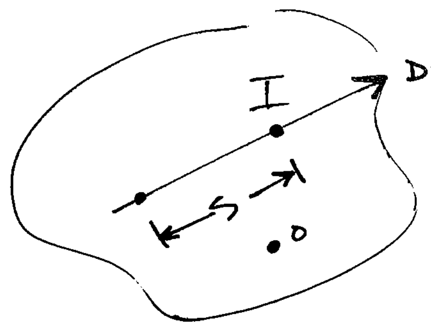
INTENSITY CHANGES ALONG THE DIRECTION OF PROPAGATION DUE TO

- SCATTERING
- EMISSION
- ABSORPTION




DIRECTION OF PROPAGATION

$$\frac{dI_\eta}{ds} = -\left(\kappa_{a\eta} + \kappa_{s\eta}\right) I_\eta + \kappa_{a\eta} I_{b\eta} + \frac{\kappa_{s\eta}}{4\pi} \underbrace{\int_{4\pi} \Phi(\Omega' \to \Omega)\, I_\eta'(\Omega')\, d\Omega'}$$

$\Downarrow$ (phase space)

$$\hat{e}_\Omega \cdot \overline{\nabla}_r I_\eta$$

POINTS OFF THE LINE, LIKE "o" AFFECT INTENSITY THRU THIS TERM VIA THE PHASE FUNCTION.

POINTS OFF THE LINE AFFECT INTENSITY THRU SCATTERING

OPTICAL DEPTH (PATHLENGTH)

$$d\tau_\eta = \left(\kappa_{a\eta} + \kappa_{s\eta}\right) ds = \kappa_{e\eta}\, ds$$

$$\tau_\eta(L) = L\kappa_{e\eta} = \frac{L}{1/\kappa_{e\eta}} \; ; \quad \Lambda = \frac{1}{\kappa_{e\eta}} \quad \text{MEAN FREE PATH}$$

$$\frac{\Lambda}{L} \equiv \text{KNUDSEN \#}$$

$$\therefore \quad \frac{L}{\frac{1}{\kappa_{e\eta}}} \equiv \text{INVERSE KNUD. \#}$$

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + \underbrace{(1-\omega_\eta) I_{b\eta} + \frac{\omega_\eta}{4\pi} \int_{4\pi} \Phi(\Omega' - \Omega) I'(\Omega') \, d\Omega'}_{S_\eta}$$

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + S_\eta(\tau_\eta)$$

$$\frac{1}{I_\eta} \frac{dI_\eta}{d\tau_\eta} = -1$$

$$\ln I_\eta = -\tau_\eta + c'$$

$$I_\eta(\tau_\eta) = c e^{-\tau_\eta}$$

$$I_s = c(\tau_\eta) e^{-\tau_\eta}$$

$$\frac{dc}{d\tau_\eta} e^{-\tau_\eta} - c e^{-\tau_\eta} = -c e^{-\tau_\eta} + S_\eta$$

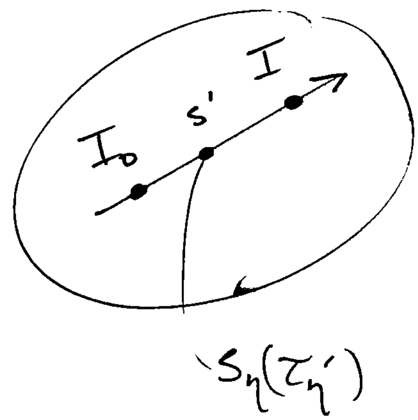
$$\frac{dc}{d\tau_\eta} = S_\eta e^{\tau_\eta}$$

$$c = c_0 + \int_0^{\tau_\eta} S_\eta(\tau_\eta') e^{\tau_\eta'} d\tau_\eta'$$

$$I_\eta(\tau_\eta) = C_o e^{-\tau_\eta} + e^{-\tau_\eta}\int_0^{\tau_\eta} S_\eta e^{\tau_\eta'}\, d\tau_\eta'$$

$$= \underbrace{C_o e^{-\tau_\eta}}_{\text{INITIAL VALUE}} + \int_0^{\tau_\eta} \underbrace{S_\eta}_{\text{SOURCE}} e^{-(\tau_\eta - \tau_\eta')}\, d\tau_\eta'$$

$C_0$ WHEN $\tau_\eta = 0$




SPECIAL CASES (PARTICULAR ASSUMPTIONS)

a) NO SCATTERING $S_\eta = I_{b\eta}$

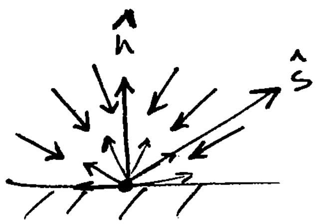
b) COLD MEDIUM $I_{b\eta} = 0$

c) ISOTROPIC MEDIUM $\Phi = 1$

$$I_\eta(\tau_\eta) = I_\eta(0)e^{-\tau_\eta} + \frac{1}{4\pi}\int_0^{\tau_\eta} \omega_\eta G_\eta(\tau_\eta') e^{-(\tau_\eta - \tau_\eta')}\, d\tau_\eta'$$

$$G_\eta \equiv \int_{4\pi} I_\eta(\Omega')\, d\Omega'$$

## BOUNDARY COND'S



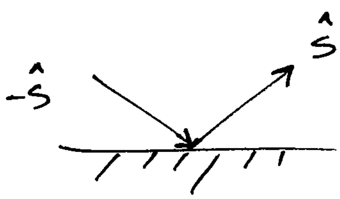

**DIFFUSE SURFACE:**

$$E_{b\eta} = \pi I_{b\eta}$$

$$I_\eta(\bar{r}_w, \hat{s}) = \varepsilon(\bar{r}_w)\, I_{b\eta}(\bar{r}_w) + \dots$$

$$\dots + \frac{\rho_w}{\pi} \int_{\hat{n}\cdot\hat{s}' < 0} I_\eta(\bar{r}_w, \hat{s}')\, |\hat{n}\cdot\hat{s}'|\, d\Omega'$$

## PARTIALLY DIFFUSE, PARTIALLY SPECULAR




$\rho$ - DIFFUSE

$$I_\eta(\bar{r}_w, \hat{s}) = \varepsilon_\eta(\bar{r}_w)\, I_{b\eta} + \frac{\rho^d(\bar{r}_w)}{\pi} \int_{\hat{n}\cdot\hat{s} < 0} I(\bar{r}_w, \hat{s}')\, |\hat{n}\cdot\hat{s}'|\, d\Omega' + \dots$$

$$\dots + \rho^s(\bar{r}_w, -\hat{s}')\, I(\bar{r}_w, \hat{s})$$

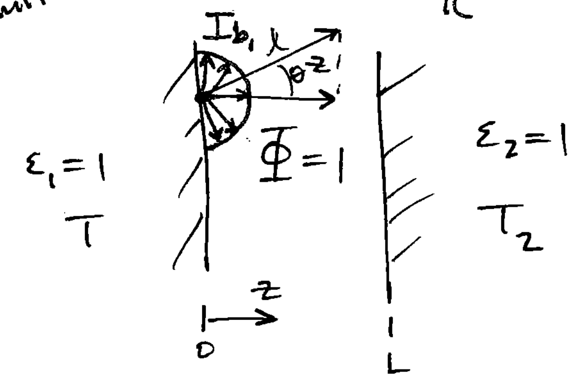
$\rho$ - SPECULAR

**Ex:** TWO PARALLEL PLATES

$\star$ $\tau_\eta \equiv$ OPTICAL PATH LENGTH

IF ABSORBING AND EMITTING

$I_{b_1} = \frac{\sigma T_1^4}{\pi}$




$$l = \frac{z}{\cos\theta}$$

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + (1-\omega_\eta) I_{b_\eta} + \frac{\omega_\eta}{4\pi}\int I_\eta' \, d\Omega'$$

$\Downarrow$

$I_{b_\eta}$ (GANG WILL ~~SHOW~~ PROVE THIS LATER)

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + I_{b_\eta}(\tau_\eta)$$

$90^\circ > \theta > 0^\circ$ AT $z=0$, $I_0 = \frac{\sigma T_1^4}{\pi} = I_{b_1}$

$$I_\eta(z) = I_{b_1} e^{-\frac{z}{\cos\theta}\kappa_{a\eta}} + \int_0^{\frac{z}{\cos\theta}\kappa_{a\eta}} I_{b_\eta}(z') e^{-\frac{(z-z')\kappa_{a\eta}}{\cos\theta}} \, d\left(\frac{z'\kappa_{a\eta}}{\cos\theta}\right)$$

DEFINE: $\mu = \cos\theta$ "DIRECTION COSINE"

$$\xi = z\,\kappa_{a\eta}$$

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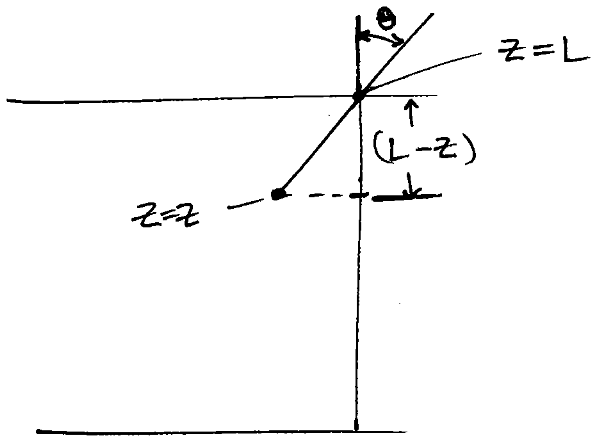
IN (+)VE DIRECTION, i.e. IN RANGE OF $90° < \theta < 0°$, ALSO $\cos\theta$ (+)VE

$$I^{+}(\xi,\mu) = I_{b_1} e^{-\xi/\mu} + \int_0^{\xi} I_b(\xi') e^{-\frac{\xi-\xi'}{\mu}} \frac{d\xi}{\mu}$$




$$I^{-}(\xi,\mu) = I_{b_2} e^{\frac{\xi_L-\xi}{\mu}} + \int_{\xi_L}^{\xi} I_b(\xi') e^{-\frac{\xi-\xi'}{\mu}} \frac{d\xi'}{\mu}$$

$90° < \theta < 180°$

OR

$-1 < \mu < 0$