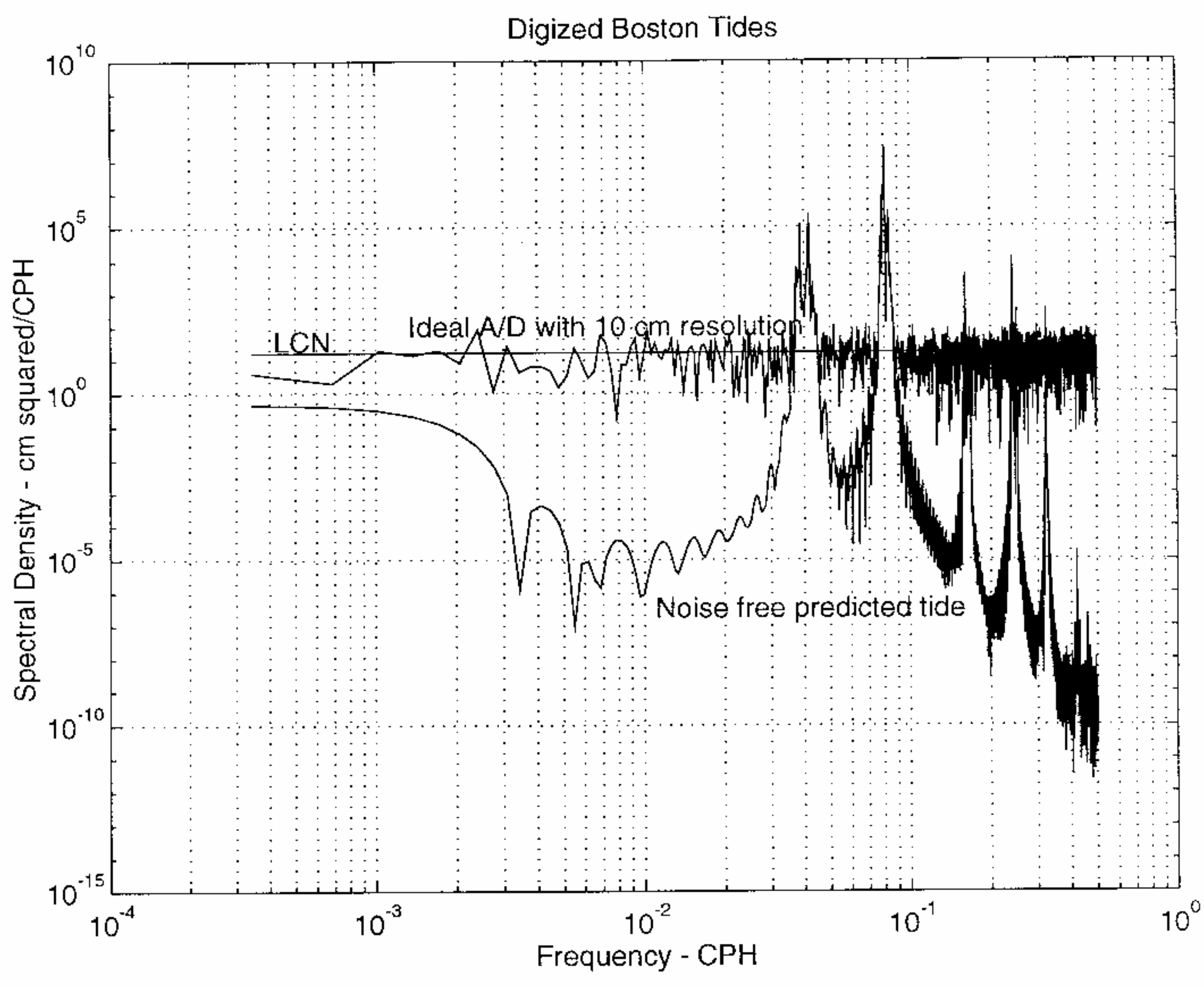
Here we have taken the ideal A/D converter which gives the closest point to the continuous curve x(t). Some A/D's give the highest number that is <u>below</u> the actual curve, e.g. the old Aanderaa 10 bit mechanical digitizer is an upper bound approximation type of digitizer. It should be obvious that the error is now always positive, and thus the mean is non-zero and our estimate has a "bias" which is not desirable. Therefore, beware of the type of digitizing that is being done so that you are sure that no biases are added to the data. Modern digitizers are actually quite good. If you have a biased type of A/D, make sure that you have an extra least significant bit that you don’t need and can discard, i.e. make sure that you have enough bits so that any bias of a fraction of a least significant bit is not important.

If the energy in our error were evenly distributed in frequency (or what we call "white"), then the spectral density would be

$$P(f) = \frac{\delta x^2}{12\, f_N} \qquad \text{Eq 79}$$

This gives a spectral "noise" level or a lower bound on the geophysical spectra that can be resolved because of the digitizing error of our measurements. This is just the error variance spread evenly from 0 to the Nyquist frequency.

This is illustrated on the following page where a computer generated tide for Boston, MA was "digitized" by the computer with a quantization interval or least count, δx = 10 cm. The power spectral density of the original noise free predicted tide and the digitized tide was then calculated and is plotted. At the diurnal and semidiurnal tidal frequencies the signals agree, but the digitized tide appears to have a constant level of "noise" below which the spectra does not go. This is the digitizing noise. Note the dynamic range of the plot, each tick on the ordinate is equal to 10 db, so the whole spectral plot is 250 db. The theoretical tide has energy only at the tidal lines around 0.04, 0.08, 0.16, 0.24, and 0.32 cph (1, 2, 4, 6, and 8 cycles per day). The energy that shows up at other frequencies in the spectra is due to windowing and resolution of the PC using MATLAB to do the calculation.



Spectra of a noise free predicted tide for Boston Harbor, and an artificial, digitized spectra of the same record, with digitizing resolution, δ=10 cm. The least count noise (LCN) predicted from theory for an ideal A/D converter is also plotted, showing the agreement of observation and theory is excellent.

The digitizing spectral noise level or Least Count Noise (LCN) was calculated from Equation 79 and plotted as the solid, straight line for comparison with the observed results. The agreement is excellent.

$$LCN = \frac{10*10}{12*½} = 16.66666..\text{cm}^2/\text{cph}$$

The numerical average of digitized spectra from 0 to 0.03 cph = 16.667 cm^2/cph, exactly the predicted value. The difference between the two time series is the error, ε, and we find the following statistics:

mean (ε) = 0.1024 cm (instead of predicted 0.0 cm)
maximum (ε) = 4.9982 cm (instead of predicted 5.0 cm)
minimum (ε) = -4.9962 cm (instead of predicted -5.0 cm)
standard deviation (ε) = 2.9294 cm (instead of 2.8867 cm)

Digitizing error in Frequency and Period Counting Techniques:

Many older oceanographic sensors output a frequency-modulated (FM) signal whose frequency is proportional to the quantity being measured. Common oceanographic examples of these types of sensors we will examine later include the Paroscientific "Digiquartz" pressure sensor, and the Sea Bird "Wein Bridge Oscillator" temperature and conductivity sensors. The output of these sensors is then digitized by a frequency or period counting technique, which also has an error associated with it. Irish & Levine, 1978 and Payne and Smith, 1980, studied this error and determined that it is just twice that of the ideal A/D converter we discussed above. We will examine this work in detail below.

As an example of using the pressure sensor to measure the instrument depth, sea level and wave activity, we invoke the Hydrostatic Equation, and for most cases assume that the depth is small enough and the frequency of the surface elevation fluctuations small enough that we can make the simplification to the vertical equation of motion to obtain:

$$dp/dz = -\rho g$$

The pressure we observe at a depth, z, is really the integral of this from the top of the atmosphere to the instrument.

$$P = P_{atm} + \rho_0 g \eta + \int \rho' g \, dz + \rho_a g z_0$$

Where the first term, P_{atm}, is the atmospheric pressure at the sea surface, the second, $\rho_0 g \eta$, is the fluctuation due to sea surface, η, changes (where ρ_0 is the sea surface density), the third term, $\int \rho' g \, dz$, is due to the internal density field changes (where ρ′ is the deviation from the averaged density ρ_a) and the fourth, $\rho_a g z_0$, the constant mean depth term at depth z_0.

Let us assume that we can model the pressure sensor as a linear system measuring sea level, η, as above, then we can say that the sensor frequency, f, is

$$f = A + B \cdot \eta$$

where A and B are the calibration constants of our sensor, η is the surface elevation and f the sensor's output frequency. The output period of the sensor is just 1/f, or

$$t = 1/f = (A + B \cdot \eta)^{-1}$$

We then digitize the sensor's frequency with either a frequency or period counting technique. For example, we can count the number of periods, t, which occur in a 1-second interval, and output the resulting count as cycles/sec or Hz. (This is what a standard laboratory frequency counter that we used in the laboratory does). The 1-second sampling interval is called the gate time, and the count is the average of the frequency during this gate time. Thus, an averaging filter is applied to the data during the frequency counting process. Note that this is a prefilter reducing unwanted high frequencies.

The number of sensor periods, t, which occurs during a fixed gate time, g, is just the sample count, k

$$k = g/t = g\ f$$

The count is an integer and therefore has an error similar to our A/D digitizer. We can write the sea level height, η, in terms of the count, k, as

$$\eta = \frac{(k-g)\ A}{g\ B} = \frac{k}{gB} - \frac{A}{B}$$

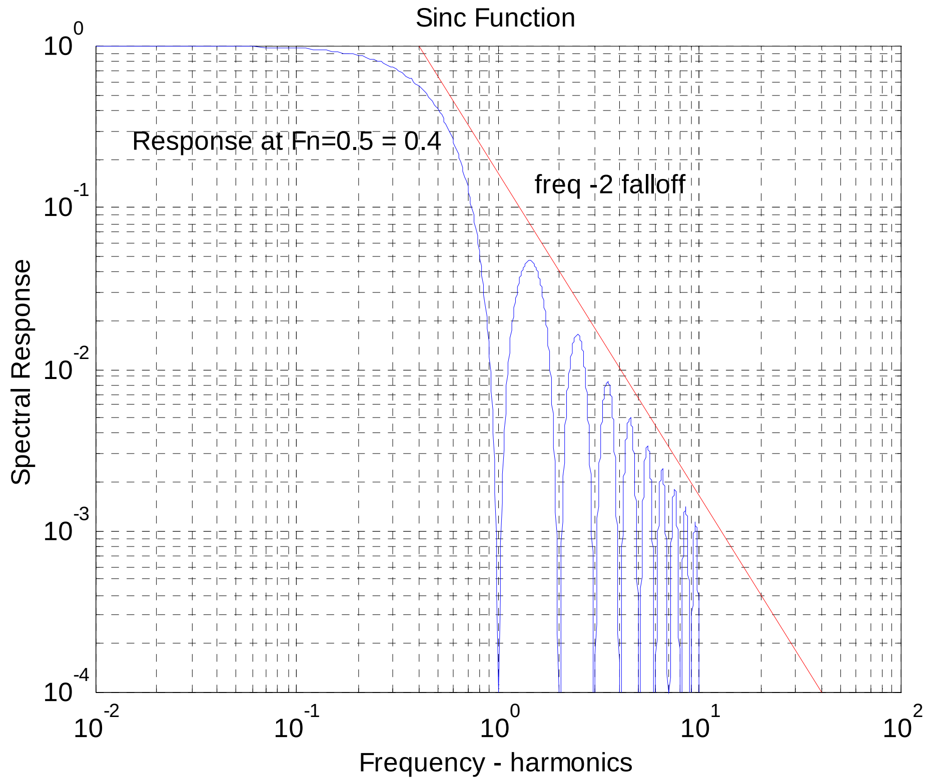
Now in reality we record the count as integer numbers, where the resolution or least count (LC) is the geophysical value of one count and is just the derivative of η with respect to k,

$$LC = \frac{\partial \eta}{\partial k} = \frac{1}{gB}$$

This is dependent on the gate time, g, and the sensor's sensitivity, B. The longer the gate time, g, or the more sensitive the sensor (the larger B is), the smaller the least count or resolution.

Therefore, to improve the resolution with a given sensor, the gate time, g, must be increased. However, it is often not possible to increase the counting time for several reasons, including, a faster sample interval than possible is required, or the sensor need to be powered down to conserve battery power. To improve the resolution by a factor of 10 from the 1-second count used above, we need to count for 10 seconds. To improve by a factor of 100, we need to count for 100 seconds, and the time quickly become too long. (Remember that to prevent aliasing we need to do some prefiltering. If this is not possible by some method such as a thermal mass for temperature, then the sensor must be sampled at the highest rate to prevent aliasing. Counting over the entire sample interval is a good method to use to avoid aliasing.)

Counting the sensor's frequency over the gate time is an averaging filter in time space that acts like a low-pass filter, suppressing high frequencies. We can consider how this behaves in frequency space by taking the transform of the gate function, which, as we saw above, is a SINC function. The figure on the following page shows the SINC function squared or the spectral response, with the first null at a frequency of one cycle per gate time, or a frequency of 1, and subsequent notches at integral gate times, i.e. 2, 3, 4, 5, . Note that the envelope of the side lobes decreases with increasing frequency as f^{-2}. This is the normal suppression of energy. The filter's response falls off as f^{-2}. This is the low-pass filter that we have applied to our data by the frequency counting technique. Note that at the Nyquist frequency (0.5) the filter has only supressed the high frequency Spectral Density to 0.4 of the low frequency limit of 1, or the signal is down 4 db (actually 3.97), which is 0.63 times the amplitude.



The spectral response of the Sinc function showing the nuls at the inverse record length, and the general fall-off of the side lobes at a rate of f^{-2}.

For a frequency counter, if the gate time is set for 1 second, then the frequency is measured with an accuracy, or least count of 1 Hz. To increase the resolution of the measurement, we must increase the gate time. For a 10-second gate, the least count now becomes 0.1 Hz. It is obvious, that to get increased, accuracy, long averaging intervals or gate times must be used. This is often the case in oceanographic instrumentation where one desires a reading after many minutes, and can spend the time (at the expense of sensor power) counting for better resolution.

To improve the resolution and/or to increase the sample rate over the frequency counting technique discussed above, we often use a period counting technique. The period count, K, is the number of periods of a clock frequency, f_{cl}, of period, T, which occurs during a gate time, G, which is a fixed number, M, of sensor periods, t. Therefore, we can write the gate time as

$$G = M \cdot t = M/f_s$$

and the period count K as

$$K = G/T = M \cdot t/T = M \cdot (f_{cl}/f_s)$$

then the sea surface elevation, η, can be found from the period count, K, by

$$\eta = M/(KTB) - A/B$$

and our least count is now

$$LC = \frac{\partial \eta}{\partial K} = \frac{-T \cdot (A + B\cdot\eta)^2}{MB}$$

Now the least count is a function of the sea level, η, or is data dependent. Also, the averaging interval of the gate, G, is dependent on the sensor's output, so the averaging filter can not be exactly written, i.e it also changes with η. If the sensor is used over a significantly large portion of its frequency range, then the least count can change significantly over the experiment. This is not as nice as with the frequency counting technique. However, normally we will use a sensor over a small part of its frequency range, so that its period, t, changes by a small fraction of its value and we then assume that G is nearly constant and the LC is nearly constant. The advantage of the period counting technique is that it gives better resolution. It divides the sensor's frequency up into parts that are equal in length to the clock period, so improves the resolution or LC by the ratio of the clock to sensor frequency. For example, if we have a 10 kHz Sea Bird temperature sensor being counted by a period counting circuit with a 131,072 Hz clock, then the improvement in resolution is 13, and with a 1 MHz clock, the improvement is 100. Also, we can think of retaining the same resolution and sampling 13 or 100 times faster!

However, as with the A/D converter, there is an error associated with the frequency or period counting technique. Consider the frequency counting technique as illustrated on the next page with Fig 6 from Irish and Levine (1979). The gate time g is a fixed interval of time, and the count is the number of sensor rising edges which occurs during this time. There is a fraction of a count error at the start since the gate opens a time δ (where δ is between 0 and 1) times the period, t, after the last sensor count. Also there is another error at the end where another interval γ (where γ is between 0 and 1) times t occurs. Note that if the gate opens just before a pulse and closes just after a pulse, there will be one extra count present. If the gate opens just after a pulse, then closes just before a pulse, the count will be one short. Therefore, our error appears to be ±1 count, rather than the ±1/2 count we had in the ideal A/D considered above.

Let us considering the period counting technique in detail. The situation is illustrated in Fig 3 from Irish and Levine (1979) below. The gate opens a time δT after a clock pulse. Again $0 \le \delta \le 1$, and T is the clock frequency. The gate is opened at the start of a sensor pulse and remains open for an integral number of sensor pulses. During this time, the clock pulses are counted. The integer count, C, is then

$$C = INT\{(G + \delta T)/T\} = INT\{G/T + \delta\}$$

where the INT function is the largest integer smaller than the number, or a truncating integerizing function rather than a rounding function. The error associate with this measurement is then just

$$\varepsilon = G/T - INT\{G/T + \delta).$$

Now the gate, G, is open for an integer number of clock cycles, N, plus some additional γT extra, where $0 \le \gamma \le 1$, or

$$G = NT + \gamma T.$$

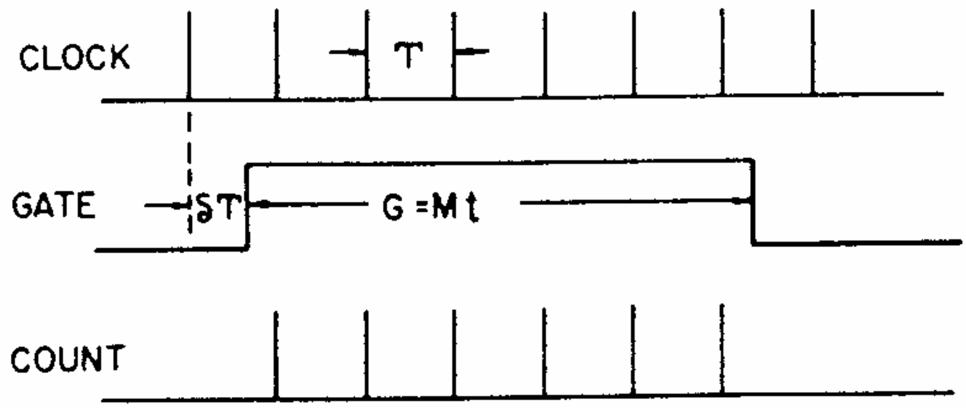


Fig. 3. Period counting technique.

Figure 3 from Irish and Levine showing the period counting technique .

Then the error can be rewritten as

$$\varepsilon = N + \gamma - INT\{N + \gamma + \delta) = \gamma - INT\{\gamma + \delta)$$

Therefore, the INT{$\gamma + \delta$} is either 0 or 1, depending on the values of γ and δ.

$$\varepsilon = \gamma \text{ or } \gamma - 1$$

where γ ranges from 0 to 1. Again the error is ±1, or twice our ideal A/D.

If the uncertainty in the start δ and the integral number of clock periods remaining, γ are uncorrelated with the gate time, G, and uniformly distributed between 0 and 1, then

$$\text{mean of } \varepsilon = \mu = 0$$

and the period counter is an unbiased estimator of the frequency. And further we find that

$$\text{variance of } \varepsilon = \sigma^2 = 1/6 \text{ of a count}$$

or twice our ideal A/D. If the energy is white, or spread evenly between 0 and the Nyquist frequency then the least count noise, LCN is

$$LCN = \frac{LC^2}{6 * f_N}$$

If the uncertainties in start and end (γ and δ) are correlated with the gate time, G, then the variance is still 1/6, but now it is not white or uniformly distributed in frequency. This is the case for (1) the Sea Data bottom pressure recorder which is normally used with the Paroscientific pressure sensor, and (2) the temperature sensor in the Vector Averaging Current Meter (VACM). The VACM averages the output of a thermistor that has been converted from a resistance to a frequency over the entire sample interval. Now, the amount missed at the end of one count plus the amount missed at the start of the next count is just the clock period, or in our notation, δ +

$\gamma = 1$. Obviously, δ and γ are related! Note that the variance is still the same, but now it is not white, or evenly distributed with frequency, but now becomes,

$$LCN = \frac{LC^2}{6\ f_N}\left[1 - \mathrm{Cos}(\pi\ k/m)\right]$$

where k is the index of frequency running from 0 to the number of points, m, in the transform. This is shown by the figure on the following page from Payne and Smith, 1980 which shows the spectra of a VACM thermistor and the estimated noise assuming correlated errors, and again the agreement is good. The variance is now concentrated at high frequencies where the spectral energy is low, not where we would prefer to have the most error.

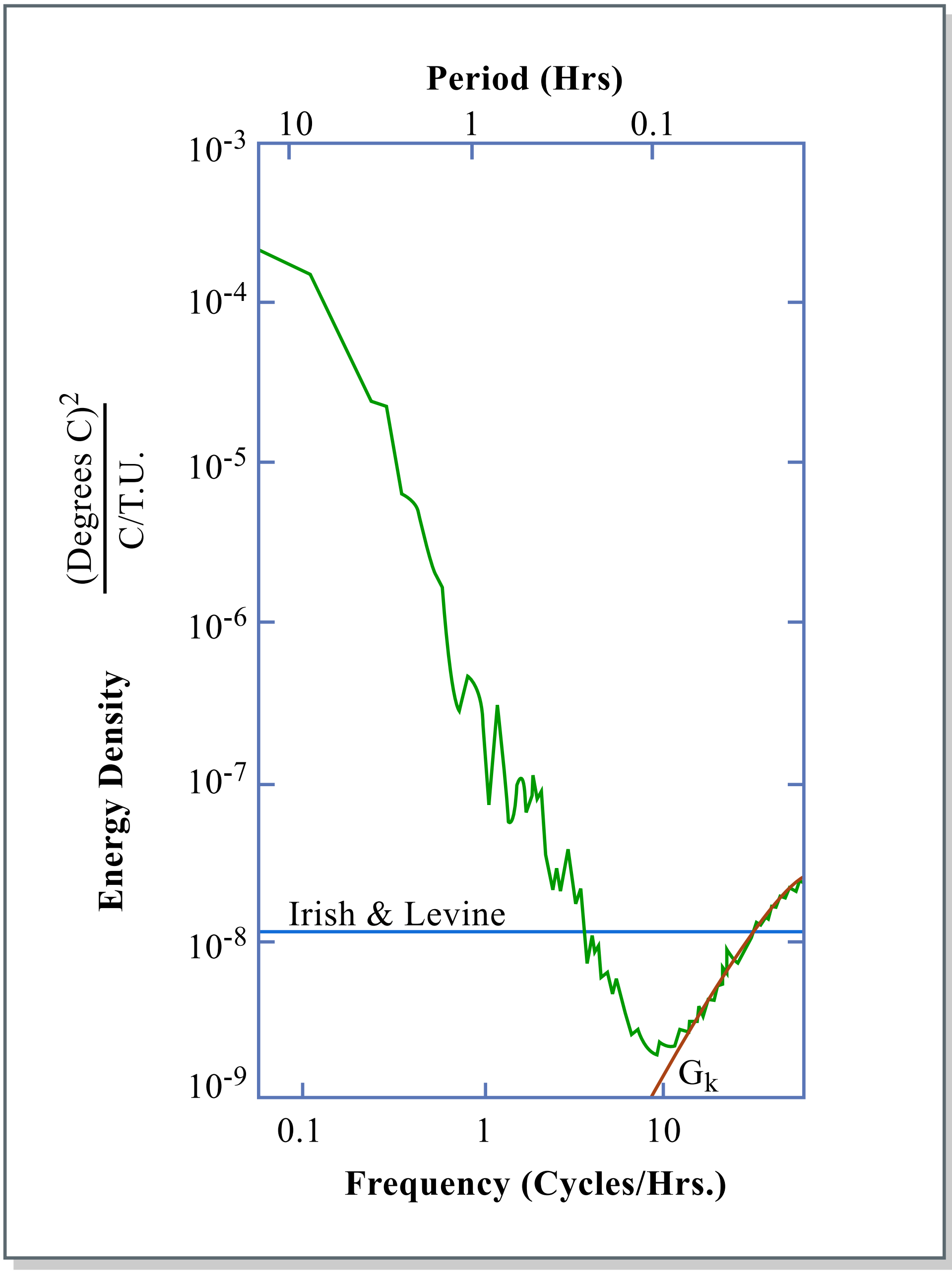


Figure by MIT OCW. After Payne and Smith, 1980.

Spectra of VACM temperature from a frequency counting technique averaging over the entire sample interval so the error in counting between the start and end is correlated.

Going back to Levine and Irish, 1978, the result for uncorrelated, white digitizing noise was tested by a computer simulation on top of the following page, and by actual data recorded with a pressure sensor at the bottom. The simulated digitizing had a digitizing interval of 11.11 cm, and a least count of 41.1 cm²/cph, which agrees well with the test. The more realistic test is the later case. Here the period counting technique was optimized for a temperature sensor, and so the pressure sensor was recorded with the same circuitry with a least count of 11.5 cm. This is not adequate for anything but a measure of the instrument's depth, and crude tides (i.e. the low frequency portion of the spectrum). The high frequency part of the spectrum of the bottom pressure record appears to level out at low frequencies. To see if this is consistent with digitizing error, the estimated least count noise plotted as a dashed line. The pressure observation is clearly limited by noise, and the least count should be reduced or the sample interval increased to about 1 per hour to save on data storage space since the high frequency signals recorded are just digitizing noise. However, as both cases show, the spectra of the digitized signals and the predicted least count noise are in good agreement, showing that we have adequately modeled the period counting technique.

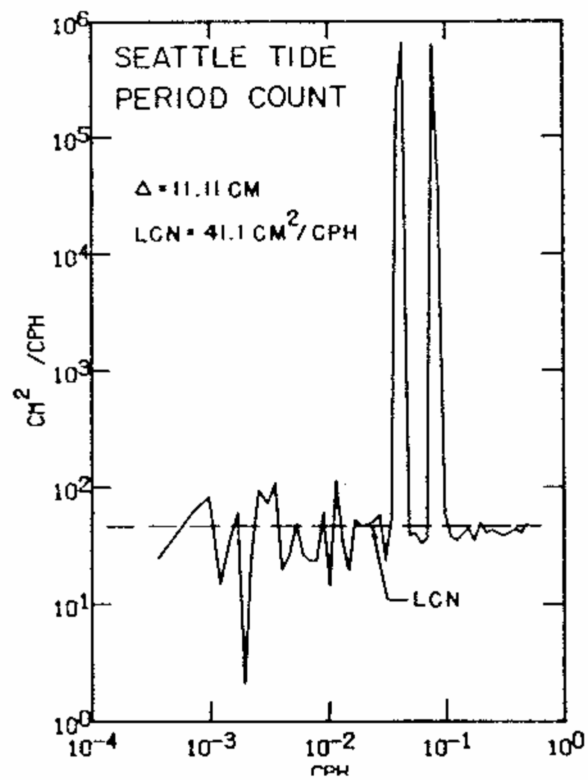
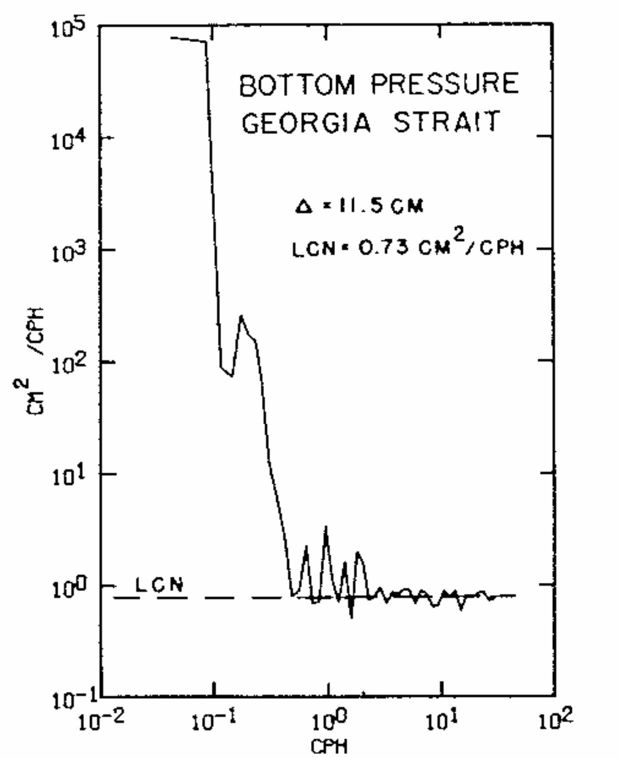


Figure from Levine and Irish showing the simulated digitizing by frequency counting technique and the least count error resulting compared with the theoretical prediction.

The period counting technique is used because it increases the resolution that can be obtained with a given sample interval, and is especially good for rapid sampling, such as in a CTD profiler. However, there are some major difficulties. To optimize the count, the gate time, G, should be as long as possible without exceeding the sample interval. If G is greater than the sample interval, the gate does not close before the start of the next sample, and data is generally lost. Therefore, M must be selected so that M*t is shorter than the sample interval. Now, to get

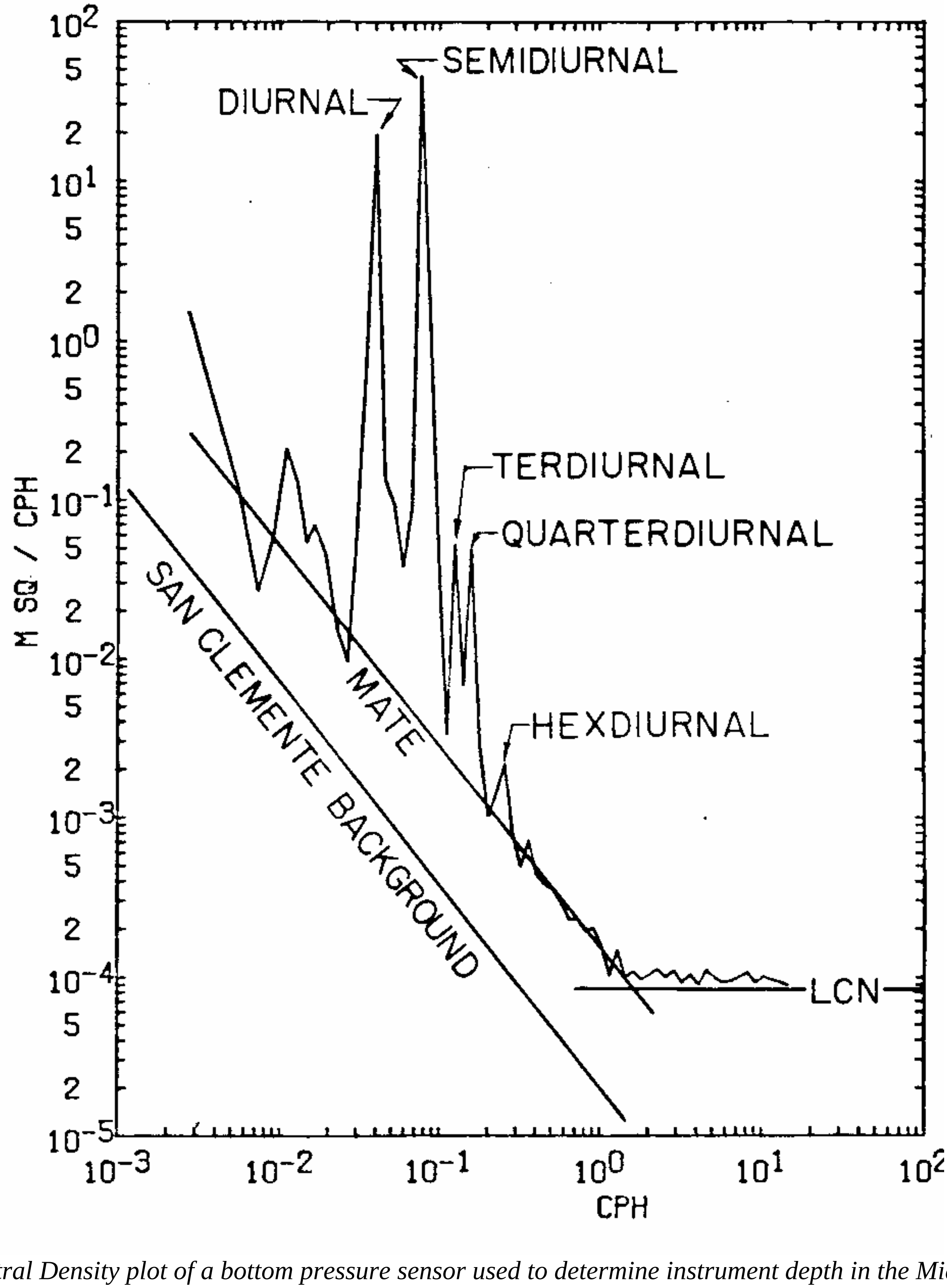


Pressure spectrum which shows a leveling off in spectral level at high frequencies which agrees well with the predicted least count noise from a period counting technique.

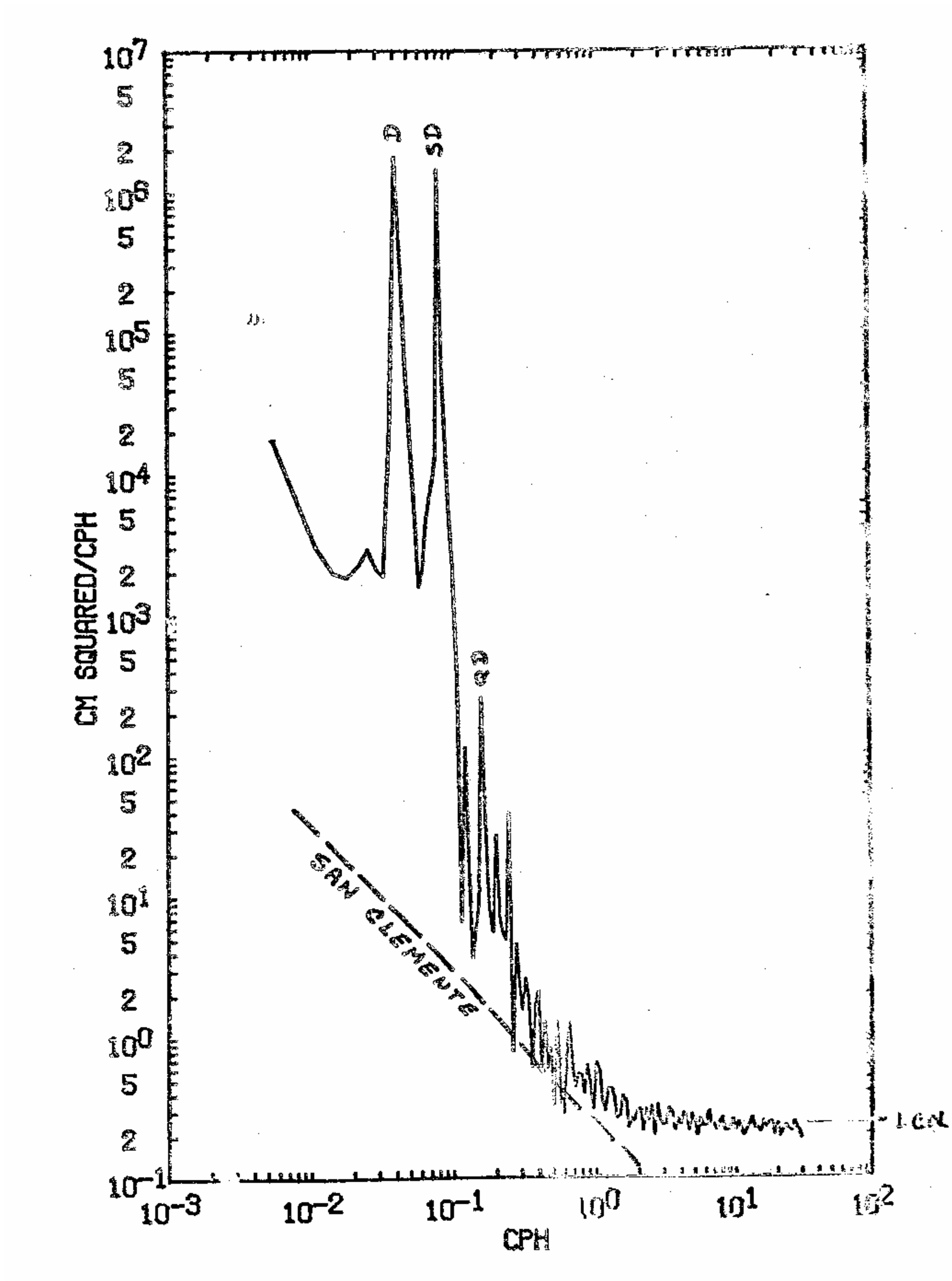
the best least count, M should be as large as possible. Also, to suppress the high frequencies in the best manor, we want G to be as large as possible. The implication of this is that you must know the expected signal range to make the optimum selection of M.

Also the least count may be a problem. If the clock frequency is too large, then the counts will be too large and storage space will be used up with recording too many least significant digits which do not have any geophysical significance. This emphasizes that to design the optimum sampling program, you need to know what the signal is that you are going to measure. Therefore, you may want to divide the clock frequency down, or "throw away" some lower or higher significant bits before recording the data to reduce storage requirements. This pre-processing is simple with modern microprocessor controller based instrumentation. However, if you are going to “throw away” some bits, particularly significant bits, you should know the effect of what you are discarding and any uncertainties you are inserting into the data.

In some moored applications, a period count is used to make the measurement quickly, so that power can be shut off to the sensors. If a mechanical pre-filter is used to suppress unwanted high frequencies, then this technique works well. What we used the moored application which will be discussed below was a few seconds to sample 12 sensors every two minutes. If we did not have the mechanical low-pass filter, then the period counting technique would not average over enough time to suppress the unwanted high frequencies and we would most likely have an aliasing problem. The alternative is to have the sensors powered up and count over the 2 minutes to get the same resolution. However, now the power is up by a factor of 60.



Spectral Density plot of a bottom pressure sensor used to determine instrument depth in the Mid-Ocean Acoustic Transmission Experiment at Cobb Seamount in the North Pacific. The southern California sea level background is shown as is the LCN for the period count used. The tides are the signal that exceeds background.



Another Spectral Density plot of measurements of pressure on a moored array in Puget Sound showing the southern California background and the LCN limitation.