

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g}$$

Cauchy Momentum Equation

INVISCID
 $\boldsymbol{\tau} = -p\mathbf{I}$

VISCOUS → ?

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g}$$

-OR-

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla B = \mathbf{v} \times \boldsymbol{\omega}$$

Euler's Equation

Tools for solving inviscid linear momentum equations

Streamline Coordinates

Generalized Bernoulli

$$\int_1^2 \frac{\partial \mathbf{v}}{\partial t} \cdot d\mathbf{s} + \int_1^2 \frac{dp}{\rho} + \frac{1}{2} (v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

Steady, constant density

Bernoulli

$$(p_2 - p_1) + \rho g(z_2 - z_1) + \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\frac{\partial}{\partial n} (p + \rho g z) = \frac{\rho V^2}{R}$$

$$\frac{\partial}{\partial \ell} (p + \rho g z) = 0$$

Important limits

Rigid body motion (hydrostatics)

Steady Irrotational

$$\nabla p = \rho(\mathbf{g} - \mathbf{a})$$

$$\nabla B = 0$$

$$B = \frac{1}{2} \mathbf{v} \cdot \mathbf{v} + \int \frac{dp}{\rho} + gz$$

Control Volume

FORM A

$$\sum \mathbf{F} = \frac{d}{dt} \int_{CV(t)} \rho \mathbf{v} dV + \int_{CS(t)} \rho \mathbf{v} (\mathbf{v} - \mathbf{v}_c) \cdot \mathbf{n} dA$$

FORM B

$$\sum \mathbf{F} = \int_{CV(t)} \frac{\partial}{\partial t} (\rho \mathbf{v}) dV + \int_{CS(t)} \rho \mathbf{v} (\mathbf{v} \cdot \mathbf{n}) dA$$

where

$$\sum \mathbf{F} = \mathbf{F}_{\text{ext}} + \int_{CV(t)} \rho \mathbf{g} dV + \int_{CS(t)} -p \mathbf{n} dA$$

MIT OpenCourseWare
<http://ocw.mit.edu>

2.25 Advanced Fluid Mechanics
Fall 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.