Bond Graph notation for physical system models

One of our first concerns in developing a modelling formalism is notation. To allow a concise representation of our models, we will use pictorial diagrams, similar to the electrical network diagram used above. Several well-established sets of pictorial symbols already exist for depicting electrical systems, mechanical systems, and so on, but none of these notations are adequate to be applied to all energetic systems. To emphasize the generality of energetic considerations we will use a notation introduced by Paynter in 1959 and developed by him and his students in the sixties — bond graphs.

The net power flow between two interacting systems results in an interdependence between the energetic states of the two systems: it bonds the two systems together into one. Consequently, the basic symbol of the bond graph notation is a line called a *bond* (somewhat reminiscent of the way chemical bonds are represented). It depicts the exchange of power between the two systems or subsystems or elements at each end of the bond.

Figure 3.6: A *bond* denoting energetic interaction between two systems.

Other elements of the bond graph notation are depicted by letters and/or numbers placed at the ends of the bond. We will introduce these symbols as we encounter the corresponding modelling elements.

The power flow between the two systems will usually be represented as a product of two real variables, an effort and a flow. As needed, the corresponding symbols are written adjacent to the bond as shown below. For clarity and efficiency, we omit the box or outline representing the boundary of each system (which is purely conceptual, anyway) and represent the interaction as follows.

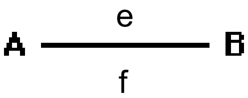

Figure 3.7: A basic bond graph.

Bond Graphs and Block Diagrams

The most important feature of the bond graph notation is that a bond explicitly represents power flow or energy transport and distinguishes it from signal flow, the transfer of information. Generally, the behavior of an element or system will be described mathematically as an operation on an input variable to produce a corresponding output variable. The operator may be dynamic, acting on one time function to produce another, or it may be static (algebraic), simply mapping one number onto another. Those mathematical operations may be represented as the flow of signals (e.g. the input and output) and the transformation of one into the other.

To represent signal flow, a familiar and versatile graphical notation is available: block diagrams. Indeed, we have used block diagrams freely up to this point and will continue to use them. However, before proceeding further we must recognize that block diagrams do not provide a suitable notation for depicting physical system models because *not all block diagrams represent physical processes.*

One of the most important consequences of energetic coupling between elements or systems is that *energy exchange implies interaction;* a bilateral, two-way influence of each system on the other. In contrast, block diagrams fundamentally depict a unilateral influence of one system on another. If we wish to describe energetic interaction of two systems or elements in terms of signal flow, then the output of one must be the input to the other *and vice versa.*

Usually the input to an element will be one of the power dual variables, effort or flow (though, to reiterate, that is not essential). Consequently, because the bond represents power flow which is determined by both of the power dual variables, the output must be the dual or conjugate of the input; if effort is input, flow must be output and vice versa. Then, when two systems interact energetically, we *must* have the situation represented by the block diagram shown in figure 3.8 (or its converse, obtained by switching A and B).

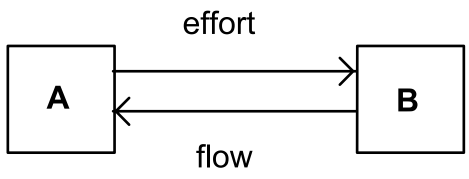

Figure 3.8: Block diagram of energetic interaction.

In contrast, the block diagrams shown in figure 3.9 might represent possible operations on signals or information, but neither represents any possible energetic interaction between two physical systems.

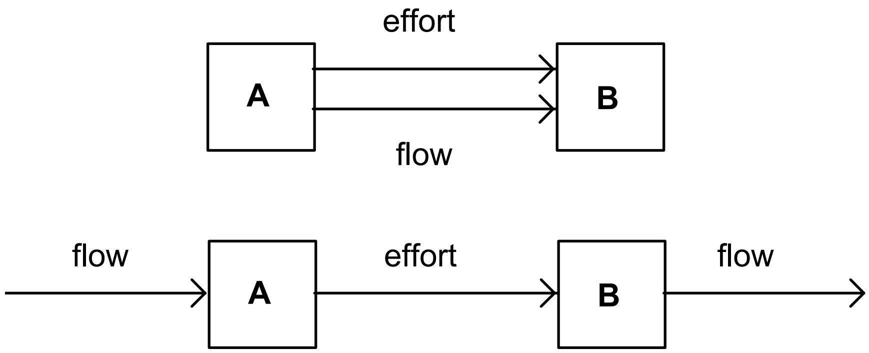

Figure 3.9: Two block diagrams with no physical counterparts.

Causality

Because energetic interaction is a function of two variables, when we come to describe a system in terms of mathematical operations on numbers (i.e. signals), there are two possible choices for the input and output of each element (or subsystem). In making these choices we are assigning one variable to the role of cause (or input) and the other to the role of effect (or output), so this choice is referred to as *causality assignment.* To represent this choice on a bond graph we add a *causal stroke* at one end of the bond[1] as shown in figure 3.10.

Figure 3.10: Representation of causal assignment.

This graphical symbol means that the system nearest the causal stroke has effort impressed on it as input and produces flow as output. Of necessity, the system at the other end of the bond has flow imposed on it as input and produces effort as output. In terms of signal flow, the bond graph of figure 3.10 is equivalent to the block diagram of figure 3.8.

We refer to the two ways of describing an element's behavior (e.g. effort in, flow out vs. flow in, effort out) as different *causal forms.* Note that the two alternative causal forms may, in general, require quite different mathematical operations. As we will see later, the causal form we use, i.e. which variable we select as input and which we select as output, can make a lot of difference. For example, the required mathematical operations may be well-defined in one causal form, but not defined at all in the other.

Sign Convention

One of the important details of formulating any model is establishing a sign convention. While this may appear to be a trivial detail, and there are no associated conceptual problems, in practice sign conventions require considerable attention. We know that when we depress the accelerator pedal, the speed of a car should change, and it usually does. But that information is insufficient for almost all purposes, and especially if we wish to design a control system. It is important to know the *sign* of the change: whether the car speeds up or slows down. In complex systems, determining the sign of the effect of a given cause can require careful attention. The toil can be minimized by working with a consistent sign convention for all energy domains, a sign convention based on power flow. In a bond graph, the direction of positive power flow is indicated by a half-arrow at the appropriate end of the bond.

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[1] If you need a mnemonic to help you to remember the convention, try this: with a little imagination the bond with the causal stroke looks like a nail. Obviously, you would push — exert effort — with the flat end of the nail.

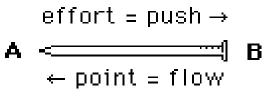

Figure 3.11: Representation of power sign convention

This symbol means that the direction of positive power flow is out of system A and into system B.

Active vs. Passive Systems

The key to our modelling formalism is to keep track of the flow of energy. We will find it useful to distinguish between *active* and *passive* systems or elements. The precise mathematical definition of passivity is quite subtle, but the basic idea is that a passive system (or subsystem) is one which cannot supply an infinite amount of energy to its environment. In contrast, an active system may supply energy to its environment indefinitely. Of course, you should realize from this definition that an active system is an idealization, a fiction; but, as we will see, it is a very useful one.

The sign convention we will use is that power is positive into a passive element. Much needless confusion can be avoided by adhering to this convention. Referring to figure 3.11, by this convention, A cannot represent a passive system, whereas B may.

The sign convention we will use for active elements is more flexible. Usually, power will be positive out of an active element, as that makes the most physical sense; the power into the passive elements has to come from somewhere, usually out of an active element. However, on occasion we may wish to model an active element which is a sink, not a source, an element which removes energy from the system independent of the system's internal state. In those cases, we may represent power as positive into an active element.

Augmented Bond Graphs

Because the basic bond graph (figure 3.7) depicts energetic interaction independent of choice of sign convention or assignment of causality, a graph with half-arrows (for power sign convention) and/or strokes (for causal assignment) is sometimes referred to as an *augmented bond graph.* Note that the choices — power sign convention and causal assignment — are quite independent of each other.