$$\rho \frac{D\mathbf{v}}{Dt} = \boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \rho \mathbf{g}$$

Cauchy Momentum Equation

INVISCID $\boldsymbol{\tau} = -p\mathbf{I}$

VISCOUS → ?

$$\rho \frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla} p + \rho \mathbf{g}$$

-OR-

$$\frac{\partial \mathbf{v}}{\partial t} + \boldsymbol{\nabla} B = \mathbf{v} \times \boldsymbol{\omega}$$

Euler's Equation

Important limits

Rigid body motion (hydrostatics)

$$\boldsymbol{\nabla} p = \rho(\mathbf{g} - \mathbf{a})$$

Steady Irrotational

$$\boldsymbol{\nabla} B = 0$$

$$B = \frac{1}{2}\mathbf{v} \cdot \mathbf{v} + \int \frac{dp}{\rho} + gz$$

Tools for solving inviscid linear momentum equations

Streamline Coordinates

Generalized Bernoulli

$$\int_1^2 \frac{\partial \mathbf{v}}{\partial t} \cdot d\mathbf{s} + \int_1^2 \frac{dp}{\rho} + \frac{1}{2}\left(v_2^2 - v_1^2\right) + g\left(z_2 - z_1\right) = 0$$

Steady, constant density

Bernoulli

$$(p_2 - p_1) + \rho g(z_2 - z_1) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\frac{\partial}{\partial n}(p + \rho g z) = \frac{\rho V^2}{R}$$

$$\frac{\partial}{\partial \ell}(p + \rho g z) = 0$$

Control Volume

FORM A

$$\sum \mathbf{F} = \frac{d}{dt}\int_{CV(t)} \rho \mathbf{v}\, dV + \int_{CS(t)} \rho \mathbf{v}(\mathbf{v} - \mathbf{v_c}) \cdot \mathbf{n}\, dA$$

FORM B

$$\sum \mathbf{F} = \int_{CV(t)} \frac{\partial}{\partial t}(\rho \mathbf{v})\, dV + \int_{CS(t)} \rho \mathbf{v}(\mathbf{v} \cdot \mathbf{n})\, dA$$

where

$$\sum \mathbf{F} = \mathbf{F_{ext}} + \int_{CV(t)} \rho \mathbf{g}\, dV + \int_{CS(t)} -p\mathbf{n}\, dA$$

Hosoi (2006)