2.20 Problem Set 11B **Name: _____**

1. Consider an approximate velocity profile within a laminar boundary layer over a wall given by:

$$\frac{u(y;x)}{U(x)} = \begin{cases} a(x)\eta + b(x)\eta^2 & y < \delta(x) \\ 1 & y \geq \delta(x) \end{cases} \qquad (1)$$

where $\eta \equiv y/\delta(x)$, and $a(x)$ and $b(x)$ are constants at any given x, and $\delta(x)$ is a measure of the boundary layer thickness at x.

(a) By applying *one* boundary condition at $y = \delta$ which matches the boundary layer flow with the outer flow $U(x)$, obtain an algebraic relationship between a and b.

(b) Now write both $a(x)$ and $b(x)$ in terms of a new parameter $\Lambda(x)$ such that the relationship between a and b found in (a) is maintained. In other words, find $a(\Lambda)$ and $b(\Lambda)$.

(c) Write the velocity profile (1) in terms of Λ instead of a and b.

(d) Using the assumed velocity profile and the x-momentum boundary layer equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

evaluated at $y = 0$, obtain an expression for Λ in terms of U, δ, and ν. Also obtain an expression for the pressure gradient dp/dx in terms of ρ, U, Λ, δ, and ν.

(*Hint*: Polhausen approach, but a *different profile* than the Lecture 17 notes - be careful)

(e) For what value of Λ is dp/dx zero? The pressure gradient is favorable when Λ is [greater than, less than] the value _____ .

(f) For the velocity profile corresponding to $dp/dx = dU/dx = 0$, compute the (i) displacement thickness δ^*, (ii) momentum thickness θ, and (iii) wall shear stress τ_0 at any location x in terms of $\delta(x)$. Let $U(x) = U_0 = constant$ in this case.

Assume $U = U_0$ *from this point on.* This corresponds to uniform flow over a flat plate.

(g) Derive a differential equation for $\delta(x)$ using the von Karman momentum integral equation.

(h) Solve the differential equation in (g) for $\delta(x)$ with the boundary condition $\delta(0) = 0$. How do $\delta^*(x)/x$ and θ/x depend on the local Reynolds number Re_x? How does this compare with the Blasius flat plate laminar boundary layer result?

2. Supplementary Problem Va22.

3. Supplementary Problem Va 24.