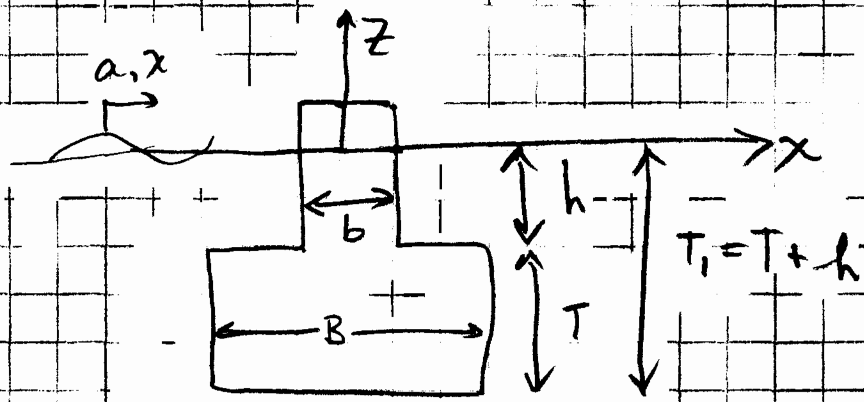
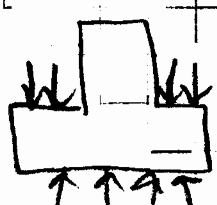
Problem 1



GIVEN : Deepwater
$\lambda >> B$ $\quad b < B$
$a < h, b$

$$F_z(t) = \underbrace{F_z^{FK}(t)}_{\text{Froude Krylov}} + \underbrace{F_z^{D}(t)}_{\text{difraction}}$$

a) FROUDE KRYLOV FORCE



$$F_z^{FK}(t) \cong B \cdot p(x=0, z=-T_1, t) \quad \text{Bottom face}$$
$$-(B-b)\, p(x=0, z=-h, t) \quad \text{Top face}$$

(force/length)

$$p(x,z,t) = \rho g a e^{kz} \cos(kx - \omega t)$$

$$F_z^{FK} = \rho g a \cos\omega t \left\{ B e^{-kT_1} - (B-b) e^{-kh} \right\}$$

②

$$F_z^{FK}(t) = \rho g a e^{-k(h+T/2)} \cos\omega t \left\{ B e^{-kT/2} - (B-b) e^{kT/2} \right\}$$

if $\lambda >> T$ $\quad e^{\pm kT/2} \simeq 1 \pm \frac{kT}{2}$

so $\quad B e^{-kT/2} - (B-b) e^{+kT/2} \simeq B\left(1 - \frac{kT}{2}\right) - (B-b)\left(1 + \frac{kT}{2}\right)$

$$= B\left\{ \cancel{1} - \cancel{1} + b/B - \frac{kT}{2}\left(1 + (1 - b/B)\right)\right\}$$

$$\therefore \quad F_z^{FK}(t) = \rho g a e^{-k(h+T/2)} \cos\omega t \left\{ B\left(\varepsilon - \frac{kT}{2}(2-\varepsilon)\right)\right\}$$

where $\varepsilon \equiv b/B$

(in 3D Force $= p \cdot$ AREA $= P_{bot} \cdot \pi \frac{B^2}{4} - P_{TOP} \frac{\pi}{4}(B^2 - b^2)$)

ⓑ $\quad (m + A_{33})\ddot{x}_3 + B_{33}\dot{x}_3 + C_{33} x_3 = F_3(t)$

$A_{33} \simeq 2\, Ma_{sphere} = 2 \cdot \frac{1}{2} \rho \forall_s \quad \forall_s = \frac{4}{3}\pi r^3$

in 2D $\rightarrow$ $a_{33} \simeq a_{33}|_{circle} = \frac{\pi}{4} B^2 \rho$

b_{33} = from viscous effects

$\hat{c}_{33} = \rho g b$ (per unit length)

$m = \rho B T\left(1 + \frac{bh}{BT}\right)$ mass/unit length

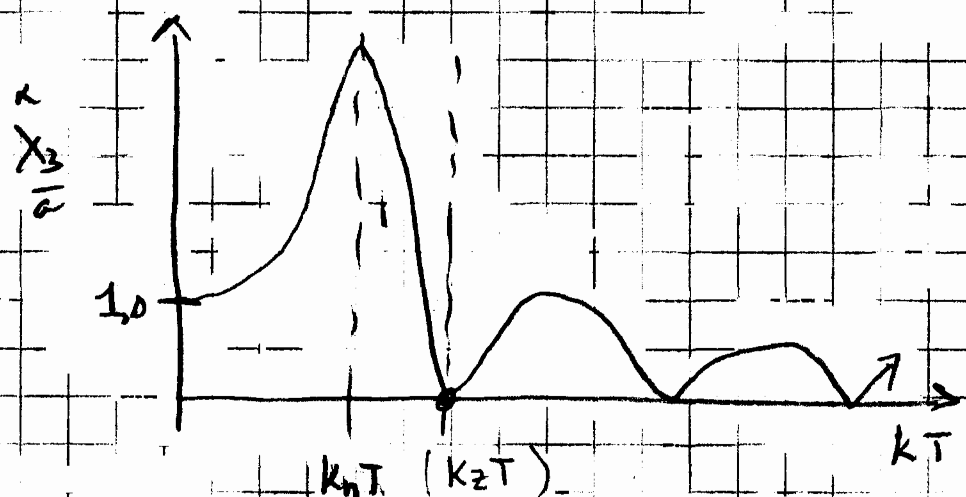
$$-\omega^2\left(\rho B T\left(1 + \frac{bh}{BT}\right) + \frac{\pi}{4} B^2 \rho\right)\hat{X}_3 + i\omega b_{33}\hat{X}_3 + \rho g b \hat{X}_3 = a \hat{F}_3$$

Natural frequency

$$\omega_{n_3} = \sqrt{\frac{\rho g b}{\rho B T + \rho b h + \rho \frac{\pi}{4} B^2}}$$

$$\frac{\hat{X}_3}{a} = \frac{\hat{F}_3}{-\omega^2 \rho B T \left(1 + \frac{bh}{BT} + \frac{\pi}{4}\frac{B}{T}\right) + \rho g b + i\omega b_{33}}$$

$$\frac{\hat{X}_3}{a} = \frac{\rho g a e^{-k(h+T/2)} B \left\{ \epsilon - \frac{kT}{2}(2-\epsilon) \right\}}{-\rho\omega^2 B T \left(1 + \frac{bh}{BT} + \frac{\pi}{4}\frac{B}{T}\right) + \rho g b + i\omega b_{33}}$$



↑ minimum force ⇒ wavenumber when force is zero (k_{zero})

$$\omega_n^2 = k_n g \Rightarrow k_n = \frac{\omega_n^2}{g}$$

Diffraction force →

incident wave velocity

$$F_z^D(t) \simeq m_a \dot{w}(x=0, z=-h_1, t)$$

$$h_1 = h + \frac{T}{2}$$

column of width b has negligible effect

bottom section moves most of the water

$$m_a \simeq \alpha \rho T B \qquad \alpha = f\left(\frac{B}{T}\right)$$

$$\dot{w}(x,z,t) = -a\omega^2 e^{kz} \cos(kx - \omega t)$$

$$F_z^D(t) \simeq -\alpha B T \rho \omega^2 a e^{-kh_1} \cos \omega t$$

$$F_z^{TOT}(t) = F^{FK} + F^D \simeq$$

$$\rho g a e^{\underbrace{-k(h+T/2)}_{=h_1}} \cos \omega t \left\{ B\left(\varepsilon - \frac{kT}{2}(2-\varepsilon)\right)\right\}$$

$$-\alpha \rho a B T \omega^2 e^{-kh_1} \cos \omega t$$

$$g = \frac{\omega^2}{k}$$

$$F_z^{TOT} \simeq \rho a \frac{\omega^2}{k} e^{-kh_1} B \left\{ \varepsilon - kT\left(1 - \frac{\varepsilon}{2}\right) + kT\alpha \right\} \cos \omega t$$

F_z^{TOT} → zero when

$$\left\{ \varepsilon - kT\left(1 - \frac{\varepsilon}{2}\right) + kT\alpha \right\} = 0$$

$$\frac{b}{B} = kT\left(1 - \frac{b}{2B}\right) - kT\alpha$$

$$\frac{b}{B}\left(1 + \frac{kT}{2}\right) = kT(1-\alpha) \Rightarrow \boxed{b = \frac{kTB(1-\alpha)}{1+kT}} \text{ for } F_z^T = \phi$$

13.42 HW#8 SOL'NS Problem #2

1a SURGE EXCITATION FORCE:

$$dF_1 = \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \left.\frac{\partial u}{\partial t}\right|_{x=0} dz$$

$$\frac{\partial u}{\partial t}(x=0, z, t) = \omega^2 A e^{kz} \sin \omega t$$

$$F_1(t) = \int_{-T}^{0} \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A e^{kz} \sin \omega t \, dz$$

$$= \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \left[\frac{1}{k}\left(1 - e^{-kT}\right)\right]$$

PITCH EXCITATION MOMENT:

$$F_5(t) = \int_{-T}^{0} (-z) \, dF_1$$

$$= \left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \int_{-T}^{0} (-z) e^{kz} dz$$

$$= -\left(\rho \frac{\pi d^2}{4} + A_{11}\right) \omega^2 A \sin \omega t \left[\frac{1}{k} z e^{kz} - \frac{1}{k^2} e^{kz}\right]_{-T}^{0}$$

b $A_{33} = \frac{2}{3} \rho \frac{\pi d^3}{8}$

$$A_{11} = \int_{-T}^{0} a_{11} \, dz = \rho \frac{\pi d^2}{4} T$$

$$A_{55} = \int_{-T}^{0} a_{11} z^2 \, dz = \rho \frac{\pi d^2}{4} \frac{T^3}{3}$$

$$A_{15} = A_{51} = \int_{-T}^{0} a_{11} z \, dz = \rho \frac{\pi d^2}{4} \frac{T^2}{2}$$

$$A_{31} = A_{13} = A_{35} = A_{53} = 0.$$

$$C_{11} = C_{13} = C_{31} = C_{15} = C_{51} = C_{35} = C_{53} = 0.$$

$$C_{33} = \rho g A_{wp} = \rho g \frac{\pi d^2}{4}$$

$$C_{55} = \rho g \forall (z_B - z_G) + \rho g \iint_{A_{wp}} x^2 \, ds$$

$$= \rho g \forall \left[-\tfrac{T}{2} - \left(-\tfrac{3T}{4}\right) \right] + \rho g \frac{\pi d^4}{64}$$

c. $$\omega_{n_3}^2 = \frac{C_{33}}{m + A_{33}} = \frac{\rho g \frac{\pi d^2}{4}}{m + \frac{2}{3}\rho \frac{\pi d^3}{8}}.$$

d. $$\det \left[-\omega^2 \begin{pmatrix} M_{11} + A_{11} & M_{15} + A_{15} \\ M_{51} + A_{51} & M_{55} + A_{55} \end{pmatrix} + \begin{pmatrix} C_{11} & C_{15} \\ C_{51} & C_{55} \end{pmatrix} \right] = 0$$

LET $A = M_{11} + A_{11}$

$B = M_{15} + A_{15}$

$C = M_{51} + A_{51}$

$D = M_{55} + A_{55}$

$a = C_{11}$

$b = C_{15}$

$c = C_{51}$

$d = C_{55}$

$$\det \begin{vmatrix} -\omega^2 A + a & -\omega^2 B + b \\ -\omega^2 C + c & -\omega^2 D + d \end{vmatrix} = 0$$

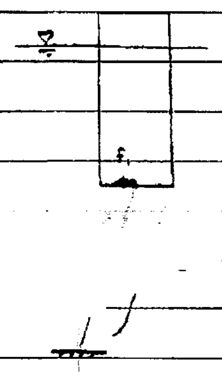
$$\omega_{n_{1,5}}^2 = -\frac{(Bc + Cb - Ad - Da)}{2(AD - BC)} \pm$$

$$\pm \frac{\sqrt{(Bc + Cb - Ad - Da)^2 - 4(AD - BC)(ad - bc)}}{2(AD - BC)}$$

e. $$\sum_{j=1,3,5}\left[(M_{ij}+A_{ij})\ddot{\xi}_j + B_{ij}\dot{\xi}_j + C_{ij}\xi_j\right] = F_i(t) \qquad (i=1,3,5)$$

f. A_{ij} SAME AS IN PART b.

$$C = \begin{bmatrix} 0 & k_{33}+\rho g\frac{\pi a^2}{4} & 0 \\ k_{51} & 0 & k_{55}+\rho g\forall z_B - mgz_G + \rho g\iint_{A_{WP}} x^2\,ds \end{bmatrix}$$

g. 

RESTORING FORCE DUE TO CABLE

$k_{11} L\theta$

$$f_1 = P\sin\theta \sim P\theta$$

$$\therefore\ k_{11} = \frac{P}{L}$$

$$\omega_{n_1}^2 = \frac{k_{11}}{m+A_{11}} = \frac{P}{L\left(m+\rho\frac{\pi d^2}{4}T\right)}$$