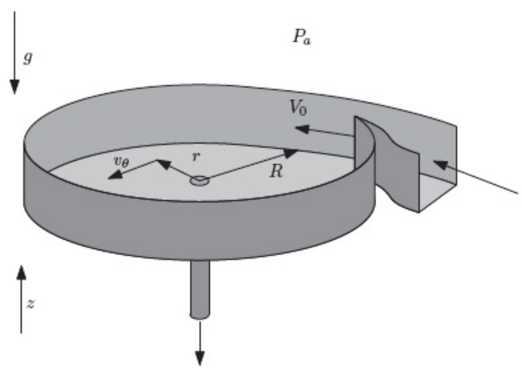
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 10.11

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



The steady sink flow in the sketch is set up by injecting water tangentially through a narrow channel near the periphery and letting it drain through a hole at the center. The vessel has a radius R. At the point of injection, the water has a velocity V and depth h_0; the width of the injection channel, b, is small compared with R. In what follows, we consider the region of the flow not too close to the drain, and assume that everywhere in that region (i) the flow is essentially incompressible and inviscid, (ii) the radial velocity component $|v_r|$ i small compared with the circumferential velocity component v_{theta}, and (iii) the water depth does not differ much from its value h_0 at the periphery.

- (a) Starting with Kelvin's theorem on circulation, show that

$$v_\theta = \frac{VR}{r}. \tag{10.11a}$$

  This equation states that the angular momentum of a fluid particle remains constant in this flow. Is the angular momentum of a particle always constant? Why is it constant in this case.

- (b) Obtain result (a) from Helmoltz's vortex laws.
- (c) Obtain the result of (a) directly from Euler's equation of motion.
- (d) Show that the assumption that $|v_r| \ll v_\theta$ is satisfied if $b \ll R$.
- (e) Derive an expression for the actual distribution of water depth, given the velocity distribution of part (a), and show that the water depth is essentially constant, as we assume, provided that

$$\left(\frac{r}{R}\right)^2 \gg \frac{V^2}{2gh_0}. \tag{10.11b}$$

