

## 13.42 Homework #6

Spring 2005

Out: Thursday, March 17, 2005

Due: Thursday, March 31, 2005

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### Problem 1: Ocean Waves:

- Discuss the evolution of water waves generated when wind blows over the ocean surface from developing seas to decaying seas. What are the main physical mechanisms for wave production by wind? Think in terms of the boundary conditions at the air-sea interface and basic fluid mechanics.
- Sketch a typical spectrum for developing, fully-developed, and decaying ocean waves on one plot. Discuss briefly why the spectra differ in shape.
- Explain what significant wave height is and why it is so widely used to describe ocean waves and sea states.
- Why is the Bretschneider Spectrum so widely used? What are its limitations?

### Problem 2:

A platform is to be installed in a location where the maximum storm for which operations are allowed is described by a Bretschneider spectrum with significant wave height  $\zeta = 5.75$  m and peak frequency  $\omega_m = 0.68$  rad/s.

The elevation of water near the platform, denoted by  $\eta_p(t)$  was found to be affected by the diffracted waves. Note that the transfer function,  $H(\omega)$ , between the elevation  $\eta_p(t)$  and the undisturbed incoming waves at a different location,  $\eta(t)$ , was found to have absolute value:

$$|H(\omega)| = \frac{(0.4 - \omega^3)^2 + 0.25}{(0.4 - \omega^3)^2 + 0.15}$$

*Determine the height  $h$  at which the deck must be installed above the average water level in order to have less than two times per hour green water on deck.*

Problem 3: Laboratory Experiments:

- a) If you have three wave probes describe an experimental setup and give the relevant equations you need to calculate phase speed,  $V_p$ , group velocity,  $V_g$ , wave amplitude,  $\eta$ , and wavelength of a monochromatic wave?
- b) Given the tank water depth of 1.5 meters, at determine the range of wave frequencies in order to ensure deep water waves?
- c) You are asked to determine the RAO (response amplitude operator. i.e. a plot of heave amplitude compared to wave amplitude as a function of frequency) for a model ship in waves. If you can measure wave elevation and heave motion of the ship and can generate both single-frequency and multi-frequency wave packets or wave trains describe how you would go about getting data to make this plot.

Problem 4: Analyze the Bretschneider spectrum, a continuous spectrum, using Matlab or another computational tool. Using the Bretschneider spectrum given in class:

$$S^+(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} \zeta^2 e^{\left\{-1.25\left(\frac{\omega_m}{\omega}\right)^4\right\}}$$

Where the significant wave height is  $\zeta$  and  $\omega_m$  is the modal frequency of the spectrum.

1. Plot the Bretschneider spectrum for four significant wave heights between 1 and 3 meters for *fully developed seas*. (see attached table at end and also posted online which may be easier to read)
2. Consider one significant wave height chosen in part a.
  - a. Compare the spectrum in part 1 with the JONSWAP spectrum for the same significant wave height.
  - b. Compare the B-S spectrum for developing, fully-developed, decaying sea states. Note that the spectrum shape can be altered by changing the modal frequency.
3. Using one case from part a:
  - a. Calculate the moments of the spectrum using numerical integration (like trapezoidal integration or similar). Note what happens with the 4<sup>th</sup> moment – does the integral converge? Compare these values with the approximate values given in class.
  - b. Determine the spectrum bandwidth,  $\epsilon$ . (Use the approximate value for  $M_4$ )
  - c. Calculate the 1/Nth highest wave height where  $N = 10, 50, \text{ and } 100$ .

Problem 5: A ship is being designed to operate in the conditions you have analyzed above. The transfer function between the incoming waves and heave motion (heave transfer function)  $H(\omega)$  is given by

$$H(\omega) = \frac{\omega_n^2}{-\omega^2 + 2i\beta\omega_n\omega + \omega_n^2}$$

where  $\omega_n$  is the structure's natural frequency (rad/s) and  $\beta$  is the structural damping ratio.

- a) Plot  $|H(\omega)|^2 = H(\omega) \cdot H^*(\omega)$
- b) Determine the spectrum of the heave response. Note that  $|H(\omega)|^2 = H(\omega) \cdot H^*(\omega)$ . Plot this spectrum. Use a structural damping ratio  $\beta = 0.28$  and a  $\omega_n = 1.1$  rad/s.
- c) How will the response change if the structural frequency is different – illustrate your explanation with plots of the response at a higher or lower natural frequency.

Problem 6: Use the relationship between frequency and encounter frequency to explain how the response of a ship in heave would change from the stationary ship to a ship moving forward in head seas (head seas are those moving towards the ship bow).

Table removed for copyright reasons.

Source: "Wave and Sea Scale for Fully Arisen Sea." Table 2.3, pp. 112-113 in,  
Berteaux, Henri O. *Coastal and oceanic buoy engineering*. Woods Hole, MA: The Author, 1991.