Lectures #4-5

READINGS AND FIGURES

Readings: Sections 1.1-1.4 COA, Sections 3.1, 3.2, 3.3, 3.3.1 COA, 3.7 COA (old book was 3.6), 3.7.1 COA, 3.7.2 COA, 3.3.5.2 -3.3.5.4 COA,pp 126-127 Fowles "Optics", 3.6.5 COA, Fermats Principle in Resnick and Halliday "Physics",3.5 COA, 7.3 Frisk, 7.4 Frisk

Figures: Vince Lombardi (from internet), Willebrord Snellius (from Wikipedia), Ibn Sahl (from Wikipedia), 1.7 COA, 1.10 COA, 1.12 COA, 1.11 COA, 1.14 COA, 1.13 COA, 1.15 COA, 1.17 COA, 1.18 COA, 1.19 COA, 3D Raypath Courtesy Y.T. Lin (WHOI), 3.4 COA, 3.7 COA, 3.8 COA, 4.20 Fowles "Optics", 3.15 COA, 3.16 COA, Gaussian Bean reflection off rough surface Courtesy Y.T. Lin (WHOI), 7.2 Frisk

DESCRIPTION OF NOTES

Having gone through some wave equation basics, we can now start looking at wave equation solutions and numerical modeling. In doing this, we will combine the "basic wave theory" and "numerical modeling" parts of the course (parts 1 and 2) together. This is a rather natural way to do things.

The simplest, most easily insightful form of wave/propagation theory is ray theory, and so we will start there. Since discussing Snell's Law with a group of MIT grad students borders on the insulting, I intend to: 1) show an amusing analogy to this case, and 2) convince the students that they perhaps don't know *everything* about Snell's Law (like, perhaps some of its history!)

The analogy is from American football, when legendary coach Vince Lombardi walked into the locker room after a loss by his usually dominant Green Bay Packers, and to insult them held up a football, glared at them, and said "Gentlemen, this is a football!" Pretty much the same thing as discussing Snell's law at the graduate level.

The history question is: What was Snell's first name and, by the way, did he first discover Snell's Law? (Ans: Willebrord and NO!) If you check out the notes, you will see the ancient Persians first described Snell's Law, which was very pleasing to one of our Persian heritage students this year.

Returning to Snell's Law, it simply tracks the path of the acoustic energy for us, given a certain launch angle from the source. And the "only" input it needs is the ocean soundspeed field, $c(\vec{r}, t)$, or in many cases just *c(z)*. (The "only" is in quotes since this is actually a HUGE demand if we take it literally.)

It is worth looking at a few examples of ray theory calculations. Only the figures are shown in the notes, so we will supply some surrounding verbiage here.

In Figure 1.7 from COA, the main classifications of ocean acoustic raypaths are shown, i.e. the "usual suspects" to quote the movie Casablanca. In Figure 1.10, we see the deep ocean phenomenon of strong convergence zones, where the non-bottom interacting rays with $|\theta_s| \leq 10$ degrees are seen to cycle in tight bundles. Thus, a near surface receiver will see a strong signal every 65 km or so in range.

Another deep ocean example in Figure 1.12 of COA shows "deep sound channel" propagation, where the ray hits neither the surface nor the bottom. Such rays are useful for ocean acoustic tomography, an inverse/imaging technique we will discuss later in the course.

In Figure 1.11 of COA, we revisit the Figure 1.10 example to show a little of the mechanics of ray tracing and usage. A simple trick to calculate raypaths in the ocean is to approximate the soundspeed profile by a series of constant gradient (linear) segments. For a constant gradient, the raypaths are circles (see the homework problem), so that for each segment the paths are strictly analytic. At the boundaries of the segments, we simply use Snell's Law to match to the angle of the segment above or below. This makes life simple. One use of such a program is to compute the convergence zone (CZ) length, as shown in Figure 1.11 of COA. It should be noted that difficulty is often conserved, and that there is a downside to the "linear segments" approach. The artificial soundspeed discontinuities between segments cause spurious features called "false caustics" which can be bothersome (see COA discussion.)

Since we've mentioned the word "caustics", let's look at Figure 1.14 for a beautiful example of them, caused by the upward refracting profile typical of the deep Arctic, in combination with a near surface source. The beginning edge of each raypath "matches" that of rays with higher launch angle, down to where these rays interact with the bottom. This leads to a focusing/reinforcement region, generically called a "caustic" or "hot spot". The greatly increased intensity due to a near-surface duct is also evident in this Figure.

If a convergence of rays can lead to sharp regions of high intensity, it stands to reason that a divergence of rays can lead to a sharp "shadow zone" of low intensity, and Figure 1.13 of COA shows just that.

In Figure 1.15 of COA, we turn from deep water to shallow water. Noticeable is the distinct "three layer" soundspeed profile, consisting of: 1) a warm water, faster soundspeed, isovelocity layer on top, 2) a "mixing waters" gradient layer in the middle, and 3) and a cooler water, slower soundspeed isovelocity layer at bottom. This is a "canonical" (i.e.generic, typical, usual) shallow water low latitude and mid-latitude (summer) profile. For an acoustic source in one of the lower two layers, one sees two groups of rays: 1) a lower group, with upper turning points in the gradient layer, and 2) a full water column group, with an upper turning point at the surface. (Both groups have lower turning points at the bottom.) This "two group" type of raypath pattern/arrival structure is typical of shallow water acoustics.

Though it it is not really a ray theory example, Figure 1.17 of COA shows an interesting peculiarity of shallow water acoustics – the existence of an "optimal frequency" for propagation, due to the tradeoff between water column attenuation (greater at higher frequencies) and bottom attenuation (greater at lower frequencies.) Sorry for the slight non-sequitur!

In Figure 1.18, we break away from the simplification of horizontal stratification. In this figure, the surface ducted propagation from a near surface source is disrupted by an ocean front (i.e. a horizontal change in the soundspeed profile), converting the propagation to deep ocean, CZ propagation. But again, simple rays and Snell's Law work well enough to show the paths of the energy.

Figure 1.19 of COA shows one of the most challenging environments for ocean acoustics, a seamount. For propagation directly over the top of a symmetric seamount, 2D ray acoustics can be used, and it nicely shows how the seamount cuts off the energy hitting its slopes (by either back reflection or absorption.) In general, a seamount is a fully 3D acoustics problem, and indeed, due to the complexity of the solid medium (the seamount) and the waters near it, is still somewhat of a research topic.

One final example, courtesy of Y.T.Lin of WHOI, is that of a pair of coastal soliton internal waves ducting sound in between them, a fully 3D propagation effect. The solitons push down the thermocline, creating a high speed region (averaged over the water depth), whereas the region in between the waves is of lower soundspeed. Thus a "horizontal duct" is created, and sound travels along in that duct in a fully 3D sense. This effect has been observed numerous times in the ocean.

Let's get back to theory world again. In Section 3.2 of COA (old), a very simple and elegant derivation of the ray equations is presented. It starts with the inhomogeneous Helmholtz equation (Eq. 3.1) and then postulates a series solution for $p(\vec{x})$. This solution is simply substituted into the Helmholtz equation, derivatives are taken, and a series solution of Eq. 3.1 is obtained. One then uses the standard trick of matching orders of the expansion (in inverse powers of ω in this case) to obtain an infinite series of equations for time $\tau(\vec{x})$ and amplitude $A_j(\vec{x})$. These are the familiar eikonal (raypath) and transport (ray intensity) equations of ray theory. A comment or two on this derivation might be useful here. First, the ray series will obviously converge faster as ω increases, as one has inverse power of ω in Equation 3.2. second, we have somehow created a first order nonlinear equation (the eikonal equation) and an infinite series of equations (the transport equations) from a simple second order linear PDE. Was that wise? (It turns out, yes…) and finally, we need ways to solve these seemingly ugly little equations. That comes next!

In section 3.3.1 of COA, there is a very straightforward, but complete, treatment of how one solves the eikonal equations. Following standard practice, COA solves it using the method of characteristics, where one generates a family of curves (rays) which are perpendicular to the constant time curves (wavefronts), $\tau(\vec{x})$. Starting with the definition of the ray trajectory $\vec{x}(s)$ in Eq. (3.16), a simple derivation of the ray equations is given, resulting in a Cartesian second order (vector component) equation in Eq. 3.22, or equivalently a series of first order equations for the [r(s), z(s)] cylindrical coordinates of the rays. The first order equations can be integrated easily, using the I.C.s of the source position and the ray initial launch angle. The remainder of section 3.3.1 points out the physical "features" of ray theory (features, not bugs!) such as its approximate frequency independence, the bending of rays towards cooler water, etc. Expression 3.3.2 for the ray travel time, accompanied by the simple relation $\varphi = \omega t$ for the phase, are very useful practically.

The introduction to section 3.6 of COA discusses two popular (well, in acoustics circles anyway) methods of solving the ray equations: 1) direct integration of the first order differential equations and 2) "cell methods" where one breaks the medium into layers or (for range dependent cases) triangular segments which admit to analytic solutions within the cell or layer (e.g. our linear gradient layer representation of *c(z)* which has arcs of circles as solutions). We will just discuss the direct integration

here. The simple cell methods are left as a homework problem. Section 3.6.1 of COA gives specific details about how to numerically integrate the governing equations for the rays, previously seen in Eqs. (3.15) and (3.16). A simple second order Runge-Kutta method is shown, which should suffice to let the reader try his or her hand. Handling of simple boundary interactions is also discussed.

Having identified the ray paths (and their phases and travel times), we now need the ray intensities (amplitudes) to get the complex pressure field. Before proceeding, however, the reader should be warned that the intensities obtained from ray theory are often not very accurate, particularly near caustics and shadow zones. One can "patch these regions up" to a degree, with various corrections, but at this point in time this is wasted effort, as better alternatives exist.

In deriving the ray intensities, I have paraphrased a section from Brekhovskikh and Lysanov (B&L), rather than use the sections from COA or Frisk. COA has more detailed mathematics, whereas B&L has a simpler physical picture and easier insight – I merely opt for the latter where available. The derivation presented is a standard "ray tube" derivation, and the notes should have adequate detal. The $\left|\frac{\partial r}{\partial \chi_1}\right|$ factor in the denominator of the equation for I is an important one, in that when it goes to zero, the intensity becomes infinite (at caustics or foci), whereas when it goes to infinity, the intensity goes to zero (at shadow zone boundaries). In neither case can the expression for I give the correct answer.

The next handwritten section, "eigenrays", deals with a very important set of rays – those which connect a source/receiver pair directly. The experimental ray (or mode) arrival time structure one observes at a receiver is very valuable data for a large number of purposes, and to model it with ray theory one needs the eigenrays. These are easily obtained by sending out "angular sprays" of rays, and then interpolating the arrivals, as is discussed. I should be pointed out that one potential pitfall of this seemingly trivial process of finding eigenrays is that, if the angular spray sampling is not fine enough, one can miss some of the eigenrays. Another cautionary note also should be sounded about "micro-multipathing", where one has a complicated, and often finescale, ocean soundspeed structure that results in a plethora of eigenrays. These rays are not really independent rays if they are withing a Fresnel zone radius of each other, as we will discuss.

Since we now can find the phase and amplitude of ray paths that arrive at a given receiver, we can add them up coherently (magnitude and phase) to get a total field, and also contrast that field with what the field is right near the source (usually at one meter distance.) This is done in section 3.3.5.2 of COA, and is called the "coherent transmission loss."

Though ray theory is tailored to high frequencies, and gives very good model estimates of phase, in the physical world, ocean finestructure scatters the rays more and more with increasing frequency, thus effectively randomizing the phases of the ray multipaths. Thus we really do not want a phase coherent model of a phase incoherent signal! In this case, we just ignore the phases, and add intensities, as is discussed in section 3.3.5.3 of COA.

In cases where we don't want to wash out all the phase coherent structure, but for which fully coherent addition is not appropriate either, a weighted average compromise called "semicoherent TL" is discussed in section 3.3.5.4 of COA.

The next small section, from Grant Fowles "Optics" book, treats the Fresnel zone. This is the correct way of treating the fact that, other than at the infinite frequency limit, rays are *not* infinitely thin geometric lines, but rather have a physical extent that depends on frequency. The section in Fowles is straightforward, but the rationale for why this pertains to ray paths is missing (as it was not the author's intent in this particular book.) The rationale is simply that we can vary the raypaths somewhat (very much in the spirit of the variational calculus) around the original path, and as long as the varied path is within the first Fresnel zone radius, it *is,* for all practical purposes, the original ray path! Thus, in Figure 4.20 of Fowles, we can vary h and h prime and r and r prime, and as long as we stay in the first Fresnel zone, those rays are "the same" as the straight line ray! There is one more "semi-subtlety" here (or maybe more like a "don't get confused.") Looking at Figure 4.20, one sees the raypath suddenly change direction, which looks like it was scattered physically. This is not the case, but rather a reflection of Huygens Principle, which says that each point on the irradiated plane can re=radiate Huygens wavelets – and we then merely track down the results of those wavelets at the receiver. BUT, if the path deflection is due to a real physical scattering, the same argument still works to determine if it is stil part of the same "basic ray."

There is one more little piece to looking at Fresnel zones that needs to be addressed in ocean acoustics. Unlike line of sight radar or light from distant stars, sound in the ocean does not, over any appreciable distance, travel in straight line paths, but rather bends, as we have seen in our previous raytraces, easily up to angles of ~15 degrees in the water column. This means that we should modify our straight line arguments to account for this. In my handwritten notes (paraphrased from the book edited by Flatte), we again look at the difference in paths; only in this case the paths are curved, not straight lines. A Taylor expansion of the curved path around the original path is used to express the difference. Interestingly, the difference is in the second order term (the curvature), as the first order term (the first derivative or variation) goes to zero by Fermat's Principle, which we will soon discuss. The result of this is that $R_F^2 = \lambda A^{-1}(x)$. This is seen to be just a simple extension of the previous result, and says that we should look at the second derivatives (the ray curvature) along the path to get the Fresnel zone size there. Not so hard!

As a closing comment on the Fresnel zone, it serves as a sort of limiting local scale, below which one does not have to explicitly consider finescale ocean/bottom structure, as it is "too small" to be seen by the ray.

Another important piece of "classical optics" that is useful for ocean acoustics is Fermat's Principle, which states that the travel time for a ray is stationary. This is discussed in section 3.5.5 of COA. Two simple examples (the law of reflection and Snells Law), are presented from Resnick and Hallidays elementary physics book.

The final piece of our purely ray theory picture is "Gaussian Beams", where one uses the trick of placing the source in the complex plane to describe a beamed source with a Gaussian wavefront. This technique is described in section 3.5 of COA. These "fuzzy rays" present a smoother transition through shadow zones and caustics than regular rays.

Having discussed the high frequency end of propagation theory, ray theory, we will next be looking at the low frequency end, mode theory. However, there is an obvious question that must be asked: is there something in the middle?! It turns out there is, under the name of WKB theory. WKB theory is actually a nice bridge between ray theory and mode theory, and will be used as such many times in this course. A very clean and clear derivation of WKB theory for a stratified medium is presented in Frisk in section 7.3, followed by a fully 3D treatment in section 7.4. Note that, even though WKB theory looks a lot like ray theory superficially, it is a lot more than that. If you look at Eq. 7.20 in Frisk, you will see that the horizontal phase term looks like plain old ray theory, but that the vertical phase is an interesting vertical accumulation of phase – a better treatment. With this WKB vertical phase, one can actually create normal modes, as we'll see later. Also, note that the WKB amplitude blows up at the turning points (i.e.the $\sqrt{k_z(z)}$ term in Eq. 7.20). WKB's forte is phase behavior, not amplitude!

2.682 Acoustical Oceanography

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