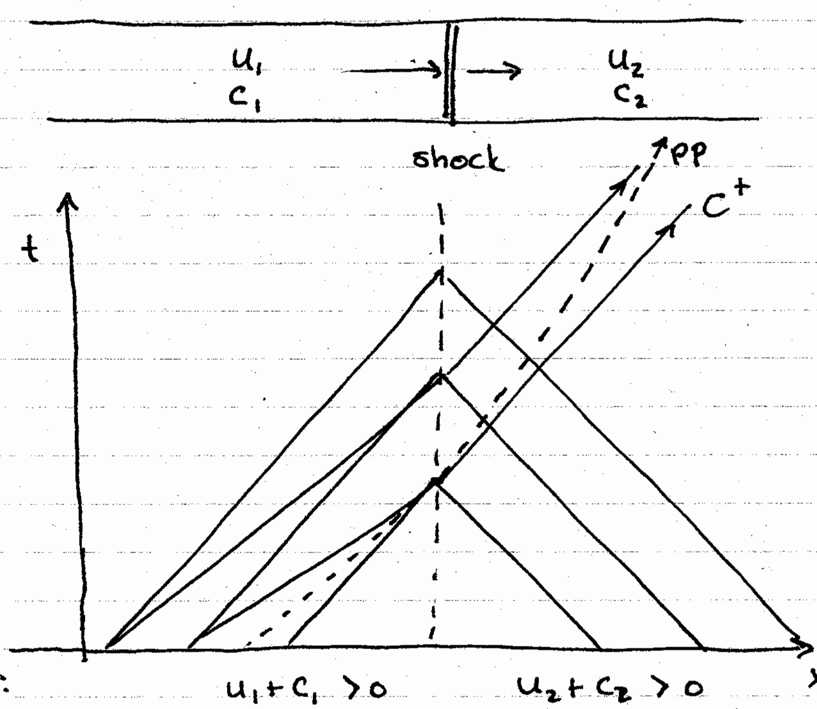
Lecture 10

Pset 3 : 8.2

Normal shock (standing) in 1D flow



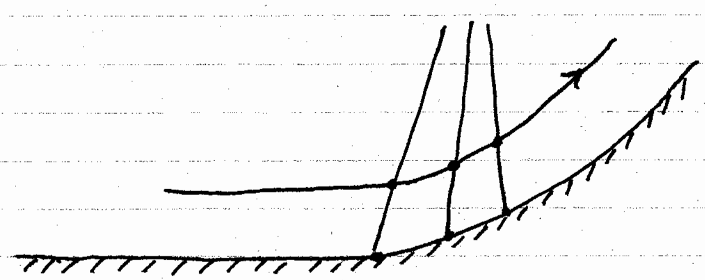
C^+: $u_1 + c_1 > 0$ $u_2 + c_2 > 0$

C^-: $(M_1 > 1)$ $u_1 - c_1 > 0$ $u_2 - c_2 < 0$ $(M_2 < 1)$

So far we've seen: quasi-1D steady
quasi-1D unsteady

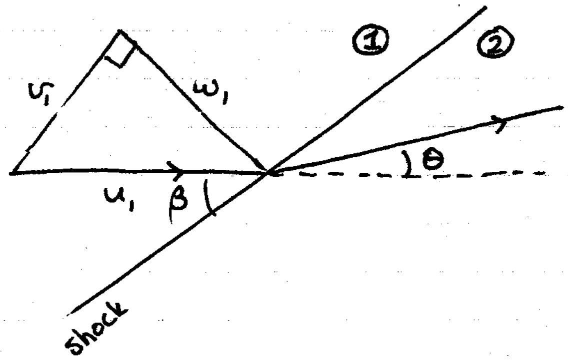
Today : 2D steady (chpt. 9)

Supersonic stream turns a corner through a series of standing oblique shocks:



$\uparrow$ # of shocks
∞ # of weak shocks
$\Rightarrow$ isentropic turn

Recall for weak oblique shocks



$$[\theta] = -\frac{\sqrt{M_1^2-1}}{M_1^2}\frac{[w]}{c_1}$$

(1st term in Taylor exp.)

Since $v_1 = v_2$,

$$u_2^2 - u_1^2 = (w_2^2 + \cancel{v_2^2}) - (w_1^2 - \cancel{v_1^2}) = w_2^2 - w_1^2$$

$$(u_2 + u_1)[u] = (w_2 + w_1)[w]$$

$$(2u_1 + \overset{\text{small}}{\cancel{[u]}})[u] = (2w_1 + \overset{\text{small}}{\cancel{[w]}})[w] \Rightarrow u_1[u] \simeq w_1[w]$$

From diagram: $\frac{w_1}{u_1} = \sin\beta$

Recall for weak oblique shocks $\beta \approx \mu_1 = \frac{1}{M_1}$

$\uparrow$ Mach angle

$$\frac{u_1[u]}{u_1} = \overset{1/M_1}{\overset{\parallel}{\frac{w_1}{u_1}}}[w] \Rightarrow \frac{[u]}{u_1} = \frac{1}{M_1^2}\frac{[w]}{c_1}$$

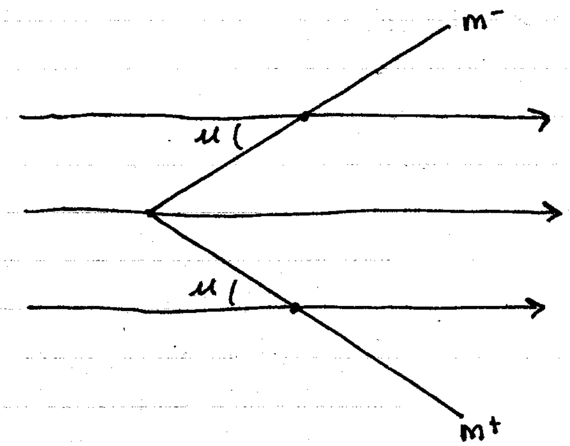
$$\Rightarrow [\theta] = -\sqrt{M_1^2 - 1}\,\frac{[u]}{u_1}$$

Infinitessimal strength shock:

$$d\theta = \pm\sqrt{M_1^2 - 1}\,\frac{du}{u_1}$$

$\uparrow$

see diagram...



$$d\theta = -\sqrt{M^2-1}\,\frac{du}{u}$$

θ

$$d\theta = +\sqrt{M^2-1}\,\frac{du}{u}$$

Recall for steady flow:

$$\frac{du}{u} = \frac{dM/M}{1+(\Gamma-1)M^2}$$

define $d\omega = \pm d\theta$

$$\boxed{d\omega = \frac{\sqrt{M^2-1}}{1+(\Gamma-1)M^2}\,\frac{dM}{M}}$$

ω = Prandtl-Meyer func.

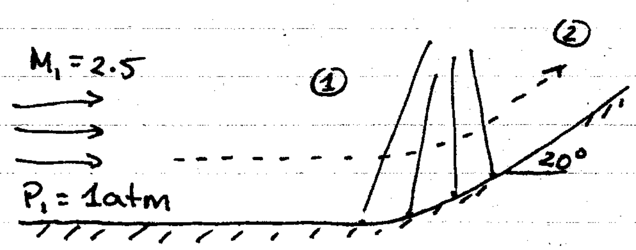
For a perfect gas, this can be integrated:

$$\omega(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\,\tan^{-1}\sqrt{\frac{\gamma+1}{\gamma-1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

Why is this useful? ω is another dimensionless measure of flow speed. (alternative to the Mach #).

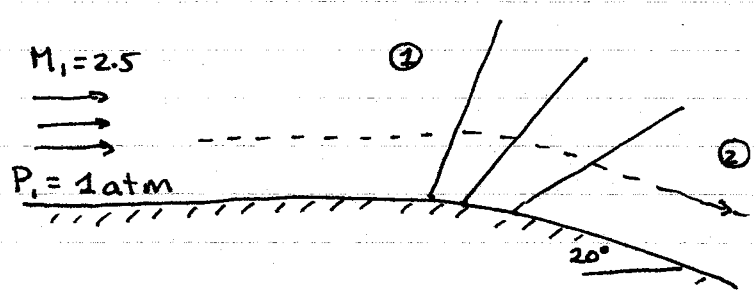
Example

a)



compressive

b)



a) $\Delta\omega = -\Delta\theta = -20°$

$M = 2.5 \Rightarrow \omega = 39.12°$

$\Rightarrow \omega_2 = 19.12° \Rightarrow M_2 = 1.75$

b) $\Delta\omega = \Delta\theta = 20°$

$M = 2.5 \Rightarrow \omega = 39.12°$

$\Rightarrow \omega_2 = 59.12° \Rightarrow M_2 = 3.54$

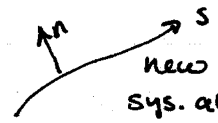
Method of Characteristics

Equations of motion:

$\nabla \cdot (\rho \bar{u}) = 0$ continuity

$\nabla \times \bar{u} = 0$ irrotationality

$\nabla \left(\frac{u^2}{2}\right) + \frac{1}{\rho} \nabla P = 0$ momentum



new coord sys. along streamlines

$\Downarrow$ lots of algebra

$$\frac{d}{dm^+}(\theta + \omega) = \frac{\sin\mu \sin\theta}{r}$$

$r \to \infty \Rightarrow$ plane flow

$$\frac{d}{dm^-}(\theta - \omega) = -\frac{\sin\mu \sin\theta}{r}$$

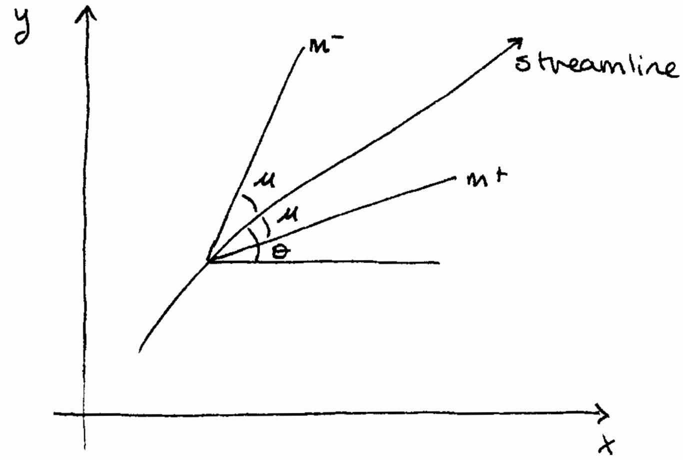
u thermodynamics

along streamline

normal to streamline

where:
$$\frac{d}{dm^+} = \cos\mu \left(\frac{\partial}{\partial s} + \tan\mu \frac{\partial}{\partial n}\right)$$

$$\frac{d}{dm^-} = \cos\mu \left(\frac{\partial}{\partial s} - \tan\mu \frac{\partial}{\partial n}\right)$$

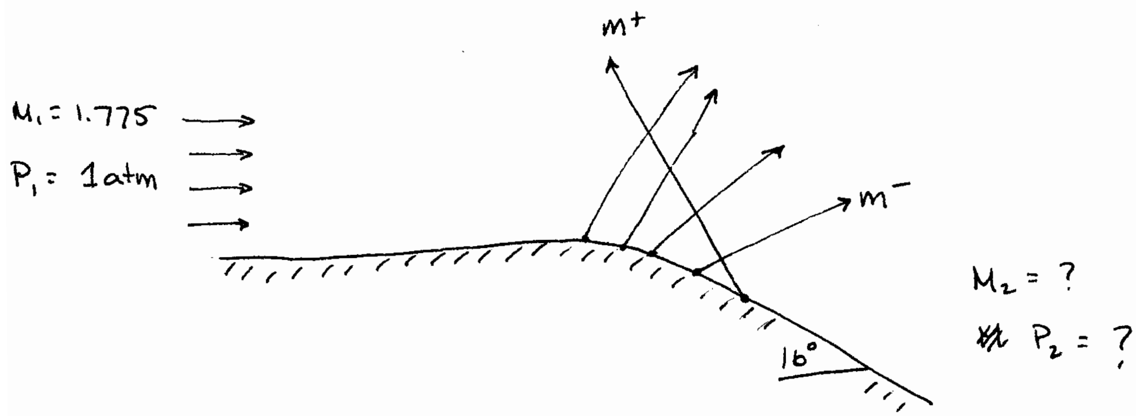


(note axes are y-x **not** t-x. analogous but not identical to 1D case.)

Invariants:

$\theta + \omega = \text{const.}$ along m^+
$\theta - \omega = \text{const.}$ along m^-
} for planar flow

Example:



$\theta + \omega = \omega_1$ everywhere (from m^+)

$\theta - \omega = \text{const} = 2\theta - \omega_1$ on m^-

$\Rightarrow \theta = \text{const.}$ on $m^- \Rightarrow \omega = \text{const}$ on m^-

$\therefore$ characteristics are straight lines

$M_1 = 1.775 \Rightarrow \omega_1 = 20$ (from table)

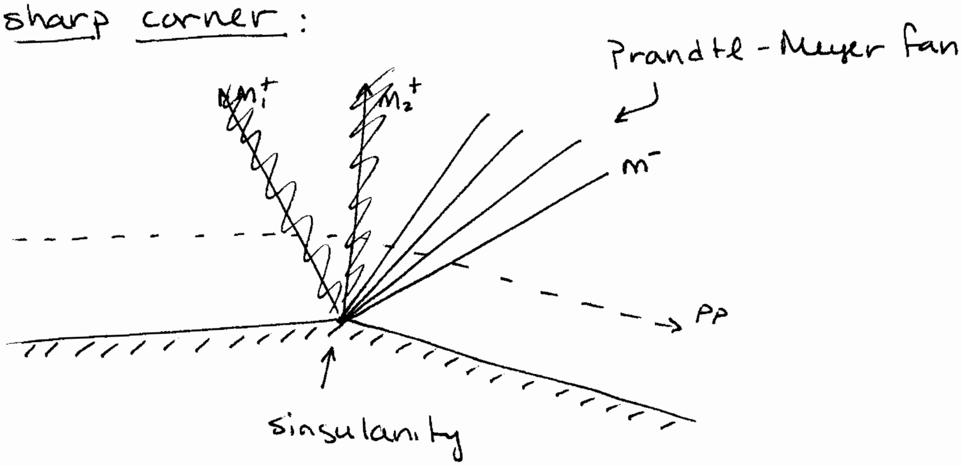
$\omega = \omega_1 - \theta$ on m^-

$= 20 - (-16) = 36 \Rightarrow \boxed{M_2 = 2.369}$ (from table)

$\frac{P_1}{P_0} = 0.18$ (from table) @ $M = 1.775 \Rightarrow P_0 = \frac{1}{0.18}$ atm

@ $M_2 = 2.369$ $\quad \frac{P_2}{P_0} = \frac{0.0717}{0.18} = \boxed{0.397 \text{ atm}}$

For a <u>sharp corner</u>:



$\theta = 0$, $M = 1.775$, $\omega = 20°$; Soln same as above.