

## Final Exam

Problem 1: This problem considers a system for which the impulse response  $h(k)$  is equal to 1 for  $N = 0, 1, \dots, N - 1$ , and equal to zero elsewhere. That is,

$$h(k) = \sum_{i=0}^{N-1} \delta(k - i) \quad (1)$$

The system has an input  $u$  and an output  $y$ .

- a) For this system, solve for the transfer function  $H(z)$ . Clearly show your reasoning.
- b) Find a difference equation in terms of  $u$  and  $y$  which has this impulse response  $h(k)$ . Explain why this type of system is often referred to as an  $N$ -point averager. (It may also be called a box-car filter, because of the shape of the impulse response.)
- c) Show that the frequency response of this system is given by

$$H(e^{j\Omega}) = \frac{\sin(N\Omega/2)}{\sin(\Omega/2)} e^{-j\Omega(N-1)/2} \quad (2)$$

Sketch a Bode plot for this system for the value  $N = 4$ . Use a linear frequency and magnitude axis, and sketch from  $\Omega = 0$  to  $\Omega = \pi$ . Be sure to include both magnitude and phase, and to label the axes with numerical values.

- d) For  $N = 4$ , at what frequencies is the frequency response equal to zero? Explain why this occurs from a time-domain perspective for an input  $\cos \Omega k$ . Support your argument with relevant sketches.

Problem 2: This problem considers a continuous-time system with a transfer function

$$H(s) = \frac{a^2}{s^2 - a^2}. \quad (3)$$

This might model the linearized behavior of a magnetic suspension. Note that both poles are real-axis, with one in the right-half plane.

- a) Calculate the zero order hold equivalent  $H_{eq}(z)$  for this system with a sampling time  $T$ .

- b) Show that for  $a = 40$ ,  $T = 0.001$  sec,  $H_{eq}(z)$  becomes

$$H_{eq}(z) = \frac{8 \times 10^{-4}(z + 1)}{z^2 - 2.0016z + 1}. \quad (4)$$

Please use this transfer function for all subsequent work even if your answer does not agree with it. Clearly state if this is the case, and then use the answer we are providing above. Plot the poles and zeros of this system on the z-plane. Do the poles map as  $z = e^{sT}$ ?

- c) Sketch a Bode plot for  $H_{eq}(z)$ . Use care in handling the pole outside the unit circle. Be sure to label and scale the axes. Hint: The magnitude will be the same as for a system with coincident poles on the real axis at the location of the single pole inside the unit circle. The phase will however be different. You can also use a few test frequencies to guide you in sketching. Does the DC gain agree with that of the continuous time system? What is the high frequency gain ( $z = -1$ )?
- d) Sketch a root locus for this system imbedded in a unity-negative-feedback loop with a proportional controller  $G(z) = G_0$ . Will any value of  $G_0$  stabilize the system?

- e) Develop a Nyquist stability analysis for this system under the proportional control above, and describe how it confirms the root locus analysis in d). It will be easiest to choose a z-plane contour which does not intersect the real-axis pole outside the unit circle in order to avoid constructing detours. Note that it *is* OK for the contour to intersect with a zero, since the loop gain magnitude is finite there.

Clearly show both your chosen z-plane contour and the resulting GH-plane image. Label corresponding points on the contours as A, B, C, etc. Also clearly indicate how you handle the presence of the unstable loop-transmission pole.

- f) Design a lead compensator for this system of the form

$$G(z) = \frac{K(z - \alpha)}{z} \quad (5)$$

in order to stabilize the loop with a phase margin greater than 45 degrees at as high a crossover frequency as possible. Clearly indicate your design approach. What are the resulting values of phase margin  $\phi_m$ , and crossover frequency  $\Omega_c$ ? For what range of gains  $K$  will your design be stable?

- g) Sketch a root locus for your design above assuming that  $K$  varies from 0 to infinity. Estimate the point at which the locus last crosses the unit circle, and any other features of the locus which you find pertinent. Do not explicitly calculate breakaway and reentry points