

Lecture #8: Background

A ray is not a mode, and a mode is not a ray; but the two are hooked together, in a very clever way. OK, really bad verse, but the message is correct!

To link high frequency rays with low frequency modes, a pretty obvious solution is to look at the “intermediate frequency” solution, which we saw was WKB. In the short paper “Rays as interfering modes in underwater acoustics,” Tindle and Guthrie do just that. They set up a normal mode solution with WKB mode functions, and then look at the accumulated modal phases φ_n from source to receiver. The key to this is to have lots of modes (mid frequency!) and then set the condition that groups of neighboring modes interfere constructively, i.e. $(\Delta\varphi/\Delta n) = 2m\pi$, whereas m is an integer. Doing this, one recovers the equation for the ray paths from source to receiver, which for non-math majors is as close to QED as you need!

The next topic, horizontal refraction of modal paths, is really our introduction to full wave (i.e., not ray theory) 3-D acoustics. While much of ocean acoustics can be done adequately in 2-D, i.e., the x-z plane between source and receiver, it has become apparent that much cannot, particularly in coastal environments.

In COA, section 5.13 and 5.13.1, the development used to generate the coupled/adiabatic solutions in 2-D (r,z) is now followed in 3-D (x,y,z), using the same local modes/partial separation of variables procedure. In this case, we generate a fully 3-D coupled mode equation (5.318), and a 2-D (x,y) set of local modes, $\Psi_m(x, y; z)$. If we throw away the mode coupling term, we are back to the adiabatic solution, but now along out-of-plane paths connecting source and receiver. In the next section of COA 5.13.2 (which we don’t pursue, but is worth reading), COA shows an example of 3-D propagation over global scales. However, you don’t have to go so far to see 3-D ocean acoustics effects – the nearest beach will do, as we will see!

In section 7.3 of B&L a very simple procedure is shown for calculating the horizontal, out of plane path that each mode takes in going from source to receiver. Given that we’ve calculated a 2-D (x,y) set of local modes as part of the full 3-D equations we just discussed, we have a nice map of $k_l(x,y)$ at our disposal (or ζ_l in B&L’s notation). If we define $\vec{r}=(x,y)$ and $v_l = \omega/k_l$, we can use the $k_l(\vec{r})$ for each mode to define a 2-D (relative) index of refraction field $n_l(\vec{r}) = k_l(\vec{r})/k_l(0) = v_l(0)/v_l(\vec{r})$ for each mode. Hmmm! Given a 2-D index of refraction field, doing simple ray traces comes to mind, and indeed that was exactly what was (and still is) done to track the out of plane mode paths (called “modal rays” – go figger!). As a note, this procedure is often called “Weinberg and Burridge’s vertical modes – horizontal rays” formalism. It was actually first published on by Allan Pierce, but the Weinberg-Burridge paper showed a more complete picture of how the vertical modes and horizontal “modal rays” combined into a 3-D adiabatic field. One further note – ray theory need not be used here in the (x,y) plane – one can also use parabolic equation, as we’ll be seeing.

As mentioned, one can just go to the beach “to see 3-D acoustics effects, and that is what B&L’s next section, 7.3.1, discusses. Using a coastal wedge and (what else?) hard bottom waveguide modes, B&L easily set up the index of refraction field in Eq. 7.3.2. From there, it is just a matter of applying Snell’s law to the various launch angles and getting analytic

expressions. The modal-ray paths form a series of hyperbolae; these can be significantly curved. Such 3-D path curvature due to bathymetry has been seen many times at this point, and is a well-established effect in ocean acoustics.

Before leaving this section, we will note that seamounts, canyons, and continental slopes also move the modal ray tracks in deeper water – the coastal wedge is just a nice, easy shallow water example.

Also, we will note that, especially in shallow water, the x-y variation of the oceanography (even with a flat bottom) can also bend energy out of place. The “horizontal Lloyds mirror” is one such effect, and is given for homework.

As has been mentioned repeatedly, the similarity of the Helmholtz equation to the Schroedinger equation of quantum mechanics allows one to easily transcribe some quantum mechanics techniques to the acoustics world. One of the most important of these techniques is perturbation theory, which (strangely to me) is not so well known to many engineering students. In perturbation theory, we can look at an unknown problem that is in some sense “close” to one we know the answer to, and get the answer to it. In quantum mechanics, we perturb the potential term in the Hamiltonian. In ocean acoustics, the term corresponding to the potential is the wavenumber $k(z)=\omega/c(z)$. This wavenumber is frequency dependent via ω , and also depends on $c(z)$ which has real and imaginary parts. We can (and will) perturb ω , $\text{Re } c(z)$ and $\text{Im } c(z)$, which will give us results for modal group velocity, a “perturbed sound speed” system, and the modal attenuation coefficient, respectively.

In the class notes, the derivation of time independent perturbation theory is shown, where one makes a small change in $H^{(0)}$ (and to the wavenumber in ocean acoustics), which makes a small change in the modal eigenvalues and eigenvectors. The changes in the eigenvalue and eigenvectors are expressed as a power series in the small parameter λ , and then one matches powers of λ to get a hierarchy of perturbations (zeroth, first, second, etc. order). The zeroth order in the expansion is just our old problem, and the higher orders are successively better approximations to the “new” problem. So, what seems rather impressively entitled a “time independent perturbation theory” is just a simple power series!

Now we look, in the class notes, at how to get the attenuation correction from the perturbation forms. We start by simply adding an imaginary part to k , i.e. $k \rightarrow k + i\alpha$. Using this, we can find the form for H in the equations, and also the ΔE , all via simple algebra. When we’re done, we have an integral form for the imagery part of the modal wavenumber (the attenuation), all in terms of known quantities! This is very simple, very useful.

Before going further, we should also note that the integral perturbation form we have is a Fredholm integral equation of the first kind, and these form a large part of the basis for inverse problems!

Knowing the modal attenuation, which in low frequency ocean acoustics is generally due to losses to the bottom, is a good start. Next, we look at perturbing the real part of the wavenumber. This is also useful, in that it lets us look at more complicated $c(z)$ profiles without

having to necessarily turn to a numerical calculation. Moreover, it can give us analytical forms if the integrals are doable exactly.

The real part perturbation to $k(z)$ is accomplished by letting $k(z) = \omega / (c_0(z) + \Delta c(z))$, where $\Delta c(z)$ is a new piece we add to an old (known answer) profile. The algebra is again simple, and now we get a perturbation to the real part of the modal eigenvalue as an answer, again in integral form.

Both the real and imaginary wavenumber perturbation forms are used extensively in ocean acoustics, and also in inverse theory.

In section 5.9 and 5.91 of COA, the attenuation argument we discussed above is again recounted, and one can see a similar deviation to my own. In COA, the modal attenuation perturbation is also applied to the case of small bottom roughness, an interesting application.

The final application of (modal) perturbation theory we will discuss is perturbing the frequency. This seems odd, until you remember that $\frac{\partial \omega}{\partial k_n}$ is the group velocity of the n th mode, and that this perturbation will thus give us an integral expression for the group velocity. This still seems like an odd thing to want until you also remember that obtaining the group velocity via the usual derivative operation is noisy and error prone! Derivatives are noise amplifiers! However, having an integral form cures this noisiness problem, at the price of a very slight bit more calculation (the integral). Integrals are smoother! The mechanics of the perturbation for ω are the same as before, so one can just read section 5.9.2 of COA for details.

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