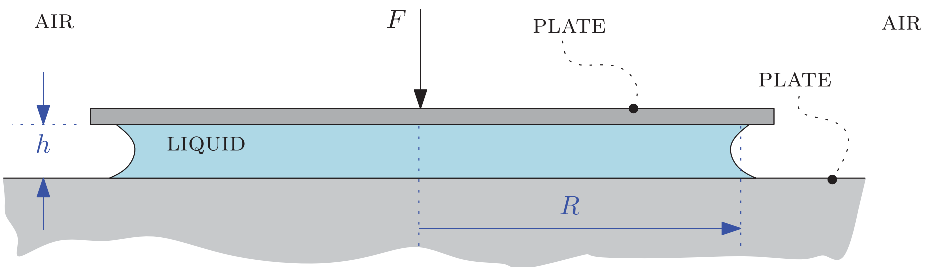
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 2.07

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A drop of liquid of volume V is squeezed between two parallel smooth plates until the liquid thickness h is very small compared with the liquid's radial extent R. The liquid/plate/air contact angle α, and the liquid/air surface tension is σ. Gravitational effects are negligible.

(a) Derive an expression for the downward force F required to hold the plates in position. Express F in terms V, α, σ, and R.

(b) If $\alpha = \pi$ radians (a perfectly nonwetting situation) and $T = 0.07\,\mathrm{N/m}$, say (representing a clean air-water interface), what downward force is required to press a $3\,\mathrm{mm}^3$ drop of liquid into a thin disc or radius $R = 2\,\mathrm{cm}$?

Solution:

Given: liquid volume, V

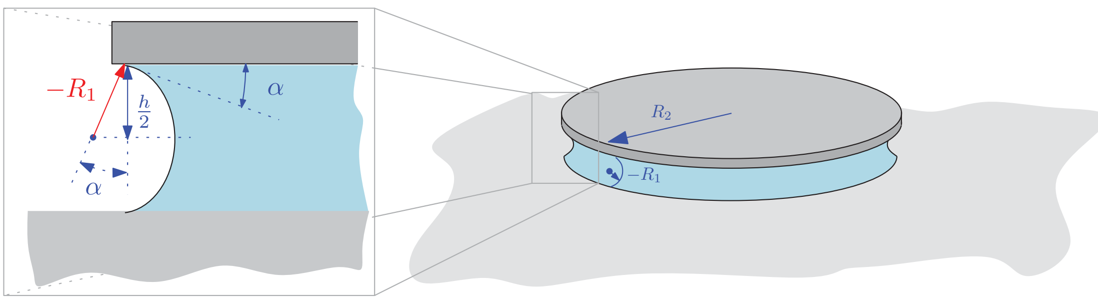
liquid/plate/air contact angle, α

surface tension, σ

Unknown: downward force, F

Find $P_o - P_i$ by considering equilibrium across an interface:

$$P_o - P_i = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



In this specific problem, (Assuming that the Bo number is small and therefore we can neglect gravity effects on the shape of the free surface)

$$\left.\begin{aligned} R_2 &= R \\ -R_1 &= \frac{h}{2\cos\alpha} \qquad (Bo \ll 1)\end{aligned}\right\} \Rightarrow P_o - P_i = \sigma\left(-\frac{2\cos\alpha}{h} + \cancel{\frac{1}{R}}\right) \qquad (\text{since } R \gg h)$$

$$\Rightarrow P_o - P_i = \Delta P = -\frac{2\sigma\cos\alpha}{h}$$

Now apply force balance on upper plate [1]:

$$\begin{aligned}\sum F_y = 0 \quad \Rightarrow \quad F &= -F_{\Delta P} \\ &= \Delta P(\pi R^2) \\ &= -\frac{2\sigma\pi R^2\cos\alpha}{h}\end{aligned}$$

where h represents the gap between the plate and the solid surface. The above equation says: the smaller the gap, the greater the force. This effect is known as *capillary stiction*.

(a) Since $h = V/\pi R^2$,

$$\boxed{F = -\frac{2\sigma\pi^2 R^4\cos\alpha}{V}} \qquad (2.07a)$$

[1] Notice that the direction of the applied force was taken as going downwards

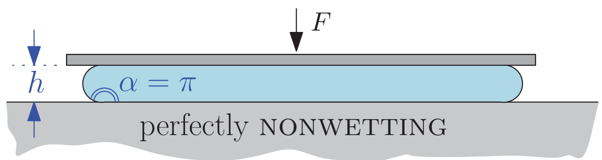
$F < 0$: (UPWARD) when $0 < \alpha < \frac{\pi}{2}$
$\rightarrow$ Liquid is <u>WETTING</u>

$F > 0$: (DOWNWARD) when $\frac{\pi}{2} < \alpha < \pi$
$\rightarrow$ Liquid is <u>NONWETTING</u>

(b) Plug in given values:

$$\alpha = \pi, \quad \sigma = T = 0.07\,\mathrm{N/m}, \quad V = 3\,\mathrm{mm}^3 = 3\times10^{-9}\,\mathrm{m}^3, \quad R = 2\,\mathrm{cm} = 0.02\,\mathrm{m}$$

$$F = -\frac{2(0.07)\pi^2(0.02)^4\cos\pi}{3\times10^{-9}}\left[\mathrm{Nm}^{-1}\right]\left[\mathrm{m}^4\right]\left[\mathrm{m}^{-3}\right]$$
$$= \boxed{74\,\mathrm{N}}$$



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Problem Solution by Sungyon Lee, MC(Updated), Fall 2008