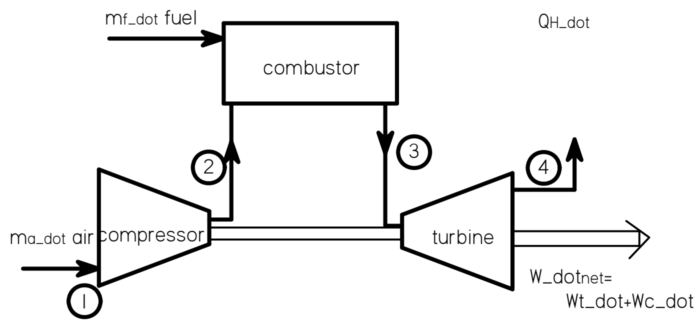
Open Cycle



power ... compressor ... $$W_{c\_dot} = -m_{a\_dot} \cdot (h_2 - h_1) = -m_{a\_dot} \cdot c_{p\_air} \cdot (T_2 - T_1)$$

turbine ...

$$W_{t\_dot} = (-m_{a\_dot} + m_{f\_dot}) \cdot (h_3 - h_4) = -m_{a\_dot} \cdot \left(1 + \frac{m_{f\_dot}}{m_{a\_dot}}\right) \cdot c_{p\_prod} \cdot (T_3 - T_4)$$

Jet engine

as a side note: if the net work were converted to velocity via a nozzle (jet engine) the relationships would be

$$W_{net\_dot} = W_{t\_dot} - W_{c\_dot}$$

determines state 4 out of turbine at $p_4 > p_1$ atmosphere is state 5

T_4 determined from equation for net work $$w_{net} = c_p \cdot (T_3 - T_4)$$

could determine p_4 from $$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

determine T_5 from $$\frac{T_5}{T_3} = \left(\frac{p_5}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$ or ... $$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

nozzle anlysis:
First law, Q = W = 0 $$h_4 = h_5 + \frac{V^2}{2}$$ determines V, thrust from momentum change

combustor ...

1 = atmosphere ... adiabatic combustion Q = W = 0

$$0 = H_{R2} - H_{P3}$$

0 = Enthaply of reactants at combustor inlet, compressor outlet
- Enthalpy of products out of combustor - first law

rewrite using LHV ... $$0 = H_{R2} - H_{R0} - (H_{P3} - H_{P0}) + LHV$$

rewrite using specifi enthalpy and mass flows ... on a per unit mass flow of fuel ...

$$0 = h_{f2} - h_{f0} + \frac{m_{a\_dot}}{m_{f\_dot}} \cdot (h_{a2} - h_{a0}) - \left(1 + \frac{m_{a\_dot}}{m_{f\_dot}}\right) \cdot (h_{p3} - h_{p0}) + LHV$$

to account for incomplete combustion introduce combustion efficiency ... only obtain $\eta_{comb} \cdot HV$

Given

$$0 = h_{f2} - h_{f0} + \frac{m_{a\_dot}}{m_{f\_dot}} \cdot (h_{a2} - h_{a0}) - \left(1 + \frac{m_{a\_dot}}{m_{f\_dot}}\right) \cdot (h_{p3} - h_{p0}) + \eta_{comb} \cdot LHV$$

can solve for

m_{a_dot}/m_{f_dot}

$$\text{Find}(m_{a\_dot}) \rightarrow (h_{f2} - h_{f0} - h_{p3} + h_{p0} + \eta_{comb} \cdot LHV) \cdot \frac{m_{f\_dot}}{-h_{a2} + h_{a0} + h_{p3} - h_{p0}}$$

$$\frac{m_{a\_dot}}{m_{f\_dot}} = \frac{\eta_{comb} \cdot LHV + (h_{f2} - h_{f0}) - (h_{p3} - h_{p0})}{h_{p3} - h_{p0} - (h_{a2} - h_{a0})}$$

introduce average specific heat ...

$$c_{p\_bar\_air} = \frac{h_{a2} - h_{a0}}{T_2 - T_0} \qquad c_{p\_bar\_prod} = \frac{h_{p3} - h_{p0}}{T_3 - T_0} \qquad c_{p\_bar\_fuel} = \frac{h_{f2} - h_{f0}}{T_2 - T_0}$$

$$\frac{m_{a\_dot}}{m_{f\_dot}} = \frac{\eta_{comb} \cdot LHV + c_{p\_bar\_fuel} \cdot (T_2 - T_0) - c_{p\_bar\_prod} \cdot (T_3 - T_0)}{c_{p\_bar\_prod} \cdot (T_3 - T_0) - c_{p\_bar\_air} \cdot (T_2 - T_0)}$$

or ... inverting

$$\frac{m_{f\_dot}}{m_{a\_dot}} = \frac{c_{p\_bar\_prod} \cdot (T_3 - T_0) - c_{p\_bar\_air} \cdot (T_2 - T_0)}{\eta_{comb} \cdot LHV + c_{p\_bar\_fuel} \cdot (T_2 - T_0) - c_{p\_bar\_prod} \cdot (T_3 - T_0)}$$

gas turbine efficiency efficiency dividing by m_{a_dot}

$$\eta = \frac{W_{net\_dot}}{m_{f\_dot} \cdot LHV} = \frac{W_{t\_dot} + W_{c\_dot}}{m_{f\_dot} \cdot LHV} = \frac{\left(1 + \frac{m_{f\_dot}}{m_{a\_dot}}\right) \cdot c_{p\_bar\_prod} \cdot (T_3 - T_4) - c_{p\_bar\_air} \cdot (T_2 - T_1)}{\frac{m_{f\_dot}}{m_{a\_dot}} \cdot LHV}$$

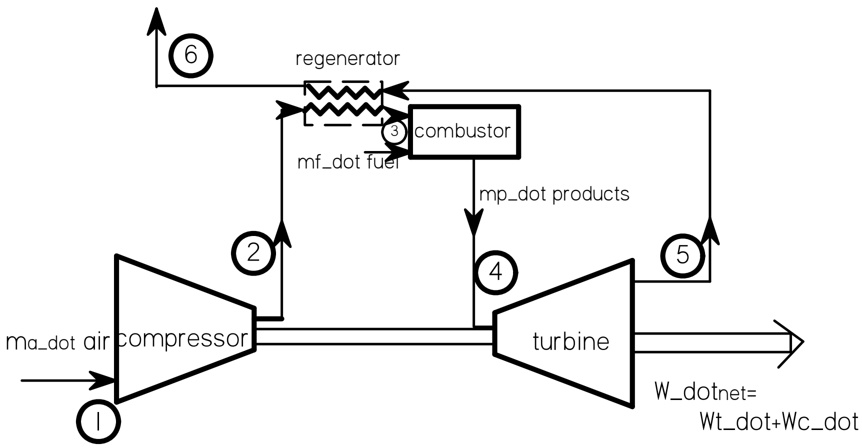
$$SFC = \frac{kg \cdot \frac{fuel}{hr}}{power = kW} = \frac{kg}{kW \cdot hr} = \frac{lb}{hp \cdot hr}$$

not equality ...

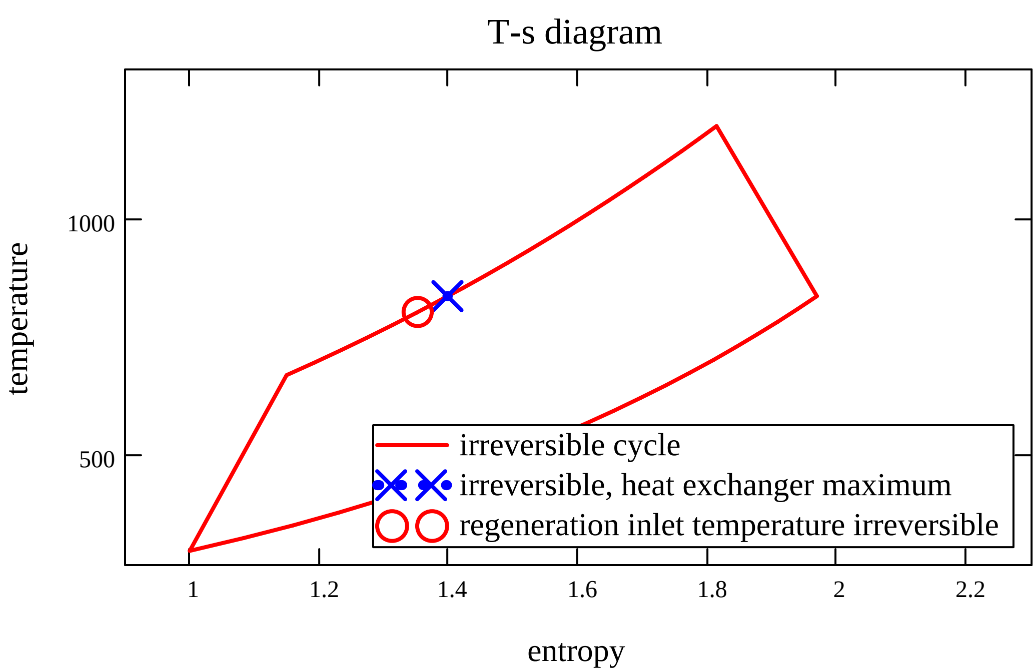
$$SFC = \frac{m_{f\_dot}}{W_{net\_dot}} = \frac{m_{f\_dot}}{W_{net\_dot}} \cdot \frac{LHV}{LHV} = \frac{1}{\eta \cdot LHV}$$

Open cycles have similar alternatives to closed
analysis would be similar as well so not repeated here

Open Cycle Regenerative (Recouperative)

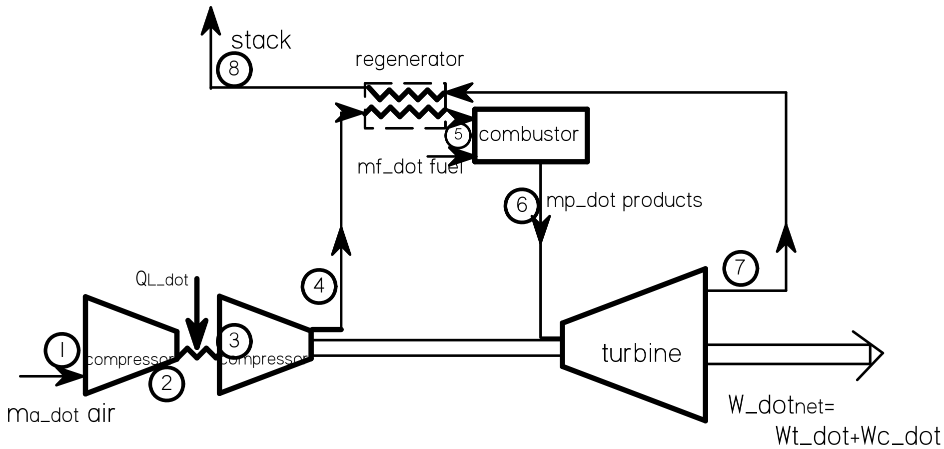


▸ static data for plot

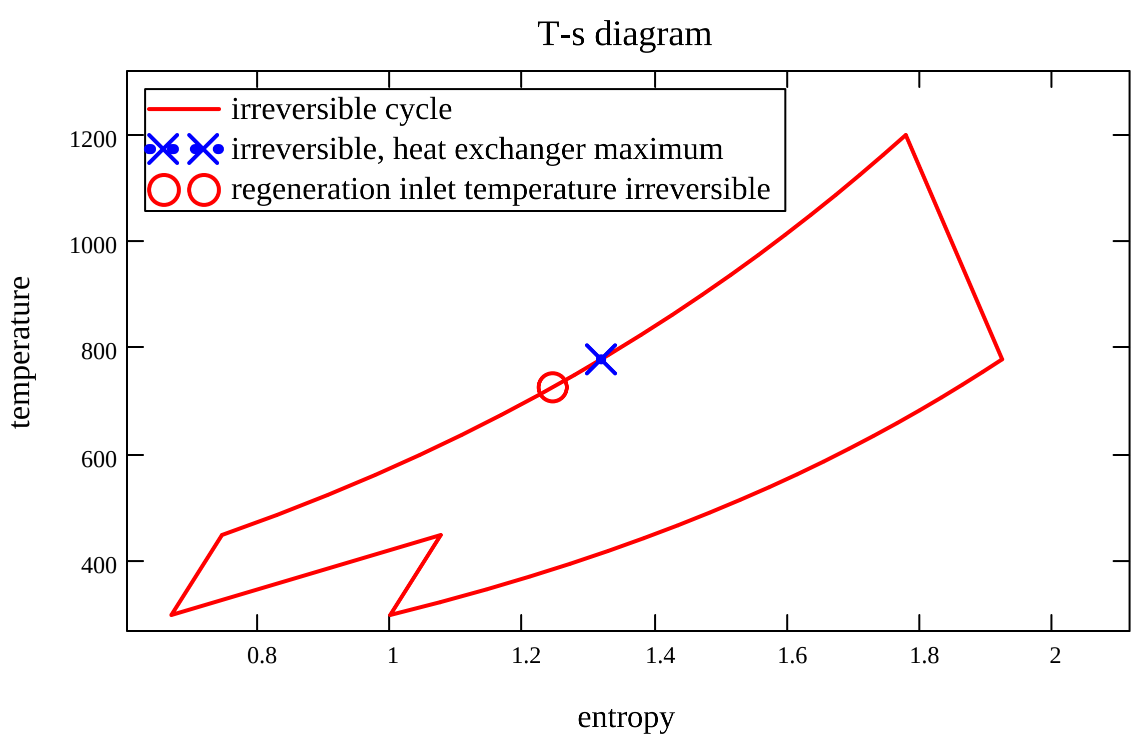


N.B. cycle is drawn closed from state 6 to 1 but is taking place in atmosphere

Intercooled Regenerative (Recouperative) Cycle Rolls Royce WR-21 is an example see links



static data for plot



thermodynamic models for combustion

Various thermodynamic models can be used for analysis of products of combustion:

1. Single gas model
   perfect gas, constant c_p (1 kJ/kg*K close enough), γ = 1.4
2. Two gas model
   a) perfect gas - air for compression, c_p = 1.0035 kJ/kg*K, γ_a = 1.4
   b) perfect gas combustion products; c_{pp} = 1.13 kJ/kg*K, γ_p = 1.3
3. Tabulated data (e.g. Keenan & Kaye Gas Tables)
   property data for air:
   Table 1: Air at low pressure: T deg F abs, t deg F, h, pr, u, v_r, ϕ
   Table 2. Air at low pressures: T, t, c_p, c_v, k = c_p/c_v, a, G_{max}/p_i , μ, λ, Pr
   Table 3: R Log N for air
   Table 4: Products - 400% Theoretical Air (for One Pound Mole)
   Table 5: Products - 400% Theoretical Air (for One Pound Mole) fuel data
   Table 6: Products - R_bar Log_e N +4.57263 n
   Table 7: Products - 200% Theoretical Air (for One Pound Mole)
   etc. data for oxygen, hydrogen, carbon monoxide, dioxide etc.

T = deg F abs
t = deg F
h = enthalpy per unit mass
p_r = relative pressure
u = internal energy per unit mass
v_r = relative volume

$$\phi = \int_{T_0}^{T} \frac{c_p}{T}\, dT$$

c_p = specific heat at constant pressure
c_v = specific heat at constant volume
G = flow per unit area or mass velocity
k = c_p/c_v
p = pressure
Pr = Prandtl number = cp*μ/λ
R = gas constant for air
a = velocity of sound
λ = thermal conductivity
μ = viscosity

Notes: Appendix (Sources and methods
- " ...calculated for one particular composition of the hydrocarbon fuel, it has been shown that it represents with high precision the properties of the productsof combustion of fuels of a wide range of composition - all for 400% theoretical air." page 205 bottom
- problems involving intermediate mixtures to Table B:
can be solved by interpolation based on theoretical air
or ... extrapolated to 100% for products is valid except for effects of disassociation

Table_B =

.	.products	.	reactants	air_and
Table	%theor	%theor	%theor	water_vapor
Number	air	fuel	fuel	mass_%_water
1	inf	0	0	0
4	400	25	14	6.7
7	200	50	28	.

4. Polynomial equations

- example in combustion example $c_p = f(\theta$

isentropic process

$$ds = c_{po}\cdot\frac{dT}{T} - R\cdot\frac{dp}{p}$$ (7.21) in gas relationships

$$\frac{dp}{p} = \frac{1}{R}\cdot c_{po}\cdot\frac{dT}{T}$$

$$\ln\left(\frac{p_1}{p_2}\right) = \frac{1}{R}\cdot\int_{T_2}^{T_{1s}}\frac{c_p}{T}\,dT$$

$$\frac{p_1}{p_2} = e^{\frac{1}{R}\cdot\int_{T_2}^{T_{1s}}\frac{c_p}{T}\,dT}$$

polytropic process

compressor

$$R\cdot\frac{dp}{p} = \eta_{pc}\cdot c_{po}\cdot\frac{dT}{T}$$

$$\frac{p_1}{p_2} = e^{\frac{\eta_{pc}}{R}\cdot\int_{T_2}^{T_{1s}}\frac{c_p}{T}\,dT}$$

turbine

$$\frac{p_1}{p_2} = e^{\frac{1}{\eta_{pt}\cdot R}\cdot\int_{T_2}^{T_{1s}}\frac{c_p}{T}\,dT}$$

High Temperature Gas Turbines

Advantages:

high efficiency - low specific fuel consumption
high specific horsepower - small size and weight

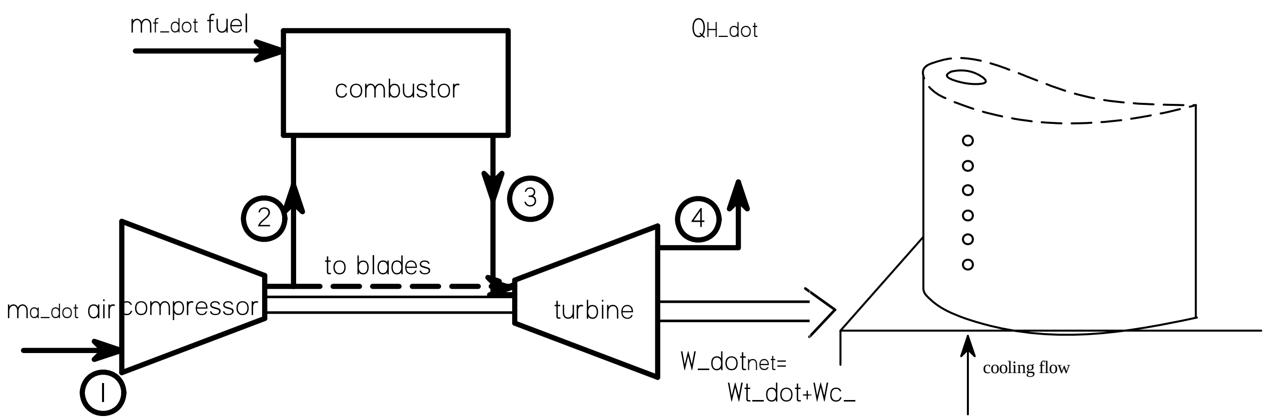
Disavantages:

materials strength problems (Creep) see separate notes re: creep
corrosion

Solutions:

better materials
blade and combustor cooling
ceramic materials

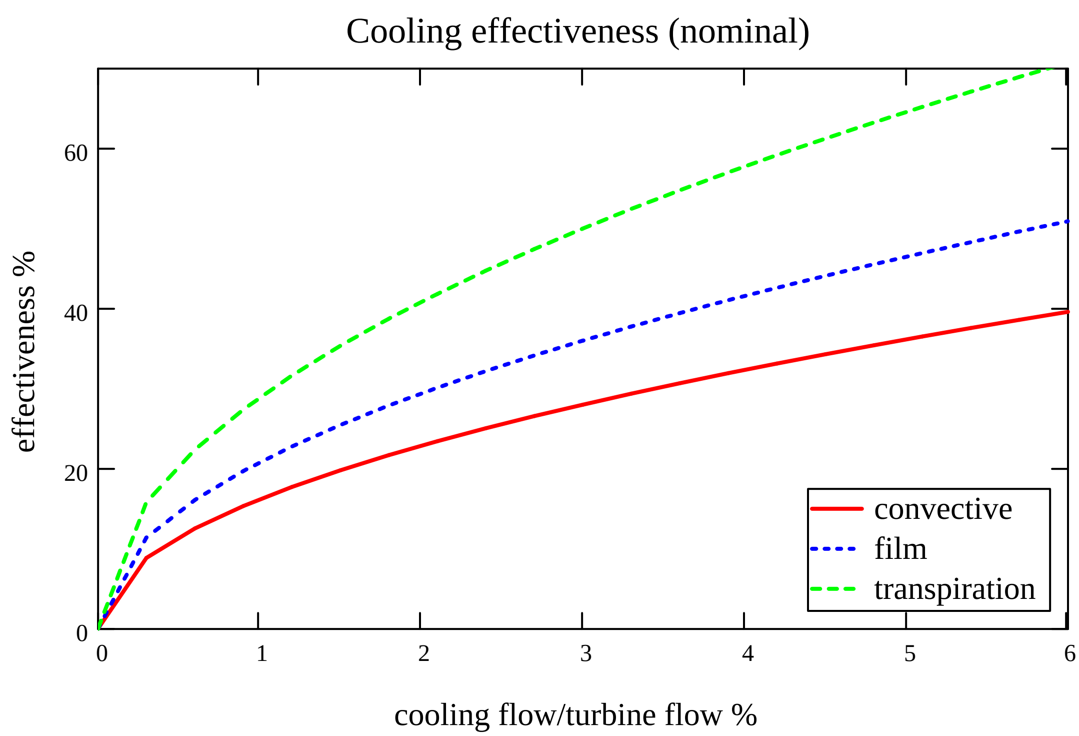
blade cooling



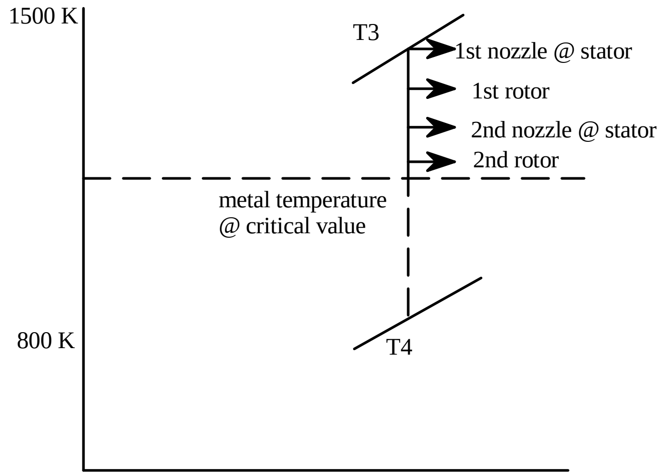
compressed air ducted into stationary AND rotor blades. Temperature reuduced by:

- convective heat transfer
- transpiration (evaporation of water from surface)
- film

▸ nominal data for plot



$$\text{cooling\_effectiveness} = \frac{T_{blade\_gas} - T_{blade\_metal}}{T_{blade\_gas} - T_{cooling\_air}}$$



nominal ΔT over stages defining where cooling is required

Ceramic materials

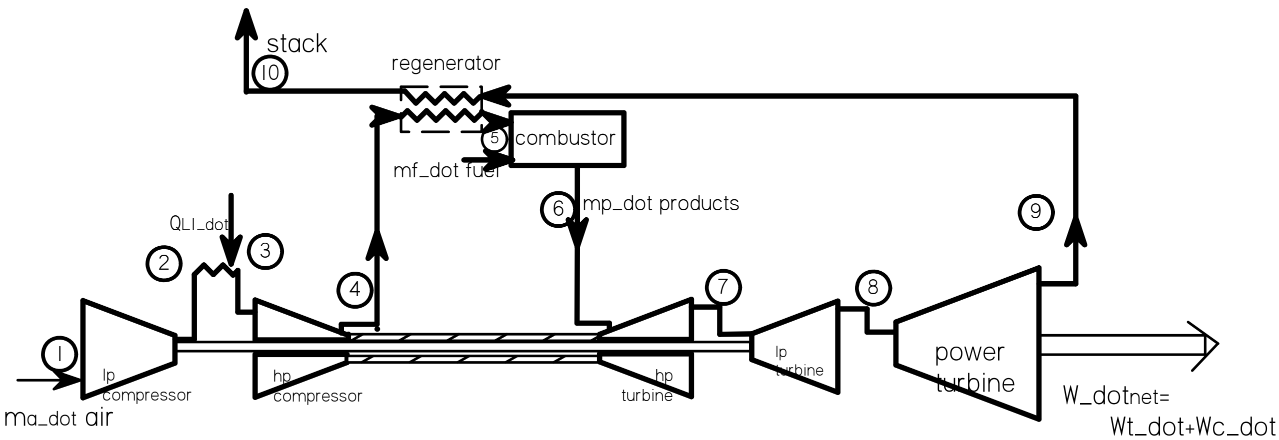
examples silicon nitride, silicon carbide
can be pressed, bonded and/or sintered to produce complete rotor system

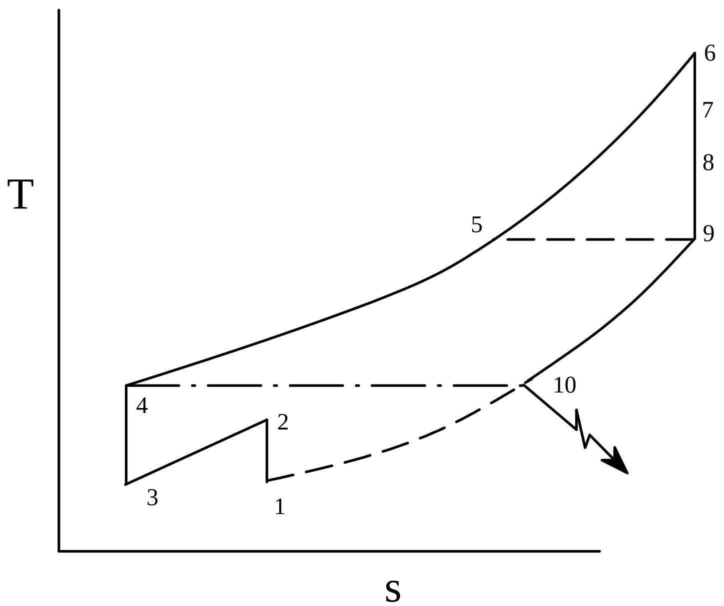
tensile_strength =

.	25_deg_C(75_deg_F)	.	.	1400_deg_C(2500_deg_F)
.	MPa	ksi	MPa	ksi
$Si_3 \cdot N_4$	552	80	172	25
$Si \cdot C$	193	28	138	20

Intercooled Regenerative Gas Turbine

typically two spool design





powers ... (review) reversible

$LP\_comp = -m_{air\_dot} \cdot (h_2 - h_1)$

$HP\_comp = -m_{air\_dot} \cdot (h_4 - h_3)$

$HP\_turb = (m_{air\_dot} + m_{fuel\_dot}) \cdot (h_6 - h_7)$

$LP\_turb = (m_{air\_dot} + m_{fuel\_dot}) \cdot (h_7 - h_8)$

$Power\_turb = (m_{air\_dot} + m_{fuel\_dot}) \cdot (h_8 - h_9)$

$Q_{H\_dot} = (m_{air\_dot} + m_{fuel\_dot}) \cdot (h_6 - h_5)$

$w\_dot_{LP\_comp} = -w\_dot_{LP\_turb}$

$w\_dot_{HP\_comp} = -w\_dot_{HP\_turb}$

$\frac{m_{air\_dot}}{m_{fuel\_dot}}$ from combustion analysis

Marinization

Problems:

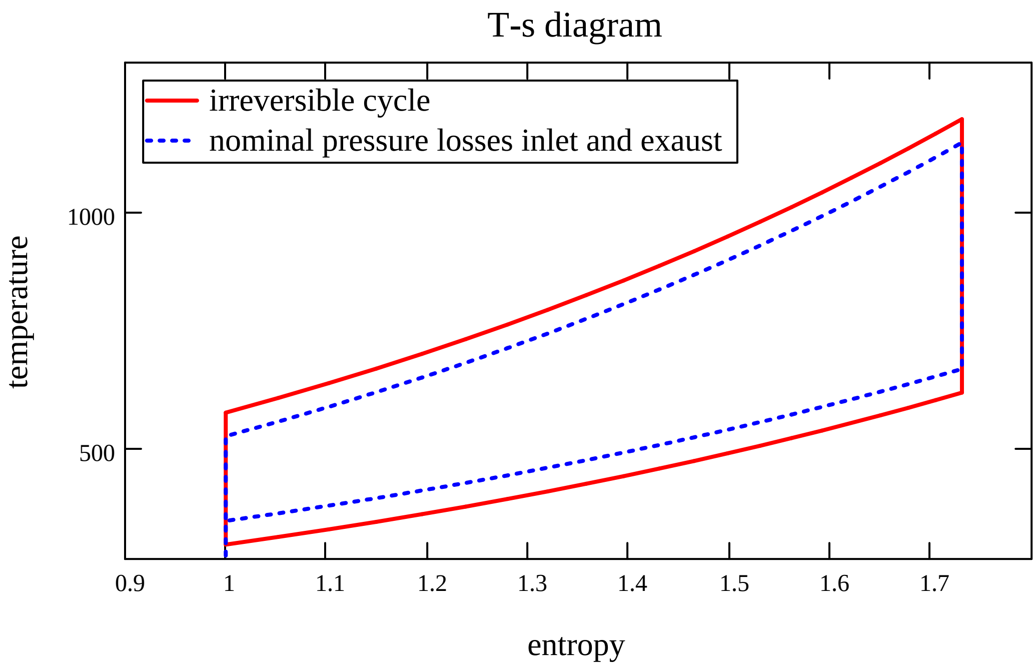
1. sea water droplets in air (inlet)
2. sea water in fuel
3. coupling to the propeller
4. long ducting

Solutions:

1. sea water in air
    1. design of inlet - demisters to remove droplets
        - demisters
            - wire mesh
            - inertial separation
    2. select corrosion resistant materials
    3. surface treatment of components - plating to improve corrosion resistance
    4. water washing and abrasive cleaning
2. sea water in fuel
    1. treat to remove sodium
3. coupling to propeller (later)
4. long ducting
    - inlet and exit pressures reduce the pressure ratio across turbine
        - reduction in power
        - increase in fuel consumption
    - additional effect from inlet density

$$p \cdot v = R \cdot T \quad \Rightarrow \quad \frac{p}{\rho} = R \cdot T \qquad \rho = \frac{p}{R \cdot T}$$

static data for plot



similar effect for T inlet > nominal

cycle will walk up p_1 curve

normally cannot increase T_H to account for these losses

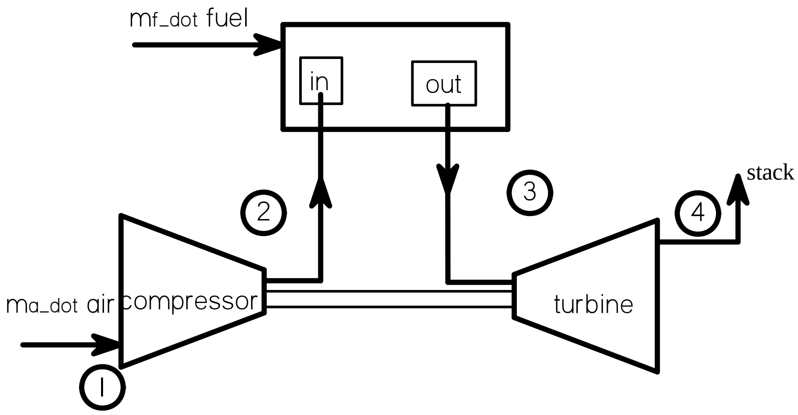
other issues/topics

Materials
- coatings
- use of titanium

fuel treatment
- sodium - bad - corrosion from products
  - remove by washing
  - add agents such as demulsifiers
    - water combines with sodium - remove by centrifuge
- vanadium - in Bunker C combines with sulfur - creates corrosive combustion products
  - GE fro example has an additive to modify ash to prevent adhering to blades

problem 3 above: coupling to propeller
1. Controllable Reversible Pitch Propeller (CRP)
2. reversing gearbox
3. electric drive
4. reversing turbine
   - concentric opposite direction direction blade annuli

Brayton cycle applied to turbocharging reciprocating engines



power ...

$$\text{comp} = -m_{air\_dot}\cdot(h_2 - h_1)$$

$$\text{turb} = (m_{air\_dot} + m_{fuel\_dot})\cdot(h_3 - h_4)$$

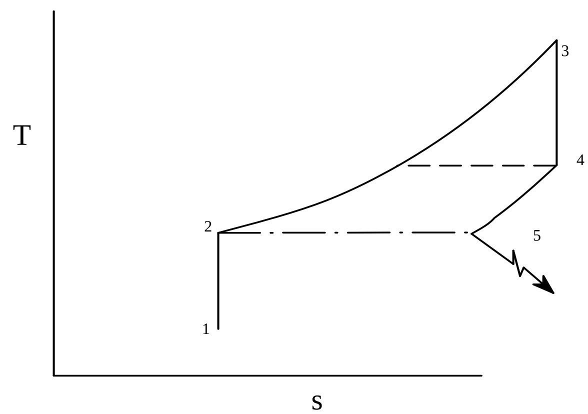
$$w\_dot_{comp} + w\_dot_{turb} = 0 = (m_{air\_dot} + m_{fuel\_dot})\cdot(h_3 - h_4) - m_{air\_dot}\cdot(h_2 - h_1)$$

$$h_2 - h_1 = \left(1 + \frac{m_{fuel\_dot}}{m_{air\_dot}}\right)\cdot(h_3 - h_4)$$

p_3 may be > or < p_2 depending on what happens in engine

combined cycles - gas turbine and Rankine - or other

maximum available power from T_4 -> T_5



$$\left(\frac{w\_dot_{rev}}{m\_dot}\right)_{max} = \psi_4 - \psi_5 = h_4 - T_0\cdot s_4 - (h_5 - T_0\cdot s_5) = h_4 - h_5 - T_0\cdot(s_4 - s_5)$$

second law ... $T\cdot ds = dh - v\cdot dp$ if ... $p_4 = p_5 = p_{atmos}$ $dp = 0$ $ds = \frac{dh}{T}$

assuming c_{pp} constant $ds = \frac{dh}{T} = \frac{c_{pp}\cdot dT}{T}$ => $s_4 - s_5 = c_{pp}\cdot\ln\left(\frac{T_4}{T_5}\right)$

$$\left(\frac{w\_dot_{rev}}{m\_dot}\right)_{max} = \psi_4 - \psi_5 = c_{pp}\cdot\left(T_4 - T_5 - T_0\cdot\ln\left(\frac{T_4}{T_5}\right)\right)$$

example LM 2500

$T_4 := 825$ K

$T_0 := 300$ K

$GT\_power := 330 \frac{kW}{\frac{kg}{s}}$

$kJ := 1000J$

$1 < c_{p\_prod} < 1.33$

$c_{p\_prod} := 1.08 \frac{kJ}{kg \cdot K}$

$$T_5 := \begin{pmatrix} 500 \\ 400 \\ 300 \end{pmatrix}$$

$$\left(\frac{w\_dot_{rev}}{m\_dot}\right)_{max} = W\_m\_dot\_max$$

$$T_4 - T_5 = \begin{pmatrix} 325 \\ 425 \\ 525 \end{pmatrix} \qquad T_0 \cdot \ln\left(\frac{T_4}{T_5}\right) = \begin{pmatrix} 150.233 \\ 217.176 \\ 303.48 \end{pmatrix}$$

$$W\_m\_dot\_max := c_{p\_prod} \cdot \left(T_4 - T_5 - T_0 \cdot \ln\left(\frac{T_4}{T_5}\right)\right) \cdot K$$

$$W\_m\_dot\_max = \begin{pmatrix} 188.749 \\ 224.45 \\ 239.241 \end{pmatrix} \frac{kJ}{kg}$$

$$\frac{W\_m\_dot\_max}{GT\_power} = \begin{pmatrix} 0.572 \\ 0.68 \\ 0.725 \end{pmatrix}$$