ENTROPY PRODUCTION AND NONLINEARITY.

Is entropy production an exclusively nonlinear phenomenon? Must it always vanish in a linearized model?

Consider simple heat transfer modeled by Fourier's law:

$$\dot{Q} = (kA/l)(T_1 - T_2)$$

where $\dot{Q}$ is heat flow rate, k is thermal conductivity, A is a surface area, l is length and T_1 and T_2 are absolute temperatures.

Entropy flow and heat flow are related by temperature.

$$\dot{Q} = T_1\dot{S}_1 = T_2\dot{S}_2$$

$$\dot{S}_1 = (kA/l)\,\frac{(T_1 - T_2)}{T_1}$$

$$\dot{S}_2 = (kA/l)\,\frac{(T_1 - T_2)}{T_2}$$

Net entropy production is:

$$\dot{S}_{net} = \dot{S}_{out} - \dot{S}_{in} = \dot{S}_2 - \dot{S}_1$$

$$= (kA/l)(T_1 - T_2)\left(\frac{1}{T_2} - \frac{1}{T_1}\right) = (kA/l)\left(\frac{(T_1 - T_2)^2}{T_1 T_2}\right)$$

Hence, as dictated by the second law, net entropy production is never negative.

LINEARIZE

Now linearize the two entropy flow equations. Subscript o denotes operating point.

$$\Delta \dot{S}_1 = \left.\frac{(kA/l)T_2}{T_1^2}\right|_o \Delta T_1 - \left.\frac{(kA/l)}{T_1}\right|_o \Delta T_2$$

$$\Delta \dot{S}_2 = \left.\frac{(kA/l)}{T_2}\right|_o \Delta T_1 - \left.\frac{(kA/l)T_1}{T_2^2}\right|_o \Delta T_2$$

Net <u>linearized</u> entropy production is:

$$\Delta \dot{S}_{net} = \Delta \dot{S}_2 - \Delta \dot{S}_1$$

$$= (kA/l)\left.\left(\frac{1}{T_2} - \frac{T_2}{T_1^2}\right)\right|_o \Delta T_1 + (kA/l)\left.\left(\frac{1}{T_1} - \frac{T_1}{T_2^2}\right)\right|_o \Delta T_2$$

Net <u>linearized</u> entropy production may be non-zero.

<u>Provided</u> the operating point is not at thermal equilibrium.

However, if the operating point is at thermal equilibrium,

$$T_{1,o} = T_{2,o}$$

$$\left.\left(\frac{1}{T_2} - \frac{T_2}{T_1{}^2}\right)\right|_o = 0$$

$$\left.\left(\frac{1}{T_1} - \frac{T_1}{T_2{}^2}\right)\right|_o = 0$$

Net entropy *production* vanishes.

The entropy *flow* need not vanish, but the input flow must equal the output flow.

CAUTION!

Another difficulty of the linearized model:

Used sufficiently far from the operating point, the linearized equations may describe a *negative* net entropy production.

(remember ΔT_1 or ΔT_2 may be positive or negative)

That would violate the second law.

The exception when the operating point is at thermal equilibrium, in which case the model describes no entropy production.

Clearly, linearized models should be interpreted with caution.